

Energy from nuclear fission

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Summary. — The physics of nuclear fission will be briefly illustrated, from the basic mechanism behind this phenomenon to the relevant physical quantities like nuclear cross sections, neutron flux and reaction products, together with the accompanying phenomenon of neutron capture and its role in determining how the fuel transforms in a nuclear reactor. The basic concepts underlying the operation of different types of nuclear reactors will be illustrated, along with the concept of fuel cycle. The aspects of radioactive waste, fuel resources and safety will also be briefly illustrated.

1. – Physics of fission

What is fission and why can it produce energy? The basic reason is the behaviour of the nuclear mass and nuclear binding energy for different nuclear species. As is known from relativity, the mass of a bound system is given by the sum of the masses of its components, plus the binding energy of the system. The binding energy is by definition a negative quantity, *i.e.* the mass of the composite system is always less than the sum of the masses of its components. When applied to nuclear masses, this concept can be expressed through the equation

$$(1) \quad M(Z, A) = ZM_p + (A - Z)M_n + B(Z, A),$$

where $M(Z, A)$ is the mass of a nucleus with Z protons and A protons and neutrons (*i.e.* $A - Z$ neutrons), M_p is the proton mass, M_n is the neutron mass and $B(Z, A)$ is the

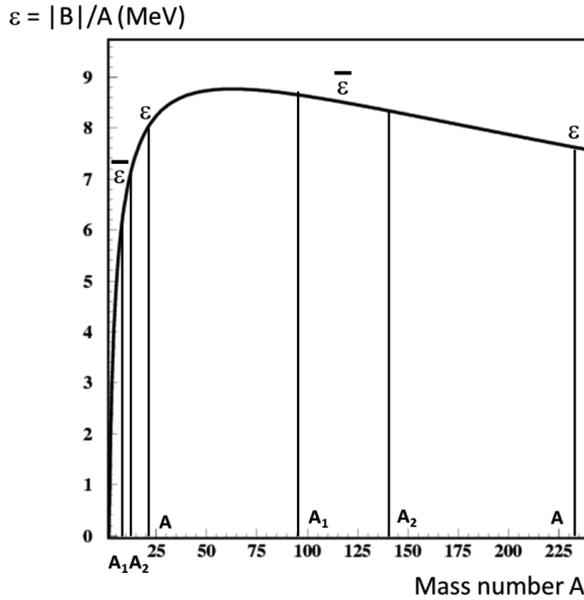
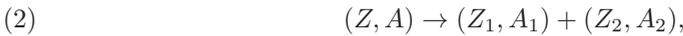


Fig. 1. – Absolute value of binding energy per nucleon ϵ , as a function of the mass number A . The figure also illustrates the case of the nuclei of mass number A_1 and A_2 undergoing fusion and merging into a nucleus of mass number A , as well as the case of a nucleus of mass number A undergoing fission and splitting into two fragments of mass number A_1 and A_2 . $\bar{\epsilon}$ is the mass-number-weighted absolute binding energy per nucleon in both the fusion and fission cases (see sect. 1.1).

binding energy of the system, with $B(Z, A) < 0$, *i.e.* $M(Z, A) < ZM_p + (A - Z)M_n$. Z is called the atomic number, while A is called the mass number. The absolute value of the binding energy represents the energy that must be given to the system from outside to overcome the binding nuclear force and set all nucleons free. We can also define the binding energy per nucleon, *i.e.* the average quantity of energy that must be supplied per single nucleon when separating all nucleons from one another. Figure 1 shows the absolute value of the binding energy per nucleon, $\epsilon = \frac{|B|}{A}$. It is seen that such value increases from the lightest nucleus, the deuteron, to the region of iron, then it decreases towards the heavier nuclei. In other words, iron is more bound than the light nuclei, while, *e.g.*, uranium is less bound than iron.

1.1. *Energy balance.* – Based on the above considerations, it is easy to see that a nuclear reaction where nuclei move from smaller to bigger binding energy will release energy. Indeed, in going from the initial to the final state in the reaction, the total mass will decrease, so that the mass defect will be transformed in energy. Looking at fig. 1, it is clear that two light nuclei undergoing nuclear fusion, *i.e.* a nuclear system moving from the left-hand side to the right-hand side of the graph, will move towards higher binding, thereby releasing nuclear energy, *i.e.* giving rise to an *exoenergetic* nuclear reaction, a

reaction which yields a net amount of energy. This is the mechanism at the basis of nuclear fusion [1], *e.g.* in the reaction ${}^2\text{H} + {}^3\text{H} \rightarrow {}^4\text{He} + n$, where about 17.5 MeV are released in the process. On the other hand, heavy nuclei above iron can undergo fission, where the nuclear system splits into two nuclei, moving from the right-hand side to the left-hand side of the graph in fig. 1, again moving towards higher binding. This is called the process of nuclear fission and can also be exoenergetic. Let us consider in detail the energy balance of a fission process, where a nucleus with Z protons and $(A - Z)$ neutrons splits into two fragments



where obviously $Z = Z_1 + Z_2$ and $A = A_1 + A_2$ and the masses of the nuclei are $M(Z, A)$, $M(Z_1, A_1)$ and $M(Z_2, A_2)$, respectively. The energy released in the reaction is called the Q -value and it is given by definition by

$$(3) \quad Q = M(Z, A) - M(Z_1, A_1) - M(Z_2, A_2).$$

By using eq. (1) we obtain (remember that ϵ is the *absolute* binding energy per nucleon, *i.e.* $\epsilon > 0$)

$$(4) \quad \begin{aligned} Q &= B(Z, A) - B(Z_1, A_1) - B(Z_2, A_2) \\ &= -\epsilon A + \epsilon_1 A_1 + \epsilon_2 A_2 = -\epsilon A + \bar{\epsilon} A = (\bar{\epsilon} - \epsilon)A, \end{aligned}$$

where

$$(5) \quad \bar{\epsilon} = \frac{\epsilon_1 A_1 + \epsilon_2 A_2}{A_1 + A_2}.$$

Figure 1 tells us that if the initial nucleus is a heavy one (as indicated at the right of the picture), its fission into two medium-mass nuclei is such that $Q > 0$. By repeating a similar calculation for two light nuclei undergoing fusion, it is easy to see that also in that case one can get $Q > 0$.

The fact that fission of heavy nuclei is possible in terms of energy balance does not mean that fission will easily happen spontaneously. To understand that, imagine the two daughter nuclei (called *fission fragments*) being bound together in a potential well (the basketball in fig. 2). In order to be separated from each other and transform into the smaller basketball and the tennis ball on the right of fig. 2, they will have to pass through a Coulomb repulsion barrier. This would be impossible in classical physics but it is possible in quantum mechanics, thanks to the quantum tunneling effect, which leads to the so-called *spontaneous fission*. The lifetime for the nucleus to decay via spontaneous fission will vary depending on the probability for the two nuclei to pass through the barrier. If energy is supplied from the outside to the initial heavy nucleus, the system will jump to a more excited quantum state, *i.e.* a higher energy state of the two fragments in the potential well, accompanied by a deformation of the nucleus schematically illustrated by

