Development of a two-dimensional mathematical model of flow boiling heat transfer in micro- and minichannels

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Abstract. The paper concerns flow boiling heat transfer in micro- and minichannels. In the mathematical model, the steady state heat transfer process in a single asymmetrically heated minichannel was considered. Calculations with the use of Trefftz functions were based on the data from own experiments. The temperature of the heater and the refrigerant were assumed to satisfy the Laplace equations and the energy equation respectively. The problem was solved by the Trefftz method using two sets of Trefftz functions. The known heater and refrigerant temperature distributions were used to determine the heat transfer coefficient at the heater – refrigerant contact. To verify the proposed mathematical model, data from experiments were applied to calculations. The essential part of the experimental stand was the test section which comprises a minichannel heat sink. The heated element for Fluorinert FC-72 flowing along minichannels was a thin foil. The temperature of its outer side was measured using infrared thermography. Thermocouples and pressure transducers installed at the inlet and outlet of the test section monitored fluid temperature and pressure. Mass flow rate, the current supplied to the heater and the voltage drop were also recorded. The resulting graphs presented thermograms of measured temperature on outer surface of the heater, temperature distributions of fluid temperature and local values of the heat transfer coefficient.

1 Introduction

Two-phase microchannel heat sinks are very suitable for resolving thermal problems in electronic components. The boiling process during fluid flow in micro- and minichannels allows obtaining the highest possible heat flux at a low temperature difference between a heated surface and a working fluid on a small heat transfer area.

Main of authors earlier works concerned steady state studies on flow boiling heat transfer in minigaps of different geometry: in an annular minigap [1,2] and in minichannels of rectangular cross sections [2-6]. It was also noticed that the use of enhanced surfaces often increases the intensification of heat transfer both in flow boiling [2-6] and in pool boiling research [7-9]. Maciejewska and Piasecka focused on flow boiling heat transfer under unsteady state conditions in minichannels [10,11].

This paper focuses on building and developing an appropriate mathematical model to describe the heat transfer processes in flow boiling of a refrigerant in micro- and minichannel heat sinks. The acquired experimental data influences the form of energy equations and boundary conditions. The model has been formulated so as to avoid the occurrence of experimentally determined constants. The procedure of calculating the temperature of the fluid flowing through a minichannel is coupled with concurrent determination of temperature distribution in the heating surface. The knowledge of temperature distributions in the heating surface and fluid permits to determine local heat transfer coefficients at the heating surface - fluid interface. The proposed model of the problem consists of energy equations subject to boundary conditions corresponding to the experimental data. The solution involves two kinds of heat transfer problems: a direct (in the fluid) and an inverse problem (in the heater). A problem like that requires an effective and stable solution method. Such requirements are met by the Trefftz method [12] which applies to various types of heat transfer problems [13-15].

2 The mathematical model

In the mathematical model it was assumed that the heat transfer process in the minichannel heat sink was in steady state and the physical parameters of the minichannel heated wall were independent of temperature. Two dimensions were taken into account in the model: one in the direction of the flow (x) and one in the perpendicular direction (y) referred to the thickness of the heater (δ_H) and the depth of the minichannels (δ_M). The temperature of the outer side of the heater was monitored continually. We assumed that the heat transfer process occurring at the side surfaces of the minichannel heat sink did not affect the thermodynamic parameters of its central part. It allows

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limiting the considerations to the central channel only (along its length L). Additionally, it was assumed that heat losses to the surroundings are negligible and are not included in the model.

Assumptions for refrigerant Fluorinert FC-72 were as follows:

– fluid flow in the minichannel is laminar (Re < 2100) and steady with a constant mass flux density,

– fluid velocity in the minichannel has one component $w(y)$ parallel to the heated foil surface while the other component equals zero,

– the vapor quality value is low and is not considered in the model,

– fluid temperature at the inlet and outlet of the minichannel is known.

