

# Spectral scaling transformations of nonstationary light

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**Abstract.** We present optical systems, which transform isodiffracting nonstationary beams into fields obeying either cross-spectral purity or spectral invariance. The designs are hybrid refractive-diffractive imaging systems, which are able to perform the desired transformations over a broad spectral bandwidth and irrespective of the state of spatial coherence of the input beam.

## 1 Introduction

Cross-spectral purity [1] and spectral invariance [2] are two cornerstone concepts of optical coherence theory, which enlighten the role of correlation-induced spectral changes on propagation and interference of light. Cross-spectral purity, introduced by Mandel in 1961, amounts to interference of two light fields, where the normalized spectrum of the superposition is the same as that of the input fields. Spectral invariance, on the other hand, was proposed by Wolf in 1986, and it refers to light fields for which the normalized far-field spectrum is independent of propagation direction.

While these concepts were originally introduced for stationary light, theoretical framework in the context of nonstationary fields was recently established for both cross-spectral purity [3] and spectral invariance [4]. An especially important class of nonstationary light is formed by the isodiffracting pulsed beams [5], which are encountered, e.g., as superpositions of Hermite-Gaussian modes in spherical-mirror resonators. These modes all have the same, frequency-independent Rayleigh range. Consequently, the transversal beam width satisfies the characteristic spectral scaling of isodiffracting beams.

In this work, we present optical systems which are capable of transforming isodiffracting beams into nonstationary fields obeying either cross-spectral purity or spectral invariance [6]. The systems are chromatically compensating hybrid lens designs consisting of one refractive and one diffractive lens, and they perform the transformations accurately over a broad spectral range.

## 2 Transformation systems

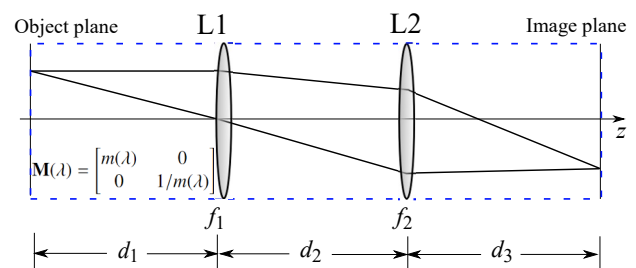
The transformation systems are based on the theory of paraxial, afocal imaging lenses. In particular, the transformations are achieved perfectly, within the paraxial approx-

imation, when the spectral magnifications of the systems are [6]

$$m(\lambda) = \sqrt{\frac{\lambda_0}{\lambda}} m_0, \quad (1) \quad m(\lambda) = \sqrt{\frac{\lambda}{\lambda_0}} m_0, \quad (2)$$

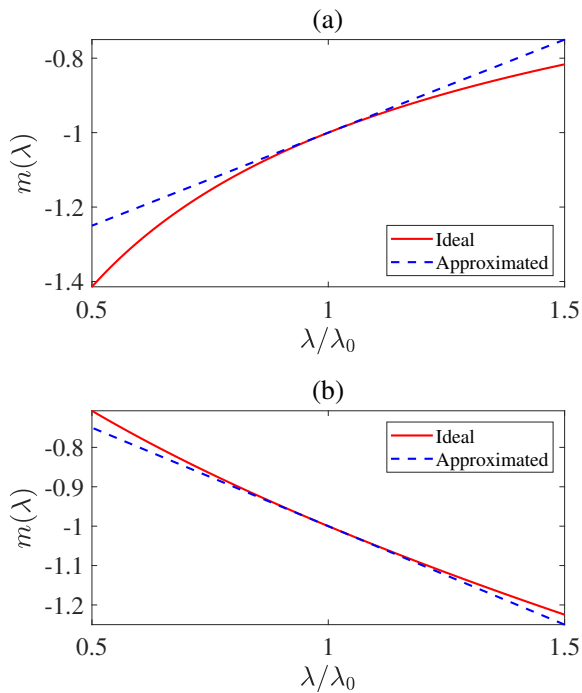
for cross-spectrally pure and spectrally invariant output beams, respectively. In these expressions,  $\lambda$  is the wavelength of light,  $\lambda_0$  is chosen as the design wavelength of the system, and  $m_0 = m(\lambda_0)$ .

