Advanced simulation of Stirling engine

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Abstract. Energy demand reaches high values with its negative impact on the environment, it presents a considerable danger. For environmental and economic reasons, the main strategic tasks in this area are associated with reducing and minimizing energy consumption. The possible solution is preceded in the development phase by the application of optimization of thermal processes using computer simulation of thermal processes of whole systems. The result is numerical model with sufficiently accurate information that help to verify the accuracy of the proposed procedures and decisions and thus contribute to the efficiency and rationalization of production. New technologies are needed to utilize and use the low potential heat, or other fuel than fossil fuels. Stirling engine, as engine with outer combustion is a possible solution.

1 Unconventional micro cogeneration unit

Today’s research priority is focused on more efficient transformation process of primary energy from renewable energy sources. There are many possibilities in low potential heat utilization from renewable energy sources and the absence of effective ways of transforming heat into electricity, thus efficient heat cycles in general. Currently there is no cost-effective energy system for converting heat of medium or low potential. Thermal cycle quality significantly affects the final energy conversion efficiency, it is currently at Organic Rankin Cycle for example about 17%, which is about 35% compared to the efficiency in the conventional Rankine cycle and a 44% efficiency in a supercritical cycle in a large facility. [1], [2] The theoretical thermodynamic cycle of Stirling engine varies considerably from the real one. This is due to imperfect isothermal conditions, heat leakage to the environment and other influences. For this reason, the efficiency of the ideal cycle is higher than the real cycle. They work on the principle of external combustion, so fuel combustion does not take place in the working cylinder. This allows, in contrast to conventional internal combustion engines, control the course of the combustion process, the related quality, which is reflected in the composition of pollutants released into the atmosphere. The air is compressed in the compressor, it passes through the heat exchanger under constant pressure and takes heat. Then the air expands adiabatically in the cylinder and makes a work. Some of this work is used to drive the compressor and the other part is converted to mechanical work with the help of the generator into electrical energy. On generating of heat energy may be used a wide range of fuel, because this is the external combustion engine and for this is very common the use of renewable sources. The fuel is combusted in a separate combustion chamber, and thermal energy is transformed with the aid of the heat exchanger to the working fluid. [3]

The proposed micro-cogeneration unit with Stirling engine consists of two heat sources, one serves as a cooler and the other as the heater. Both of them have different requirements for operation. The proposal of measurement system is in Fig. n. 1.

Fig. 1 Measurement setup

2 Numerical simulations

The thermal efficiency of the ideal Stirling cycle is just the same as the thermal efficiency of the Carnot cycle with the same inlet and outlet temperatures. For an ideal
regenerator, the heat removed during cooling period is absorbed by this regenerator and transferred to the working medium within compression – heating period. Computation of Stirling engine cycle was later generalized by Reitlinger, where he replaced the isotherms with polytropes. Significant clarification of the Stirling engine's analytical solution is the regenerative Rallis Circulation. It consists of two isotherms and two regenerative processes, which are carried out in part at constant pressure, in part at a constant volume. In the case of the expansion of gas compression in the working space of a heat engine, which are considered to be isotherms in the Stirling Circulation, they are in fact polytropes with an exponent approaching, in particular, the higher frequencies of the pistons of the motor of the exponent adiabatic engine. Rallis therefore replaced the isotherms with adiabatic changes in their final form. For the assessment of the potential electrical power of the cogeneration unit, a performance characteristic can be constructed from the inlet temperature on the heater and the average working pressure of the medium in the engine. Theoretical electrical power can also be calculated for temperature differences of heater and cooler. These simple methods are based on an ideal lossless model with experimentally determined efficiency and power correction factors applied, which represent generalized effects of various energy losses. These less accurate methods serve primarily as a starting point for estimating engine performance, suitability for use in a given application, and the relationship between engine size and power. However, they do not give a more specific view and their results are relatively optimistic. The first-order method is the frequently used Schmidt analysis. A dimensionless Beale or West number, as in Fig. 2, is most often used for preliminary performance determination.