The steady state heat transfer in the heated thin foil and flowing fluid was described using two-dimensional Poisson equation and two-dimensional energy equation, respectively:

a) for heater in the domain $\Omega_H = \{(x,y) \in R^2 : 0 < x < L, \quad 0 < y < \delta_H\}$

$$\frac{\partial^2 T_H}{\partial x^2} + \frac{\partial^2 T_H}{\partial y^2} = -\frac{q_V}{\lambda_H} \tag{1}$$

b) for fluid in the domain $\Omega_f = \{(x,y) \in R^2 : 0 < x < L, \quad \delta_H < y < \delta_H + \delta_M\}$

$$a\left(\frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2}\right) = w(y)\frac{\partial T_f}{\partial x} \tag{2}$$

where: q_V – the volumetric heat flux supplied to the heated foil, λ_H – the thermal conductivity of the heater, a – the thermal diffusivity.

For equation (1), the boundary conditions took into account the measurements of the heated foil temperature $T_{H,i}$ conducted in K points $(x_k,0)$ on the basis of measurements by an infrared camera:

$$T_H(x_k,0) = T_{H,k} \quad \text{for } k=1,2,\ldots,K \tag{3}$$

Additionally, it was assumed that the boundaries of the heater temperature for $x = 0$ and $x = L$ were isolated, i.e.

$$\frac{\partial T_H}{\partial x} = 0 \quad \text{for } 0 < y < \delta_H \text{ and a) } x=0, \text{ b) } x=L \tag{4}$$

For equation (2), the boundary conditions depend, among others, on the assumptions made earlier

$$\text{a) } T_f(0,y) = T_{f,in}, \text{ b) } T_f(L,y) = T_{f,out}$$
$$\text{for } \delta_H < y < \delta_H + \delta_M \tag{5}$$

$$T_H(x,\delta_H) = T_f(x,\delta_H) \tag{6}$$

$$\frac{\partial T_f}{\partial y} = 0 \quad \text{for } y = \delta_H + \delta_M \text{ and } 0 < x < L \tag{7}$$

In the first step, two-dimensional temperature distribution of the heater is determined by solving the system of equations (1), (3) and (4). Next, the solution of the system of equations (2), (5) - (7) allows us to determine the fluid temperature. Determining the heater temperature is an inverse heat conduction problem (IHCP) while determining the fluid temperature is a direct heat conduction problem (DHCP). Having found both temperature distributions (in the heater and fluid) one can evaluate the heat transfer coefficient from the Robin condition, which gives

$$\alpha(x) = \frac{-\lambda_H \dfrac{\partial T_H}{\partial y}(x,\delta_H)}{T_H(x,\delta_H) - T_f(x,\delta_H)} \tag{8}$$

3 Methodology of calculations

For the differential equations with suitable boundary conditions written in the mathematical model (1) – (7), approximate solutions were found using the Trefftz method [12]. The choice of the method was based on the fact that determining solutions to equations (1), (3), (4) lead to finding the solution to (IHCP).

The Trefftz method, as reported in the available literature [1, 10-17], gives stable results for the inverse problems. The Trefftz method is based on approximating the unknown solution of a differential equation using the linear combination of Trefftz functions, that is, the functions that satisfy the equation exactly. Two sets of Trefftz functions were used in the numerical calculations: harmonic polynomials $u_n(x,y)$ for two-dimensional Laplace's equation in Cartesian coordinates [12] and the Trefftz functions $h_n(x,y)$ for the energy equation (2) described in detail [14]. In this case the heater temperature T_H and fluid temperature T_f were approximated by linear combinations of adequate Trefftz functions in the following form

$$T_H(x,y) = \sum_{n=1}^{N_H} a_n u_n(x,y) \tag{9}$$

$$T_f(x,y) = \sum_{n=1}^{N_f} b_n h_n(x,y) \tag{10}$$

where N_H and N_f denote the number of adequate Trefftz functions in linear combinations (9) and (10).

The computed functions T_H and T_f satisfy exactly the governing equations (the Laplace equation and the energy equation respectively), and the assumed boundary conditions are satisfied approximately. The numerical procedure included IHCP in the heated foil and DHCP in flowing fluid.