We employ hybrid lens systems, which consist of one achromatic refractive lens and one diffractive lens. An illustrative sketch of the thin-lens arrangement is presented in Fig. 1. Both transformation systems are similar designs, where the order of the two lenses and their separations specify the nature of the transformation. For cross-spectrally pure output beam, the refractive lens is placed before the diffractive lens, and if the order of the lenses is flipped, a spectrally invariant secondary source is achieved at the image plane of the system in Fig. 1. The systems perform the desired transforms approximately, with the ideal magnifications accomplished perfectly at  $\lambda_0$ . Figure 2 illustrates the accuracy of these approximations.



**Figure 1.** The transformation system, which is described within the blue dashed lines by the ray-transfer matrix  $\mathbf{M}$ . The lenses L1 and L2 are refractive or diffractive thin lenses with respective focal lengths  $f_1$  and  $f_2$ . The lenses are separated by distances  $d_1$ ,  $d_2$ , and  $d_3$ , which, together with the order of the lenses, specify the nature of the transformation. Adapted from [6].

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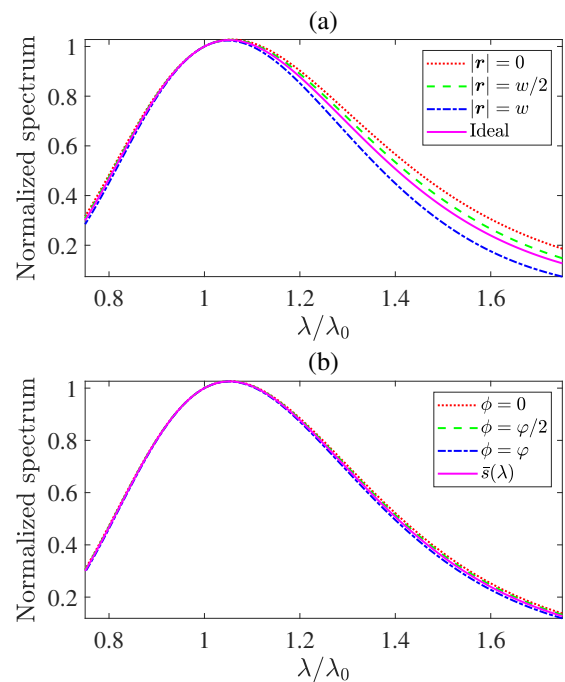


**Figure 2.** The magnifications for (a) cross-spectrally pure, and (b) spectrally invariant output fields. The solid lines show the ideal magnifications in Eqs. (1) and (2), together with the respective approximations (dashed line) accomplished with the thin-element designs in Fig. 1. Adapted from [6].

### 3 Results

Figure 3 shows the numerically simulated performance of the presented thin-lens designs, where the input beam corresponds to a pulsed isodiffracting source with a broad spectral bandwidth. Figure 3 (a) illustrates the accuracy of the system designed for generating a cross-spectrally pure beam. The figure shows the normalized spectrum at three transversal positions  $|\mathbf{r}|$  along the beam width on the image plane of the system. In addition, the ideal normalized superposition spectrum is shown. We notice that the isodiffracting beam is transformed into a nearly cross-spectrally pure field, apart from long wavelengths.

Figure 3 (b) presents the performance of the other system, which was designed to create a spectrally invariant secondary source on its image plane. The figure represents the far-field spectrum of the source for three values of the direction angle  $\phi$ , which are relative to the  $1/e^2$  beam divergence angle  $\varphi$ . In addition, the figure shows the normalized source-averaged spectral density, which is equal to the ideal, directionally invariant far-field spectrum obtained with the exact magnification in Eq. (2). We see that the results are even closer to the ideal situation over a wide range of wavelengths and the curves shown in Fig. 3 (b) are nearly indistinguishable.



**Figure 3.** Numerically simulated performance of the transformation systems for pursuing a (a) cross-spectrally pure (b) spectrally invariant output beam. Adapted from [6].

### 4 Conclusions

We presented refractive-diffractive imaging systems, which transform nonstationary isodiffracting beams into cross-spectrally pure or spectrally invariant fields accurately over a broad spectral range. Finally, we note that the systems produce inverse transformations if the setup in Fig. 1 is reversed, i.e., the order of the object and image planes is flipped. An incoming cross-spectrally pure or spectrally invariant field is then transformed into an isodiffracting field, and this holds for both stationary or non-stationary beams.

### References

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