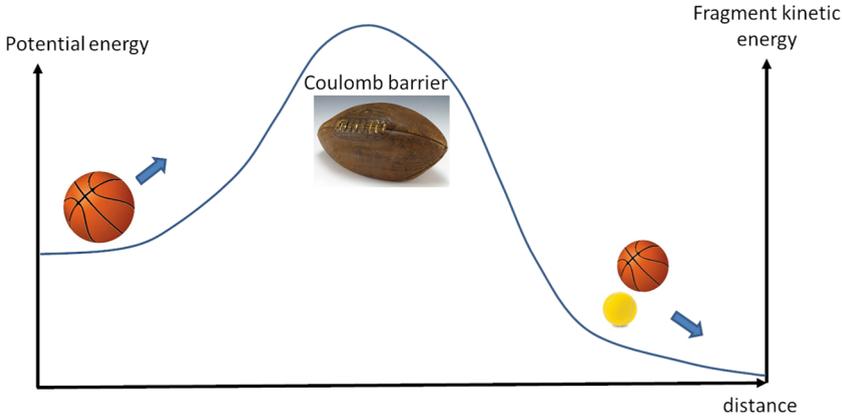


Fig. 2. – Nuclear potential energy as a function of the distance between the two fragments.

the rugby ball in fig. 2. Therefore the probability for the two fragments to penetrate the barrier will increase. This is what we call *induced fission*, which can actually happen with significant probability for nuclei with $A \gtrsim 230$.

How can we supply energy to the nucleus from the outside? For example, suppose that an incoming neutron is captured in the potential well of a heavy nucleus (Z, A) . Due to the generally significant binding energy (a few MeV) in the newly formed $(Z, A + 1)$ nucleus, a few MeV of energy will be released and made available for the nucleus to approach the top of the barrier, thereby increasing by orders of magnitude the probability of the $(Z, A + 1)$ nucleus to undergo fission. However, given the specific dependence of the binding energy on Z and A , typically for odd- A nuclei the binding energy of the extra neutron captured is higher than for even- A nuclei. The former nuclei are called *fissile* and they can undergo fission with high probability even when capturing a very slow neutron (even a neutron in thermal equilibrium with the surrounding medium). The latter nuclei are called *fissionable* and they can undergo fission with significant probability only when capturing a neutron with kinetic energy of the order of 1 MeV or more. Two famous examples of the above are the ${}^{235}_{92}\text{U}$ and ${}^{238}_{92}\text{U}$ Uranium isotopes. Indeed, uranium is found in nature as two isotopes, ${}^{238}_{92}\text{U}$ and ${}^{235}_{92}\text{U}$, whose abundances are 0.7% and 99.3%, respectively. While ${}^{238}\text{U}$ is fissionable, ${}^{235}\text{U}$ is fissile. Both uranium isotopes are radioactive and undergo α decay, with emission of an α particle, *i.e.* a helium nucleus. Radioactive decays follow an exponential law, which means that, given an initial number of radioactive nuclei N_0 , their number at a time t is given by

$$(6) \quad N(t) = N_0 e^{-t/\tau},$$

where τ is the mean decay time. Often used is the *half-life*, $t_{1/2} = \tau \ln(2)$, which is the time it takes for the number of radioactive nuclei to drop by 50%. ${}^{238}\text{U}$ has a half-life of 4.47 billion years, about the age of the Earth, while ${}^{235}\text{U}$ has a half-life of about

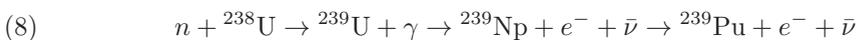
704 million years. This means that when the Earth was formed, ^{238}U was twice more abundant than today, while ^{235}U was about 84 times more abundant.

Fission is not the only exoenergetic nuclear reaction induced by neutrons. Just to make two examples, the reaction $n + {}^3\text{He} \rightarrow {}^3\text{H} + p$, when initiated by a very slow neutron (whose kinetic energy we can neglect), can release about 0.76 MeV in form of the kinetic energy of the two final nuclei. The reaction $n + {}^6\text{Li} \rightarrow {}^3\text{H} + {}^4\text{He}$ can release about 4.78 MeV, again taken by the motion of the final nuclei. However, fission has a distinctive feature that makes it different from all others. Indeed, the fission process is accompanied by the emission of a few *prompt neutrons*, *i.e.* in addition to the splitting of the initial nucleus into two fragments, a few neutrons are emitted by the fragments in a very short time after the splitting. Such neutrons are fast, *i.e.* they have high speed and high kinetic energy (about 2 MeV on average) and, as we will see shortly, if properly slowed down, can be very effective in inducing another fission. Due to the additional fission processes that can be initiated by these other neutrons, fission, under certain conditions, can give rise to a *chain reaction*, where neutrons emitted by one fission can produce additional fissions, and so on and so forth, resulting in a very significant release of energy by the fissile material.

Another important process is the *radiative neutron capture*, where the energy released during capture of a neutron excites the resulting nucleus to a quantum state that then decays by emission of electromagnetic energy in form of energetic photons (gamma rays). Neutron capture often leads to the formation of radioactive nuclei. An example is the formation of a radioactive Cobalt isotope, ^{60}Co by exposure of steel (which contains Cobalt) to neutrons, via the process



We see that radiative capture is accompanied by “prompt” gamma emission, *i.e.* the high-energy photons are emitted immediately upon capture, but in the above case the resulting nucleus ^{60}Co is radioactive and undergoes β decay, with emission of an electron and an antineutrino. ^{60}Co has a half-life of about 5.3 years, however β decay half-lives in different nuclei can vary by several orders of magnitude, depending on the involved quantum nuclear transitions. The radiative capture process can also occur on uranium nuclei and it can be a competitor to fission. The case of radiative capture on ^{238}U is a special one, as it leads to the formation of Plutonium, ^{239}Pu , a fissile element not existing in nature, via the chain of reactions



Let us discuss the energy balance of the capture process in more detail, by considering the process $n + A \rightarrow A + 1$. In particular, we want to calculate the energy excess ϵ created by the capture process.

We can write the energy conservation as

$$(9) \quad m_n + T_n + M(Z, A) = M(Z, A + 1) + T_{A+1} + \epsilon,$$

with m_n , $M(Z, A)$, $M(Z, A + 1)$ being the neutron, initial nucleus and final nucleus masses, respectively, T_n the neutron's kinetic energy, T_{A+1} the recoil nucleus' kinetic energy and ϵ the excess energy. Therefore we obtain

$$(10) \quad m_n + T_n + M_A = M_{A+1} + \frac{p_{A+1}^2}{2M_{A+1}} + \epsilon = M_{A+1} + \frac{p_n^2}{2m_n} \frac{m_n}{M_{A+1}} + \epsilon,$$

where we used momentum conservation, $|\vec{p}_n| = |\vec{p}_{A+1}|$, assuming the initial nucleus to be at rest. Approximating $M_{A+1} \sim (A + 1)m_n$ in the ratio m_n/M_{A+1} (by neglecting the proton-neutron mass difference and the binding energy, which amounts to a small correction to the final result), we get

$$(11) \quad T_n + m_n + M_A - M_{A+1} = T_n \frac{1}{A + 1} + \epsilon,$$

i.e.

$$(12) \quad T_n \left(1 - \frac{1}{A + 1} \right) - B_n = \epsilon,$$

where $B_n = M_{A+1} - m_n - M_A < 0$ is the captured neutron's binding energy in the nucleus $A + 1$. Finally, we can write

$$(13) \quad \epsilon = T_n \frac{A}{A + 1} + |B_n|.$$

We can see that even in the case of capture of slow neutrons ($T_n \rightarrow 0$) there is an energy excess $\epsilon \simeq |B_n|$. This energy excess necessarily goes into internal nuclear degrees of freedom, which means that the compound nucleus $A + 1$ is excited and must eventually release the excess energy in form of γ rays, nuclear particles, or the fission process.

