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Finite-size scaling behaviour in fully-connected equal-coupling multimode photonic networks

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The recently established connection between nonlinear multimode photonic systems and models of interacting classical spins [1], allows to now analyse dynamical equilibrium behaviour and collective phenomena in photonic systems using methods developed in statistical mechanics [2]. This unexpected synergy is highly valuable for studying effects that require a system-wide cooperation of all its degrees of freedom (DOF). In this regard we consider mean-field equal-coupling photonic networks [1], in which the elementary DOF are given by the complex-valued modes of the system. These modes are subject to a Kerr-type self-interaction and interact pairwise via uniform couplings. They can be viewed as photonic soft-spins: two-component spin vectors for which the mode-amplitude determines the length of the spin, see Figs. 1(a,b). Such a geometric picture exploits analogies to models of interacting classical spins, such as the XY model [2]. Previous studies of mean-field photonic networks unveiled [1], that, depending on the energy per mode, a second-order phase transition can be observed: if the average energy per mode is large, the system is in a disordered state in which the photonic spins are active and where there is no orientational order among the spins [Fig. 1(b)]; if the average energy per mode decreases, the system undergoes a continuous transition towards an ordered phase in which alignment forces induce global order [Fig. 1(a)].

Here, we perform long-time temporal evolution of the nonlinear equations of motion of the coupled modes using a stepsize controlled Runge-Kutta method of high order [3]. We address the issue of obtaining initial conditions for specified optical power and energy, for which we devised an effective optimization heuristic [4]. Based on extensive numerical simulations, we perform a comprehensive finite-size scaling analysis to estimate the critical points and critical exponents of the phase transition driven by the energy per mode. For this purpose we build upon the analogy to XY-type models by defining two-component and single-component order parameters, similar to those studied in statistical mechanics [2]. This allows to describe collective phenomena on equal-coupling photonic networks in terms of statistical summary measures that take the familiar form of the magnetization [Fig. 1(c)] and finite-size susceptibility [Fig. 1(d)] of statistical mechanics models. Particular attention is paid to the scaling behavior right at the critical point [Fig. 1(e)], at which we account for corrections to scaling by means of simple scaling laws. We take care to sample the statistical properties of the model in dynamical equilibrium and we compare our findings to exact results whenever possible.

In case of photonic networks, systems of interest naturally exhibit a finite number of DOF, hence finite-size effects as well as corrections to scaling can become an important feature whenever collective phenomena are addressed or systems with specific properties are to be designed. In this regard, the reported results are of fundamental interest in nonlinear optics.

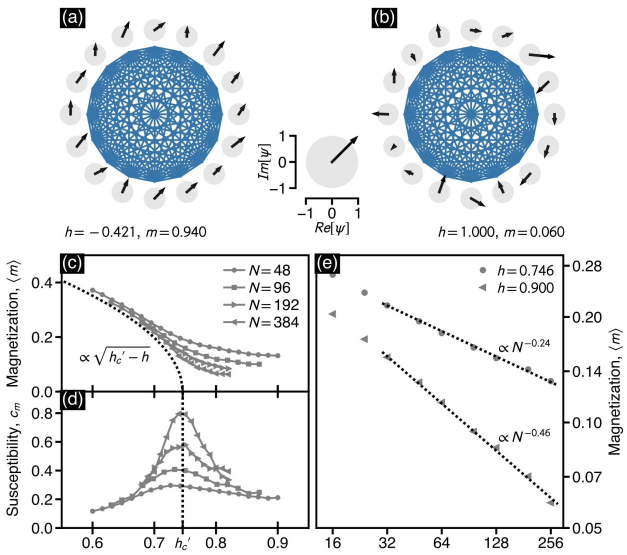

Fig. 1 Fully-connected equal-coupling photonic networks. Soft-spin configurations in (a) the ordered phase, (b) the disordered phase. (c) Scaling behaviour of the time-averaged magnetization, and, (d) scaling behaviour of the finite-size susceptibility as function of the energy per mode. (e) Finite-size scaling at fixed values of the energy per mode.

References

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