![Fig. 2 Beale number according to mean temperature](image)

The Beale number is determined as a function of the mean working gas temperature of the hot side of the engine. The upper dotted line corresponds to the engines with a smaller self-capacity and higher power and the lower ones with a larger dead volume and lower power. The assumptions under which the relationships characterizing the circulation of Stirling, Reitlinger and Rallis were derived are a great idealization of reality. Their practical use is only possible in the case of elementary preliminary calculations. A much more accurate expression of the events actually taking place is the Schmidt cycle, whose relations were derived in 1871. Modern Stirling engine is a clean device operating with considerable efficiency, which is related to the fact that heat, as an energy source, is extracted from the outside of the engine and transferred to the working medium of the engine via a heat exchanger. External combustion, which takes place under almost ideal conditions (sufficient oxygen, low time, theoretically any fuel, etc.) is not burdened with the production of undesirable emissions, especially nitrogen oxides. In connection with ecology, other sustainable sources for a modern engine, such as solar or geothermal energy, should be mentioned. Other aspects such as extra silent engine operation, vibration-free operation, or reverse operation capability make it an interesting engine for modern society.

At present, the Stirling engine is undergoing breakthroughs in several areas, particularly as a submarine drive. Last but not least, as a heat source in households, energy sources in space and solar power plants. Even the production of passenger cars is not completely closed for the Stirling engine. Other applications may be power sources for generators, onboard heaters and aircraft propulsion. Powered by a Stirling engine, the yacht's electricity generator is a practical combination of its smooth and silent operation and the disadvantage of cooling, which should certainly not be a problem on a boat surrounded by a lot of water. Also, driving a low-vibration silent engine is not a bad idea, moreover, any aircraft engine used up to now loses power to an increasing altitude. The sealed Stirling engine is not dependent on ambient air density, moreover, as the temperature decreases, the temperature difference between the hot and cold parts, and thus its performance, would increase. The main purpose of this publication is to show possible accuracy of numerical simulation in regard to real measurements.

### 3 Basic setup

Simple analysis of an ideal Stirling engine model, where the compression and expansion spaces are maintained at some designed temperatures led to the paradoxical situation that neither the heater nor the cooler produce any net heat transfer during the cycle and it is therefore redundant. All the required heat transfer took place across the boundaries of the isothermal work spaces, which is not correct because the cylinder walls are not designed to transfer heat. In real engines, the workspaces will act as adiabatic rather than isothermal, which means that the heat transferred during the cycle must be provided by heat exchangers.

An alternative adiabatic model for Stirling cycle engine, based on the previous design (Urieli, 1984) assumes the following conditions:

- volume variations are sinusoidal,
- working fluid has ideal gas behaviour,
- the temperatures on the wall of the heater and cooler remain constant,
• compression and expansion spaces are adiabatic,
• cyclic steady state equilibrium is reached,
• the pressure drop along the engine is initially neglected,
• heat and mechanical losses are initially neglected,
• no gas leakage; therefore, the total mass of the gas in the system is constant.

Depending on the model, the engine is theoretically divided into five characteristic control volumes: expansion chamber (E), heater (H), compression space (K), regenerator (R), and cooler (C). These are represented by control volumes, shown in the Fig. 3 along with the appropriate variables describing the system - volume, weight, temperature, pressure and mass flow.

![Fig. 3 Cylinder control volume](image)

### 4 Energy balance

The advanced adiabatic model, worked out according to [3], is based on the basic equations of mass and energy equilibrium, with a state equation applied to each control volume. Control volume represents every part of the engine, the expansion chamber (E), heater (H), compression space (K), regenerator (R), and cooler (C). By applying the mass balance equation to the control volume, the mass change according to the piston crank position (φ) can be expressed as in [3]:

$$\dot{m}_e - \dot{m}_c = \frac{\text{d}m}{\text{d}\phi}$$

(1)

By including mass flow into the energy equilibrium balance with neglected kinetic components, whole system energy can be expressed as:

$$\frac{\text{d}Q}{\text{d}\phi} + c_p T_i \dot{m}_i - c_p T_o \dot{m}_o = \frac{\text{d}W}{\text{d}\phi} + c_v \frac{\text{d}(mT)}{\text{d}\phi}$$

(2)

The evaluation of the pressure change along the working space of the engine shall be calculated taking into account the mass balance applied to the whole engine, where the whole mass is not changing:

$$M = m_c + m_k + m_r + m_h + m_e$$

(3)

The working gas mass is defined by ideal isothermal Schmidt analysis. [3] This analysis is taken into account as first step, where these assumptions make it possible to obtain the following analytical expressions:

$$M = \int_0^{2\pi} r \left( \frac{V_c}{T_k} + \frac{V_k}{T_r} + \frac{V_r}{T_h} + \frac{V_h}{T_e} \right) d\phi$$

(4)

$$M = \frac{p_m(s\sqrt{1 - b^2})}{r}$$

(5)

With parameters s and b defined as:

$$b = \frac{c}{g}$$

(6)

$$c = \frac{1}{2} \left( \frac{V_{swc}}{T_h} \right)^2 + \frac{2}{\kappa} \frac{V_{swc} V_{swc} \cos(\alpha)}{T_k}$$