Let's now calculate the actual value of $|B_n|$ in the two cases of interest: capture by ^{235}U and by ^{238}U . By applying the definition of binding energy and by using tabulated values of the Uranium isotope masses, we find that in the case



about 6.5 MeV are released by the capture process, while in the case



about 4.8 MeV are released. This difference is due to the fact that the captured neutron is more bound in the ^{236}U nucleus, with an even number of protons (92) and neutrons (144), than in the ^{239}U nucleus, with the same even number of protons (92) but an odd number of neutrons (147). Indeed, in a semiempirical description of nuclear masses, even-even nuclei have one more negative term in the expression for the mass as a function of Z

and A , which namely means that the binding is stronger. The opposite happens for odd-odd nuclei, where that term is positive, which means less binding. For even-odd or odd-even nuclei, such term is absent, so that the captured neutron is more bound (*i.e.*, releases more energy upon capture) in ^{236}U (even-even) than in ^{239}U (even-odd). Such additional term in the mass is related to the so-called *pairing* part of the nuclear force, a term essentially telling us that a nucleus is more bound when both protons and neutrons can be coupled in pairs.

We discussed above that we can see a heavy nucleus as being formed by two fragments that have to cross a barrier to get separated from one another in the fission process. Therefore, from a quantum mechanics point of view, if we excite the nucleus to a state which is energetically closer to the barrier, its fission probability will increase. At variance with that, the decay of the excited nucleus via gamma radiation does not depend on any barrier and so it is not so much dependent on the excitation energy obtained via neutron capture. As a result, radiative neutron capture can happen in both ^{235}U and ^{238}U no matter what the initial neutron energy is. On the contrary, while in ^{235}U the energy excess from capture of a slow neutron (which we saw above to be just the absolute value of the binding energy of that neutron in the final nucleus) is enough to cross the barrier and make fission happen, in the ^{238}U case the capture of a slow neutron does not supply enough energy and the neutron has to possess a kinetic energy of the order of 1 MeV for the fragments to cross the barrier and make fission happen.

1.2. *Energy released and reaction products.* – By calculating the specific mass balance given by eq. (3), it is easy to see that when a uranium nucleus fissions into two nuclear fragments, about 0.1% of uranium mass appears as a fission energy of ~ 200 MeV. Such an energy release is bigger than any other exoenergetic nuclear reaction by more than an order of magnitude (see for instance the above example of neutron capture on ^6Li) and bigger than chemical reactions by several orders of magnitude. Just for comparison, in the chemical reaction where a methane molecule is burnt, $\text{CH}_4 + 2\text{O}_2 \rightarrow \text{CO}_2 + 2\text{H}_2\text{O}$, about 9.2 eV of energy are released, more than seven orders of magnitude less than nuclear fission. In terms of specific energy, *i.e.* the amount of energy that can be obtained from a certain mass of material, measured in MJ/kg (MegaJoule per kilogram), Uranium produces about 8×10^7 MJ/kg, while for example methane produces about 56 MJ/kg. In terms of energy density, *i.e.* the amount of energy that can be obtained from a certain volume of material, measured in MJ/L (MegaJoule per liter), uranium produces about 1.6×10^9 MJ/L, while for example methane (at 15 °C and atmospheric pressure) produces 0.0378 MJ/L.

As anticipated above, the chain reaction is possible because along with the fission fragments an average of 2.5 prompt neutrons are emitted, with a mean kinetic energy per neutron of about 2 MeV. The average number of neutrons emitted is called ν and in general is of the order of 2-3, depending on the nucleus undergoing fission and on the incident neutron energy. Out of the 200 MeV released per fission, about 180 MeV appear as kinetic energy of the emerging two nuclei, plus kinetic energy of the prompt neutrons, plus energy in form of prompt gamma rays (photons) emitted when nuclear fragments are

produced in a nuclear excited state. Due to the longer time scale of β decay of the fission fragments, additional energy is released later after fission and is carried by the electron, the associated antineutrino and possibly additional gammas. Antineutrinos escape from the reactor due to their negligible interaction with matter and therefore do not contribute to the process thermal budget, while electrons and gammas do, contributing with about 13 MeV per fission. When radioactive fragment build-up reaches a steady state, these 13 MeV (6.5% of fission energy) contribute to the instant reactor thermal power. At the exact moment that the reactor is shutdown by stopping the chain reaction, the thermal power due to these radioactive decays, called *residual heat*, remains and decays slowly with time (see later).

1.3. *Cross sections and flux.* – The cross section is a physical quantity and an observable that characterizes a nuclear reaction (elastic, inelastic scattering, etc.). The cross section is connected to the range and strength of the involved forces and represents the effective area of a nuclear target in a certain type of reaction or collision. Here we will only consider the so-called total cross section, defined as follows. Given a flux of particles incident on a single nucleus (target) per unit area dS per unit time

$$(16) \quad \frac{dN_{in}}{dSdt}$$

and given a reaction rate, which represents the number of interacting particles (scattered or absorbed projectiles) per unit time,

$$(17) \quad R = \frac{dN_{reac}}{dt}$$

then the total reaction cross section σ is

$$(18) \quad \sigma = \frac{\frac{dN_{reac}}{dt}}{\frac{dN_{in}}{dSdt}}.$$

As is evident from the above equation, σ has the physical dimensions of a surface and can be interpreted as an effective target area.

Given a macroscopic target comprising several nuclei with mass density ρ and small thickness x , struck by a particle beam of intensity I (particles/sec), we have

$$(19) \quad R = I \frac{\rho x}{A} N_A \sigma,$$

where A is the target atomic weight and N_A is the Avogadro number. For a target of arbitrary thickness, by dividing it in thin slices of thickness dx , one obtains that

$$(20) \quad I(x) = I(0) \exp\left(-\frac{\rho}{A} N_A \sigma x\right).$$

In nuclear physics, the cross section unit is the *barn*, corresponding to 10^{-24} cm².

The quantity

$$(21) \quad \Sigma = \frac{\rho}{A} N_A \sigma$$

is called the *macroscopic cross section* and represents the probability of interaction per unit length of material crossed (and therefore has dimensions of the inverse of a length). It is easy to show that $1/\Sigma$ is the mean free path of an incident particle in the material and Σv is the frequency at which a projectile with speed v interacts with the crossed material, *i.e.* it represents the number of interactions of the projectile with the target nuclei per unit time.

Since the nuclear radius is roughly 10^{-12} cm, the geometrical cross sectional area of the nucleus is roughly 10^{-24} cm² = 1 barn. However, the combination of quantum mechanical and specific dynamical effects from the nuclear forces can make nuclear cross sections extremely variable over a few orders of magnitude. As an example, in fig. 3 are reported the cross sections for the processes of fission and radiative capture for the two uranium isotopes found in nature and used in nuclear reactors, ²³⁸U and ²³⁵U, as a function of the neutron energy [2]. It is clearly seen that in ²³⁸U, only fast neutrons with kinetic energy above 1 MeV can cause fission, with a cross section of the order of the barn, while for lower energies only radiative capture is significant. On the contrary, in ²³⁵U, both fission and radiative capture cross sections are relevant at all neutron energies, with the fission cross section being much bigger than the one of radiative capture. Radiative capture for both nuclei and fission for ²³⁵U follow a $1/v$ law, which makes fission in ²³⁵U much more likely to occur at low neutron energies or, in other words, for “slow” neutrons. This is the main characteristic of fissile nuclei: the fission process can occur at all neutron energies and the corresponding cross section falls down with a $1/v$ law. As we saw in sect. 1.1, such a difference in the fission process is due to the nuclear pairing interaction, which makes the capture neutron more bound in ²³⁶U than in ²³⁹U, thereby making ²³⁶U crossing the barrier and undergoing fission even when the captured neutron possesses a negligible kinetic energy, while ²³⁹U needs additional energy besides that obtained via the capture process, which is supplied by a fast neutron possessing at least 1 MeV kinetic energy.

