## Four wave mixing in multimode hollow core waveguides with a two-color pump for the thorium nuclear clock

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Generation of high-energy pulses at wavelengths < 200 nm is a daunting task. One of the most important applications is a nuclear clock: the low-energy resonance of Th-229 is at around 8 eV, corresponding to  $\sim 150$  nm wavelength [1]. The clock, based on transitions in nuclei is less sensitive to perturbations and therefore more precise than atomic clocks. On the other hand, high isolation from the environment makes it difficult to excite, since high-intensity fields are required. Current methods to approach frequencies in this range involve typically nonlinear processes in gases, including high ionization-based harmonic generation (HHG) [2], resonant dispersive wave (RDW) from collapsing optical pulses [3] or four wave mixing (FWM) [4-8]. HHG and RDW are inherently bounded to short pulses. FWM in gases can potentially work both with short and long pulses but up to now also showed low efficiency and minor tunability. The primary FWM approach is cascaded FWM [4-8] using collinear phase-matching (PM) [4-6] or quasi-PM [7] in waveguides, or noncollinear PM in crossing filaments [8]. Using higher order modes (HOM) of waveguides was also reported for 3rd harmonic generation [9].

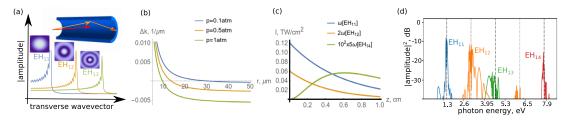


Fig. 1 (a) Higher-order modes in hollow-core waveguide for noncollinear PM. (b) PM in argon-filled capillary for different pressures and HOM given in (c). (c) Long-pulse FWM (efficiency  $\approx 1\%$ ). (d) Short pulse FWM calculated using UPPE, efficiency  $\approx 3\%$ .

Here we propose to use two- color pump pulses and HOM of hollow-core waveguides. Using HOM is equivalent to noncollinear FWM (Fig. 1a), but allows increased interaction distance in comparison to crossing beams. In our approach, the pump is located at the fundamental (FH) frequency  $\omega$  and its second harmonic (SH), giving rise to the FWM process  $2\omega + 2\omega + \omega = 5\omega$ . Every frequency is located at different transverse harmonic (see Fig. 1). Tuning SH  $\omega \to \omega + \delta$  allows also to tune the signal wavelength. PM can be achieved by tuning the diameter of the capillary and the gas pressure (see Fig. 1c).

In the simulations of narrowband nanosecond Fourier-limited pulses we used the equations for the harmonic amplitudes similar to [7]. For short pulses, we simulated the multimode extension [10] of the so called unidirectional propagation equation (UPPE). Our results (Fig. 1) show that, despite of strong losses for higher order modes and the pulse reshaping (for short pulses), it is possible to achieve few per cent conversion efficiency using practically achievable pump pulse configuration.

As a conclusion, we demonstrated theoretically efficient generation of tunable VUV pulses via FWM involving higher order modes and two-color pulses. This method is especially attractive to generate VUV light for thorium clocks.

## References

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