Unknown coefficients a_n, b_n in equations (9) and (10) were determined by minimizing the error functionals that described the mean square error with which functions T_H and T_f satisfied the assumed suitable boundary conditions. For example, error functional for fluid is given in the form

$$
\begin{aligned}
J_f = & \int_{\delta_H}^{\delta_H+\delta_M} \left(\sum_{n=1}^{N_f} b_n h_n(0,y) - T_{f,in} \right)^2 dy + \\
& + \int_{\delta_H}^{\delta_H+\delta_M} \left(\sum_{n=1}^{N_f} b_n h_n(L0,y) - T_{f,out} \right)^2 dy + \\
& + \int_{0}^{L} \left(\sum_{n=1}^{N_f} b_n h_n(x,\delta_H) - T_H(x,\delta_H) \right)^2 dx + \\
& + \int_{0}^{L} \left(\sum_{n=1}^{N_f} b_n \frac{\partial h_n(x,\delta_H+\delta_M)}{\partial y} \right)^2 dx
\end{aligned} \tag{11}
$$

Its minimization leads to solution of equations

$$
\frac{\partial J_f}{\partial b_n} = 0 \quad \text{for} \quad n=1,2,\ldots,N_f \tag{12}
$$

4 Experimental data

4.1 Experimental stand

The experimental stand consists of several loops and systems. A view of the setup is shown in Fig. 1. The main is a flow loop in which working fluid – Fluorinert FC-72 is recirculated. This loop comprises: a test section with minichannel heat sink as the main element, a mass flow meter with a controller for flow stabilizing, a pump, a filter, a deaerator, a compensating tank (a pressure regulator) and a heat exchanger. The system for data and image acquisition system consists of: an infrared camera, two data acquisition stations and a PC computer with appropriate software. Temperature and pressure of the fluid at the inlet and outlet of the test section are measured by thermocouples (K-type) and pressure transducers, respectively. The supply and control system contains a power supply unit.

4.2 The test section

The test section is illustrated in Fig. 2. It comprises a group of parallel minichannels (each is 32 mm long, 0.5 mm wide and 0.5 deep). The heated element for FC-72 flowing in channels is a foil made of Haynes-230 alloy, 0.1 mm thick. Temperature of the outer side of the foil is measured using FLIR E60 infrared camera.

4.3 Experimental methodology

During experimental series, there is FC-72 flow along minichannels. The experiments are conducted in stationary state. Mass flow rate, temperature and pressure of the fluid at the inlet and outlet of the test sections, current intensity and voltage drop across the heater are monitored by the data acquisitions stations. Thermograms of the outer foil surface are measured due to infrared thermography.

Fig. 1. A view of the experimental stand.

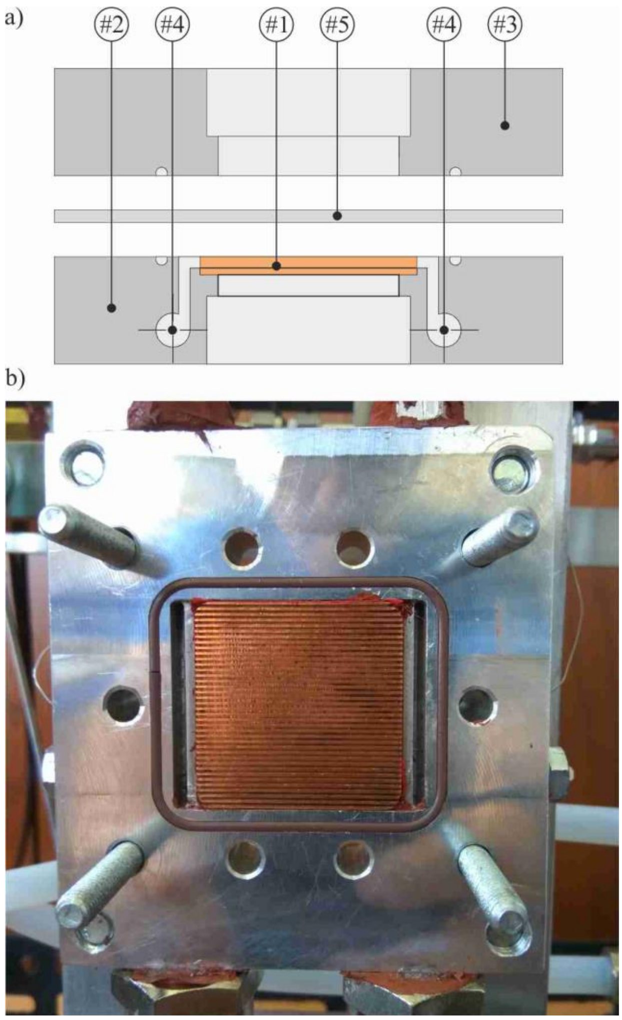

Fig. 2. The test section: a) a schematic diagram, #1 - a group of parallel minichannels, #2 - a channel body, #3 - a front cover, #4 - inlet/outlet chambers,#5 - a heated foil, b) a view.