1.4. *Neutron density and flux.* – We define as neutron density the expected number of neutrons with energy between E and $E + dE$, in the volume $d^3\vec{r}$ at the space position \vec{r} , at a time t in a nuclear assembly. This quantity is indicated by $n(\vec{r}, E, t)$ and can be measured for instance in neutrons per cm³ (or in cm⁻³). The reaction density $R(\vec{r}, E, t)$ is defined as the number of reactions in the volume $d^3\vec{r}$ about \vec{r} , at a time t , initiated by neutrons with energy between E and $E + dE$ and is given by the equation

$$(22) \quad R(\vec{r}, E, t) = n(\vec{r}, E, t) \Sigma v,$$

where again v is the neutron speed. The reaction density is measured in reactions per cm³ per second. We give a special name to the quantity $n(\vec{r}, E, t)v$: it is called the

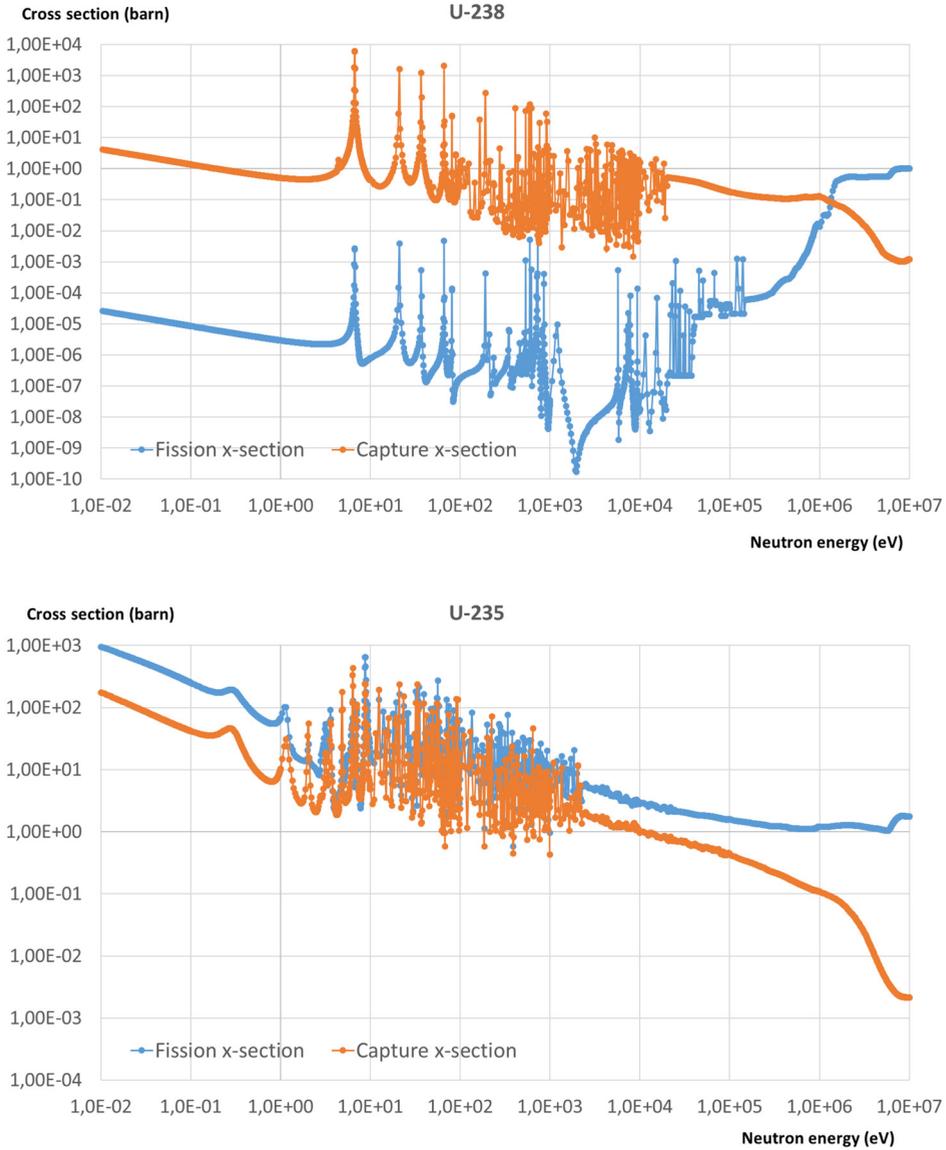


Fig. 3. – Fission (blue) and radiative capture (red) cross sections for the two isotopes of uranium.

neutron “flux” $\phi(\vec{r}, E, t)$ and is measured in neutrons per cm^2 per second. This quantity is essentially the number of neutrons arriving from all directions and crossing a unit surface in unit time (it may differ from other definitions of flux in physics but we will not discuss these differences here). Therefore, given a neutron flux ϕ and a macroscopic cross section Σ , the reaction density, number of reactions per unit volume and per unit

time, R is simply given by $\Sigma\phi$

$$(23) \quad R(\vec{r}, E, t) = \Sigma\phi.$$

As an immediate example, let us consider a thermal reactor (see later) with 3 GW thermal power = $3 \cdot 10^9$ J/s (indicated by the symbol 3 GWth, with a 30% conversion efficiency it will produce about 1 GW electric power, 1 GWe). Considering that each fission releases order of 180 MeV energy = $3 \cdot 10^{-11}$ J at the moment of fission, it follows immediately that in the whole reactor about 10^{20} fissions/s occur, which corresponds to $2\text{--}3 \cdot 10^{20}$ neutrons/s emitted and about $6 \cdot 10^{20}$ neutrinos/s emitted from fragment β decay. Assuming the reactor core to contain 27 tons of uranium oxide (UO₂, with density about 11 gr/cm³), we get a total volume of the core of about 2.5 cubic meters, or $2.5 \cdot 10^6$ cm³, which implies that the reaction density must be around $4 \cdot 10^{13}$ fissions/cm³/sec. By recalling eq. (21) and assuming an enrichment of 3.5% in fissile uranium (*i.e.* 3.5% of the uranium in the core is fissile ²³⁵U, which means that the effective fissile uranium oxide density will be 0.39 gr/cm³, see sect. 1.5 below) and by using the known value of the fission cross section for thermal neutrons⁽¹⁾ on ²³⁵U (from fig. 3 we see that at 0.025 eV, corresponding to room temperature, we have about 500 barn), we can calculate that $\Sigma \simeq 0.5$ cm⁻¹. By recalling eq. (23), we can see that the neutron flux ϕ inside the reactor must be in the order of $8 \cdot 10^{13}$ neutrons cm⁻² s⁻¹. This is a lot of neutrons incident on structural materials and since a good percentage of them are fast, energetic ones, possible damage to the materials has to be taken into consideration⁽²⁾. This is even more the case in fast reactors, where most of the neutrons are fast, as well as in fusion reactors, where neutrons are essentially only fast ones (see [1]).