(7)

$$s = \frac{V_{swc}}{2T_h} + \frac{V_{swc}}{T_h} + \frac{V_{swc} \ln T_h}{T_h} + \frac{V_h}{T_h} + \frac{V_{swc}}{2T_h} + \frac{V_{swc}}{T_h}$$

(8)

The non-isothermal mass volumes for the compression and expansion cylinder should be described in differential form:

$$\frac{\text{d}m_c}{\text{d}\phi} = \frac{p M}{V_c} \left( \frac{\partial V_c}{\partial \phi} + \frac{\partial p}{\partial \phi} \right)$$

(9)

$$\frac{\text{d}m_e}{\text{d}\phi} = \frac{p M}{V_e} \left( \frac{\partial V_e}{\partial \phi} + \frac{\partial p}{\partial \phi} \right)$$

(10)

The actual values of the temperatures in the compression and expansion cylinder during the cycle are calculated using the differential form of the working gas state equation:

$$\frac{\text{d}T_c}{\text{d}\phi} = T_c \left( \frac{\partial p}{\partial \phi} + \frac{\partial V_c}{\partial \phi} - \frac{\partial m_c}{\partial \phi} \right)$$

(11)

$$\frac{\text{d}T_e}{\text{d}\phi} = T_e \left( \frac{\partial p}{\partial \phi} + \frac{\partial V_e}{\partial \phi} - \frac{\partial m_e}{\partial \phi} \right)$$

(12)

By applying the energy equilibrium equation to each heat exchanging volume, considering that it is an isothermal space, we can according to [3], write the following expressions:

$$\frac{\text{d}Q_k}{\text{d}\phi} = V_k \left( \frac{\partial p}{\partial \phi} \right) c_v - c_p(T_k m_k - T_k m_{kr})$$

(13)

$$\frac{\text{d}Q_r}{\text{d}\phi} = V_r \left( \frac{\partial p}{\partial \phi} \right) c_v - c_p(T_r m_r - T_r m_{rh})$$

(14)

$$\frac{\text{d}Q_h}{\text{d}\phi} = V_h \left( \frac{\partial p}{\partial \phi} \right) c_v - c_p(T_h m_h - T_h m_{he})$$

(15)
Although the ideal adiabatic model is independent of the operating frequency, in order to evaluate power and other time-dependent effects, such as conduction heat loss in the regenerator, it must be defined. The system is designed as a quasi-static flow, i.e. mass flows are constant for each integration interval. For the calculation it is necessary to know at least the initial values of working temperatures, which are the result of adiabatic processes and enthalpy flow. The only clue to their correct selection is that their end-of-cycle values should be equal to their respective values at the start of the cycle. [3], [4] Due to the cyclical nature of the system, the problem is solved by entering the initial conditions and integrating them over several cycles. [4], [6] The best measure of the convergence to cyclic equilibrium is the residual amount of regenerative heat or at the end of the cycle, which should be zero. [7], [8] The fourth order Runge-Kutta method is used to solve the system of differential equations with initial conditions.

5 Results and conclusions

The measurement was carried out at different engine operating pressure settings using natural gas as fuel. The measured values were the inlet and outlet temperature from the heat exchanger, which is a part of the boiler. To measure them, wells were created on the inlet and outlet pipes, in which the sensors were placed. Temperature measurements were made with resistance thermometers, magnetic flowmeter and pressure gauges. The internal engine parameters such as the temperature of the radiator, heater, combustion chamber, and helium working pressure are recorded by the engine control panel. These values are recorded to the computer. The cogeneration unit was connected to a heat exchanger control station, which cooled the excess heat energy through an external cooler of 50 kW. The results are in Fig. 4.

A graph was constructed to compare the measured values of electrical power with the values obtained by the calculation model. The results are shown in Fig. 5. It can be observed that model power values differ only slightly from those measured, with the largest deviation of 425 W recorded at an operating pressure of 80.9 bar. The graph shows that model power at 56.3 bar is less than measured. This is due to a decrease in the instantaneous temperature of the heater head when the highest measured power is reached. The instantaneous head temperature at the measured power is set into the model, which translates the momentary drop into the total power.

Fig. 5 Comparison between model and measurement

The experimental equipment for determining the operational and performance parameters of the individual parts of the system was designed and implemented under laboratory conditions. Comparison of the measured parameters with the corresponding values obtained from the model for the same operating conditions showed small deviations and correctness of the design.

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