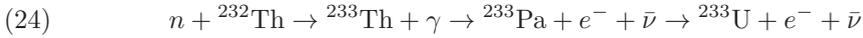
1.5. *Fuel.* – We already encountered the definition of *fissile* (*e.g.* ²³³U, ²³⁵U, ²³⁹Pu) and *fissionable* (*e.g.* ²³⁸U) nuclei. There are also nuclei that can produce a fissile isotope via neutron radiative capture and subsequent β decay. These nuclei are called *fertile*, meaning that they can be used to produce fuel and essentially comprise ²³⁸U and ²³²Th. As an example, ²³⁸U can produce a fissile nucleus via the chain of reactions in eq. (8).

Natural uranium comprises 0.7% ²³⁵U and 99.3% ²³⁸U, however reactors using mixed ²³⁵U–²³⁸U fuel typically need 3–5% ²³⁵U, so that a procedure called uranium *enrichment* is needed to fabricate reactor fuel. The process of plutonium production, where actually a mixture of several Pu isotopes is produced, is called Plutonium *breeding*. Pu breeding is important in cases where the used fuel is reprocessed to extract the Plutonium isotopes. Under certain conditions, a reactor can produce more Pu than it consumes, in which case it is called *breeder*. From the cross sections shown in fig. 3 it follows that, in a reactor using a mixed ²³⁵U–²³⁸U fuel and for neutron energies < 1 MeV, ²³⁵U provides

⁽¹⁾ Thermal neutrons at room temperature have T_n around 0.025 eV or 25 meV, *i.e.* the value of kT , where k is the Boltzmann constant and T is the temperature of the system; see sect. 1.8.

⁽²⁾ A fast neutron can collide with material nuclei and transfer a significant amount of energy, so that *e.g.* atoms in a metal can be displaced from their position in the lattice.

the dominant contribution to the fission rate. It is worth mentioning that thorium, an abundant element in nature, can be used to produce a different type of fuel. Indeed, in the reaction



the final product is ${}^{233}\text{U}$, which is fissile. Use of thorium as a breeder of ${}^{233}\text{U}$ is studied as a possible way to increase the available stock of fissile elements. The whole process of mining, extraction of the uranium mineral from the ore, purification, fuel fabrication through enrichment, irradiation and final storage or disposal is called *fuel cycle*. In the *open* or *once through* fuel cycle the spent fuel is put in temporary storage waiting for final disposal. In the (partially) *closed* fuel cycle the spent fuel is reprocessed to extract the Pu, which is then used to fabricate new fuel typically in the form of Mixed Oxide of Uranium and Plutonium (MOX).

1'6. *Nuclear waste*. – As discussed above, many of the fission fragments are radioactive, typically undergoing β decay very often accompanied by gamma emission. Such radioactive elements increase the radioactivity of the irradiated fuel, so that the fuel extracted from the reactor is a highly radioactive material that needs to be protected and shielded. While most of the radioactive fragments have relatively short half-lives, the longest being in the order of a few tens of years, a small group of fragments has very long half-lives in the order of 10^5 years, the so-called Long Lived Fission Products, LLFP. The aforementioned plutonium production is one example of process leading to the appearance of *Transuranics* in the fuel, *i.e.* chemical elements beyond uranium that do not exist in nature. Reaction (8) leads to production of the ${}^{239}\text{Pu}$ isotope, but further neutron captures produce other isotopes ${}^{240-242}\text{Pu}$, of which ${}^{241}\text{Pu}$ β -decays to americium, ${}^{241}\text{Am}$ and so on. Many of the Transuranics decay by α radioactivity and have long half-lives, from a few hundred years to a few hundred thousand years. Reprocessing of the irradiated fuel gives the possibility to recycle the Pu to fabricate new fuel, but whenever reprocessing is not applied or stopped, the resulting spent fuel is considered to be a *nuclear waste* or *radioactive waste* which needs to be properly stored and eventually disposed of. Indeed, such radioactive nuclei can be dangerous for the environment and for human health, due to direct exposure, especially in the case of the penetrating γ radiation, but also in case of skin/eyes exposure to β radiation, and due to ingestion or inhalation in the case of α and β decay. Additional radioactive waste is produced in the reactor by neutron capture in the surrounding materials, producing *activated* materials with a varying degree of radioactivity. The International Atomic Energy Agency has come up with a classification scheme that is used as a template by each country to define categories of waste depending on its radioactivity, isotope content and possible heat production due to the radioactivity itself [3].

1'7. *Fuel consumption*. – Let us go back to our ideal 1 GWe reactor. About 10^{20} fissions/s take place in the reactor, meaning that 10^{20} (${}^{235}\text{U}$ nuclei)/s disappear (actually somewhat more because of the concurrent radiative capture process). Therefore roughly

40 mg/s of ^{235}U are “burnt” in the reactor. For 1 year of operation at 80% load factor (the actual working time), this means a consumption of about 1 ton of ^{235}U : in volume of pure metallic ^{235}U , this would be a cube of about 36 cm side. For comparison, the same amount of electric power is obtained by burning 1.6 Mtoe, million tons of oil equivalent (considering a slightly higher conversion efficiency of oil-fired power plants), or about 2 billion cubic meters of natural gas, or 2.2 million tons of coal. In a real fission reactor ^{235}U consumption is partly compensated by plutonium (^{239}Pu) breeding according to eq. (8). Indeed, since ^{239}Pu is also a fissile nucleus, it is both produced and burnt in the fission core. In practice, for an initial load of about 27 tons of uranium enriched at 3.5% fissile ^{235}U (of which about 950 kg are ^{235}U), after 1 year of operation 280 kg of ^{235}U remain: about 560 kg of ^{235}U and 380 kg of other isotopes, mostly ^{239}Pu , have been burnt, which is approximately the 1 ton we have been estimating above.

1’8. *Fast and slow neutrons.* – We have seen that the cross sections for nuclear reactions induced by neutrons depend a lot on the energies involved, therefore it is important to introduce some classifications of the neutrons according to their energies. It is customary to adopt the following classification: *slow* neutrons are those with kinetic energy $T_n < 1$ eV, in particular thermal neutrons at room temperature have T_n around 0.025 eV or 25 meV (the value of kT , where k is the Boltzmann constant and T is the temperature of the system); *epithermal* neutrons are those with 1 eV $< T_n < 100$ keV (0.1 MeV); finally, *fast* neutrons have 0.1 MeV $< T_n < 20$ MeV. Obviously neutrons in general can have energies above 20 MeV but this is an extreme limit in reactor physics (*e.g.*, neutrons from fusion of a deuteron and a tritium nucleus have 14 MeV fixed energy).

It is easy to show in non-relativistic kinematics that after a scattering off a nucleus with mass number A , *on average* the kinetic energy of the neutron changes according to the ratio

$$(25) \quad \frac{T'_n}{T_n} \simeq \frac{1 + A^2}{(1 + A)^2}$$

(where T_n and T'_n are the kinetic energies before and after the scattering, respectively). For a heavy nucleus $A \gg 1$, one has that $T'_n \simeq T_n$ or in other words, the neutron has to undergo many collisions in order to significantly lose energy. Consider instead the case $A = 1$ (this is relevant for any medium containing hydrogen, where hydrogen nuclei, *i.e.* protons, will be among the targets). In this case $T'_n = \frac{T_n}{2}$, *i.e.* on average a neutron will lose half of its energy at each collision and therefore few collisions are sufficient to rapidly decrease its energy. For collisions on a light nucleus like carbon ($A = 12$), the situation will be similar to that of hydrogen, although the average loss of energy per collision will be lower. Light materials containing hydrogen (water, paraffin, plastic materials in general etc.) or containing light nuclei (graphite, etc.) are called *moderators*, for their effectiveness in slowing down neutrons. In a thermal reactor, the fuel is immersed in water, which is used both as coolant and as moderator: the fast neutrons from fission will be slowed down very quickly via collisions with protons and oxygen nuclei in water and therefore they will have a very high fission cross section for ^{235}U (see fig. 3).

2. – Chain reaction and simple reactor kinetics

For the reactor to work effectively and safely, the chain reaction must not diverge and must not die away. This means that, amongst the 2-3 neutrons emitted on average in the fission process, precisely one has to induce another fission event, not more, not less. The remaining fission neutrons will then either be absorbed by radiative capture or will leak out from the system. The character of the chain reaction depends essentially on the reactor composition and on its geometry and is defined through the quantity called k , also called multiplication coefficient

$$(26) \quad k = \frac{\text{number of neutrons in one fission generation}}{\text{number of neutrons in the preceding fission generation}}.$$

The condition $k = 1$ corresponds to a *critical* reactor, *i.e.* a reactor actually working and producing thermal power from fission. For $k > 1$ we have a supercritical reactor (the number of fission reaction diverges) and for $k < 1$ we have a subcritical reactor (fission reactions die away). In a “simple minded” reactor time evolution, called *kinetics*, it is relatively easy to show that the number of neutrons in the system evolves as

$$(27) \quad n(t) = n(0) \exp\left(\frac{k-1}{\tau}t\right),$$

where $n(0)$ is the initial number of neutrons and τ represents the average lifetime of a neutron in the system before it disappears due to fission, capture or escape (leakage). Here we clearly see that $k = 1$ corresponds to a steady state, while $k > 1$ implies that the system is diverging and $k < 1$ corresponds to a decreasing population. Based on this simple kinetics, we see that the time constant governing the response of the reactor would be $\frac{\tau}{k-1}$. Since the typical neutron lifetime in a thermal power reactor is of the order of 10^{-4} s, we see that even for $k = 1.001$, the neutron population and therefore the reactor power would increase by a factor 2.7 in 0.1 s, which would not be a manageable situation from a practical point of view. But there is one missing piece in the picture of fission discussed above. Actually, there is a small fraction (less than a percent) of neutrons originating from the fission process that are not emitted immediately, but some time later. These neutrons are emitted by highly excited fission fragments on a time scale from milliseconds to seconds and for this reason are called *delayed neutrons*. In simple terms, a nuclear reactor is made critical thanks to such small fraction of delayed neutrons and therefore they are the ones that dominate the reactor response time making it much longer, thereby allowing reactor control by neutron-absorbing control rods.

2.1. A bit more on neutron multiplication and transport. – By considering in more detail what processes can intervene along the route of the next-generation neutrons emitted after a single fission, one can come up with the following definition of multiplication

coefficient k :

$$(28) \quad k = \eta f P_{NL},$$

where

- f is the conditional probability that, if a neutron is absorbed, it will be absorbed in the fuel instead of structural elements;
- η is the average number of neutrons produced per neutron absorbed in the fuel, which is given by the average number of neutrons per fission ν multiplied by the probability that the absorption process leads to fission instead of radiative capture $\eta = \nu \frac{\sigma_f^F}{\sigma_a^F}$, where σ_a^F is the absorption cross section in the fuel (absorption=fission+radiative capture) and σ_f^F is the fission cross section in the fuel;
- P_{NL} is the probability that a neutron does not escape from the reactor (No-Leakage).

For an infinite reactor clearly $P_{NL} = 1$ and we get $k_\infty = \eta f$. In practice, an effective multiplication factor, k_{eff} is defined, which takes into account neutron leakages and the reaction dependences on neutron energy. It is interesting to look at the behavior of the quantity η as a function of neutron energy, shown in fig. 4 for the ^{235}U and the ^{239}Pu fissile nuclei. It is clear that for thermal neutrons, this number is around 2, which means that effectively there are two neutrons available for further processes, one of which can trigger another fission, while another can be lost in captures or leak out of the system. In case of capture on a fertile nucleus like ^{238}U , plutonium breeding can occur, but overall this will happen with relatively low probability. On the contrary, for high kinetic energies about 10–100 keV (fast neutrons), η becomes larger than 2 and so more neutrons will be available for breeding new fuel. Therefore, a so-called *fast reactor*, *i.e.* one in which neutrons are not moderated (see sect. 3.1: “Reactor types”), may produce more plutonium than it consumes, in which case we will have a *fast breeder*.

3. – Main features of a nuclear fission reactor

We saw that neutrons slow down when undergoing collisions with nuclei (in particular with light nuclei), therefore their energies in the core can go from about 10 MeV (usually the maximum energy of fission neutrons), down to as low as 10^{-3} eV. We also saw that neutron cross sections have a strong dependence on neutron energy, in particular fission cross sections decrease very strongly with energy for fissile nuclei like ^{235}U . Therefore, when using fuels containing these nuclei, it is easiest to maintain a fission chain reaction by using slow neutrons. Hence most nuclear reactors until now (the so-called Generation I to III+) use low mass number materials such as water or graphite to moderate the fast fission neutrons, thereby slowing them down to energies comparable to the thermal energies of the nuclei in the reactor core. By definition, a *thermal reactor* is one where the

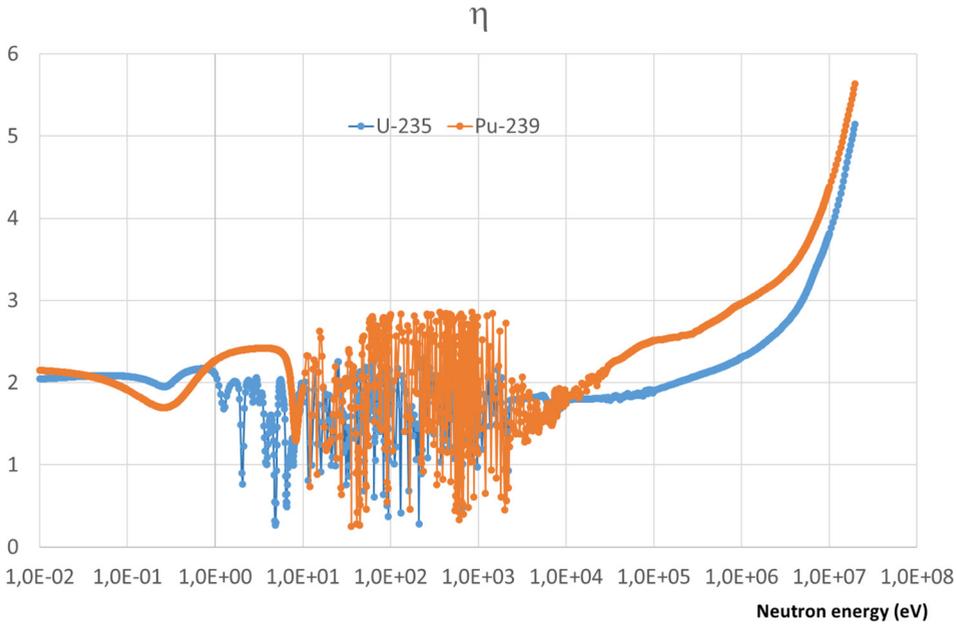


Fig. 4. – The parameter η as a function of neutron kinetic energy for the two main fissile nuclei.

average neutron energy is comparable to thermal energies (order of several tens of meV). Because of the very high-fission cross section at thermal energies (see fig. 3), this type of reactor requires the minimum amount of fissile material for fueling. As an example, a Light Water Reactor (LWR) can start with 3.5% ^{235}U + 96.5% ^{238}U (a mixture enriched with respect to natural uranium). Burn-up (consumption) of ^{235}U is partly compensated by breeding of ^{239}Pu via neutron radiative capture and subsequent β decays and after 1 year operation, the core may contain something like 1% ^{235}U + 1% plutonium (all Pu isotopes, including ^{239}Pu). At the contrary, in a *fast reactor* the average neutron energies in the core are typically above 100 keV. As previously discussed, this can be accomplished if the core contains mostly high mass-number materials. This means that for core cooling the choice will be a liquid metal like sodium, lead or a lead-bismuth mixture, or alternatively the coolant should be a low-density material like Helium gas. On the other hand, σ_f is smaller for fast neutrons, which means that a much higher percentage of fissile material is needed to sustain the chain reaction. At the same time, based on similar considerations to those in sect. 1'4, it is easy to see that the neutron flux must be a couple of orders of magnitude higher than the thermal reactor, therefore more around 10^{16} neutrons $\text{cm}^{-2} \text{s}^{-1}$. Therefore, in a fast reactor the absolute neutron flux is much higher than in a thermal reactor and it is also composed by a majority of fast, energetic neutrons. This is why the long-term damage to the structural elements by neutron bombardment is a more important issue in a fast reactor than in a thermal one. In summary, the typical conceptual schemes of a thermal and a fast reactor are illustrated in fig. 5.

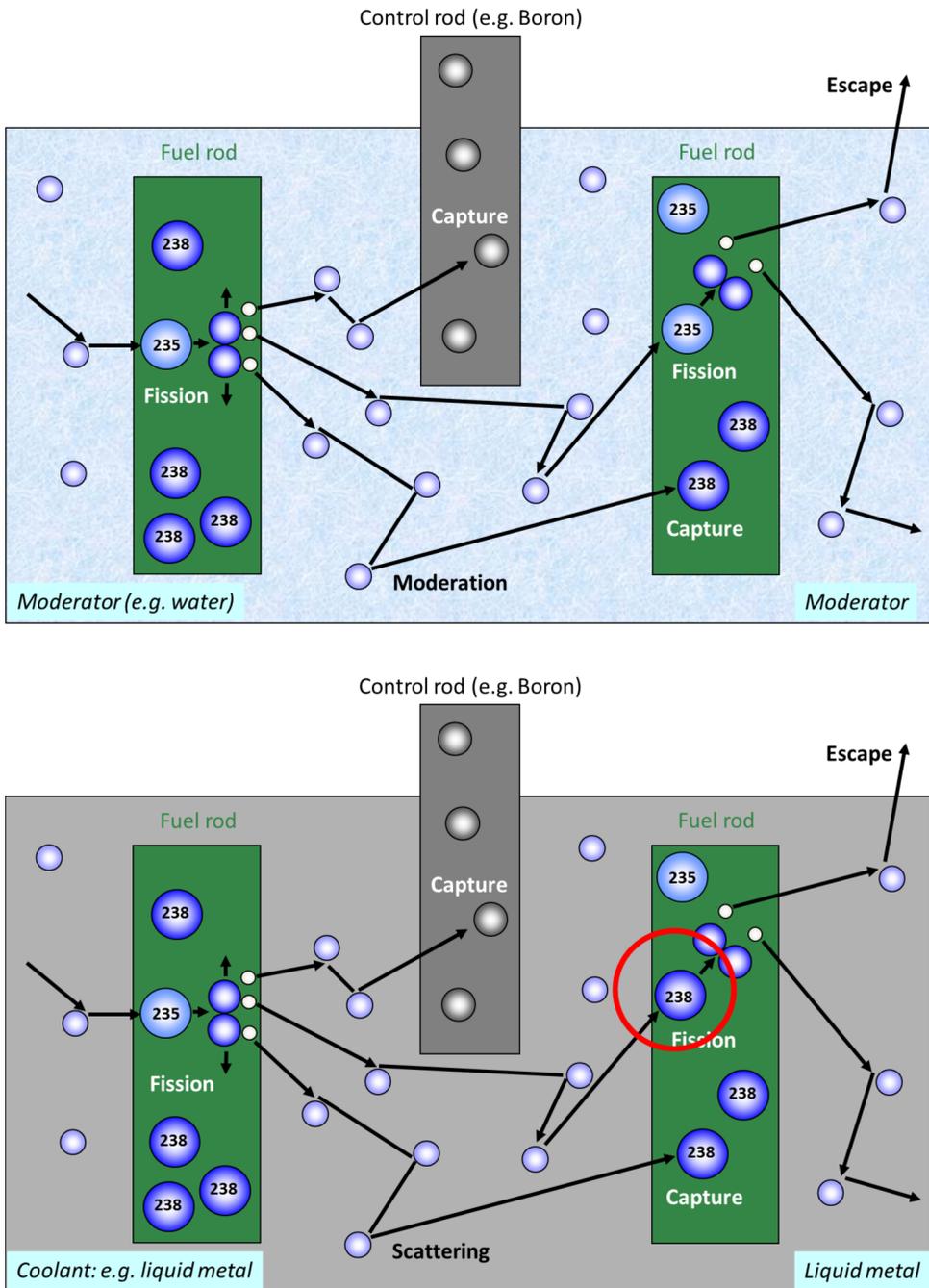


Fig. 5. – Conceptual schemes of thermal (top) and fast reactor (bottom). In the thermal reactor (top), the neutron energy spectrum exhibits a strong slow component from neutrons in thermal equilibrium with the moderator, with a kinetic energy of 25 meV at 290 K, corresponding to the most probable speed of the Maxwellian distribution. In the fast reactor (bottom), the neutron energy spectrum is much harder and most neutrons can have kinetic energies up to a few MeV.

3'1. Decay heat. – Decay heat is the heat produced in the reactor core as a result of radioactive decay: the energy of the alpha, beta or gamma radiation is converted into atomic motion. Decay of the short-lived radioisotopes created by fission continues for some time after shut down. A practical approximation is given by the formula

$$(29) \quad \frac{P}{P_0} = 6.6 \cdot 10^{-2} \left[\frac{1}{(\tau - \tau_s)^{0.2}} - \frac{1}{\tau^{0.2}} \right],$$

where P is the decay power, P_0 is the reactor power before shutdown, τ is the time since reactor startup and τ_s is the time of reactor shutdown measured from the time of startup (in seconds). At shutdown, the heat power is about 6.5% of the previous core power ($\simeq 200$ MWth for a 1 GWe reactor), which is sufficient to melt the core. About 1 hour after shutdown, the decay heat will be about 1.5% of the previous core power, after 1 day it will fall to 0.4% and after a week it will be 0.2%, much smaller but still significant. For this reason, spent fuel rods discharged from a reactor core after a period of operation are kept for long time in special cooling water pools, before being further processed. Removal of decay heat is therefore a very important aspect in designing a reactor and implementing safety measures, as the Fukushima accident has dramatically stressed.

3'2. Safety. – Safety is a very important aspect in nuclear power production. The core of a nuclear reactor produces radiation both while in operation and after shutdown, because of the high content of radioactive materials in the fuel and because of the neutron irradiation of materials in the core and surrounding the core, which produces radioactive nuclei starting from stable ones. Therefore, the safety culture is an essential aspect of modern nuclear industry. Attention to safety is present from the design stage down to operation, shutdown and decommissioning of the plants and particular care is devoted to the analysis of possible accidental scenarios. A fundamental principle is the *defence-in-depth*, which means that a series of barriers, starting from the solid fuel form itself up to the reactor containment building, are put in place to contain radioactivity as much as possible at all times and in all instances. Obviously, in this respect, the role of independent regulatory authorities is crucial in overseeing all stages of a plant lifetime.

3'3. Radioactive waste. – We saw in the preceding sections that a nuclear reactor is producing a significant amount of radioactive substances, due to fission itself, as well as due to neutron capture in the fuel and in structural elements. Fission products with a half-life until several days are responsible for the large amount of decay heat that is produced within the core immediately after reactor shutdown, *i.e.* after stopping the chain reaction. However, their radioactivity dies away in a few days or weeks, which means that in case of accidental release of radioactive products they pose a threat only for such a limited amount of time and when the spent fuel is stored in the cooling pools they do not produce anymore heat after a relatively short amount of time. Other Short-Lived Fission Products with half-lives until a few years can instead be a safety concern also after an accordingly longer period of time, as are Medium-Lived Fission Products

with half-lives until about 100 years (*e.g.*, ^{137}Cs and ^{90}Sr , with half-life of about 30 years). Such isotopes contribute to heat production in the spent fuel and can be a safety concern for decades if accidentally released into the environment. Finally, Long-Lived Fission Products (LLFP) have half-lives over about 200000 years (no fission product has half-life between about a 100 years and 200000 years). Transuranics, *i.e.* plutonium together with the so-called *Minor Actinides*, MA, neptunium, americium, curium, etc., decay predominantly by α decay and have half-lives ranging from a few hundreds of years up to above 2 million years (^{237}Np). The typical inventory from a 1 GWe PWR fuelled with about 27 tons of U (3.5% ^{235}U) at discharge (when the spent fuel is removed from the core) is approximately 280 kg of ^{235}U , 266 kg of Pu of which 156 kg of fissile ^{239}Pu , 20 kg of MA, 13 kg of ^{90}Sr and 30 kg of ^{137}Cs (both with 30 years half-life), and 63 kg of LLFP [4]. The very long decay time of both LLFP and MA is the reason why disposal of spent fuel and similar waste requires very long-term storage, for which one possible solution envisaged is storage deep underground in the so-called geological repositories. Currently, Finland, Sweden and France have advanced projects for building such repositories for final disposal. Some countries apply reprocessing of the spent fuel, where plutonium is extracted from the fuel rods and used to produce new fuel containing both uranium and plutonium, so that the spent fuel is directed to the underground repository only after partial recycling.

3'4. Waste production and incineration. – In the effort to improve the safety, security and efficiency of nuclear plants, new concepts of reactors have been developed with goals including the minimization of the production of MAs, a better and more efficient use of the fuel, a better thermodynamic efficiency, the possible production of hydrogen at high temperatures and finally improved safety features to minimize the risk of accidents. All these various concepts are considered within the so-called Generation-IV reactors, which are the subject of an international initiative [5]. As an example, we saw that in a fast reactor, by using *e.g.* a liquid metal or gas as coolant, the energy spectrum of the neutrons is harder. In such a reactor core, it becomes possible to burn not only fissile elements like ^{235}U and ^{239}Pu , but also fissionable elements like ^{238}U for which fission occurs significantly only above a certain energy threshold around 0.5 MeV, as can be seen in fig. 3. Although given the much lower fission cross section typically the fuel has to be richer in fissile content, this means that also ^{238}U can to some extent be considered a fuel as well, which has obvious implications in terms of how long uranium resources will last. Another important advantage in terms of fuel cycle in fast neutron systems is that, for energies, say, above 0.5 MeV, fission becomes dominant over capture, as can again be seen in fig. 3. This means that the MA production, which occurs namely via capture, is suppressed. Yet another important aspect is that in a fast reactor the MA produced can be partly destroyed in the reactor itself by fission. This is because many of the MA are fissionable, *i.e.* they can undergo fission only for neutron energies above around 0.5 MeV. An important physics aspect when considering fission of MA is that the amount of delayed neutrons produced upon their fission is significantly less than for ^{235}U and ^{239}Pu . This means that for instance fueling a reactor purely with MA is not possible,

as one would lose the crucial safety feature of the reactor response times being dominated by the relatively long delayed neutron emission times. However, for Accelerator Driven Systems (ADS), which are based on a subcritical reactor core that requires an external neutron source, the role of delayed neutrons is not so crucial and therefore there is the possibility to mix a larger quantity of MA into the fuel, thereby offering the possibility to use the apparatus to perform *transmutation* of the MA, *i.e.* incinerate the MA into fission products, thereby reducing the radiotoxicity of the final materials in the fuel. This is called *P&T*, *partitioning and transmutation* approach, meaning that first some nuclides are separated from the spent fuel (partitioning) and then they are irradiated in a special core to be either destroyed, transformed into stable elements or into very short-lived isotopes reaching fast the stability (transmutation [4,6]). Currently, the MYRRHA project in Belgium [7] started a staged approach to the construction of a multipurpose ADS that will also demonstrate the concept of waste incineration.

REFERENCES

- [1] ONGENA J., “Fusion: a true challenge for an enormous reward”, these proceedings , p. 143; ROMANELLI F., “Fusion energy: technological challenges”, these proceedings, p. 175.
- [2] Java-based Nuclear Data Information System, <http://www.oecd-nea.org/janis/>.
- [3] *Classification of Radioactive Waste*, No. GSG-1, General Safety Guide (IAEA) 2004, https://www-pub.iaea.org/MTCD/Publications/PDF/Pub1419_web.pdf.
- [4] NIFENECKER H., MEPLAN O. and DAVID S., *Accelerator Driven Subcritical Reactors* (Institute of Physics Publishing, Bristol and Philadelphia) 2003.
- [5] *Generation IV International Forum*, <https://www.gen-4.org/>.
- [6] ACHUTHAN P. V. *et al.*, *Implications of partitioning and transmutation in radioactive waste management*, Technical Reports Series No. 435 (IAEA) 2004, (Physics Publishing, Bristol and Philadelphia) 2003, <https://www.iaea.org/publications/7112/implications-of-partitioning-and-transmutation-in-radioactive-waste-management>.
- [7] ABDERRAHIM H. AÏT *et al.*, “Partitioning and transmutation contribution of MYRRHA to an EU strategy for HLW management and main achievements of MYRRHA related FP7 and H2020 projects: MYRTE, MARISA, MAXSIMA, SEARCH, MAX, FREYA, ARCAS”, *EPJ Nucl. Sci. Technol.*, **6** (2020) 33.