5 Results and discussion

The calculations were performed using the model described in the previous section, with data collected during experiment conducted on the set up presented above.

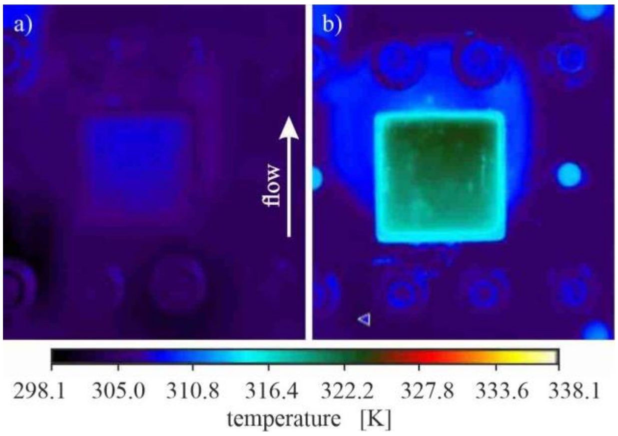

Fig. 3. Thermograms (infrared thermographic images) of the outer surface of the heated foil, for two values of the heat flux: a) q_w= 1.8 kW/m^2, b) q_w = 13.98 kW/m^2.

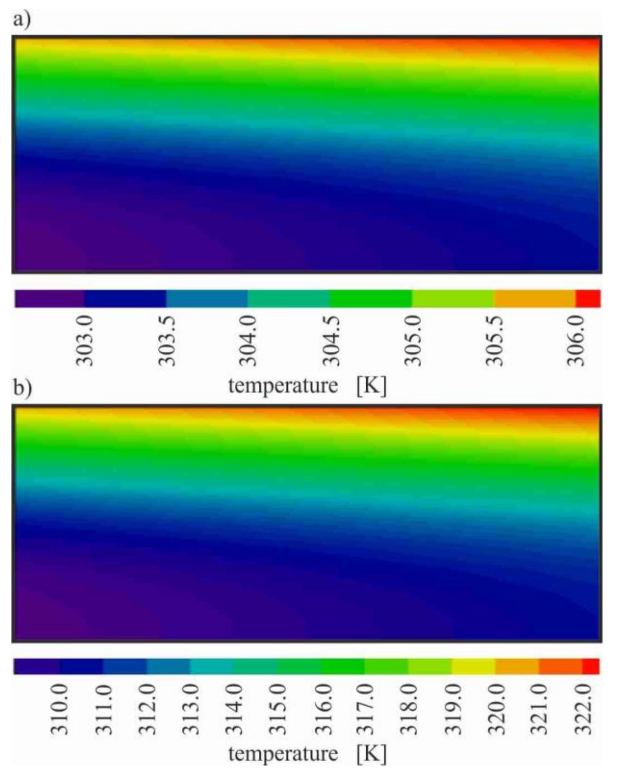

Fig. 4. Two dimensional fluid temperature distribution for two values of the heat flux: a) q_w= 1.8 kW/m^2,b) q_w = 13.98 kW/m^2.

The experimental parameters were as follows: average mass flux of 124 kg/(m^{2}s), average inlet pressure of 134.21 kPa, average inlet liquid subcooling of 31.58 K and 14 values of the heat flux in the range of 0.17 kW/m^2 ÷ 28.30 kW/m^2.

Figure 3 shows the thermograms of the outer foil surface, obtained from infrared thermography. Distribution of fluid temperature along the minichannel width are presented in Fig. 4.The heat transfer coefficient in the function of the distance from the minichannel inlet is illustrated in Fig. 5. It can be noticed that the heat transfer coefficient increased with the distance from the minichannel inlet for each heat flux values. The values of the heat transfer coefficient are in the range 0.53 ÷ 5.22 kW/(m^2 K).

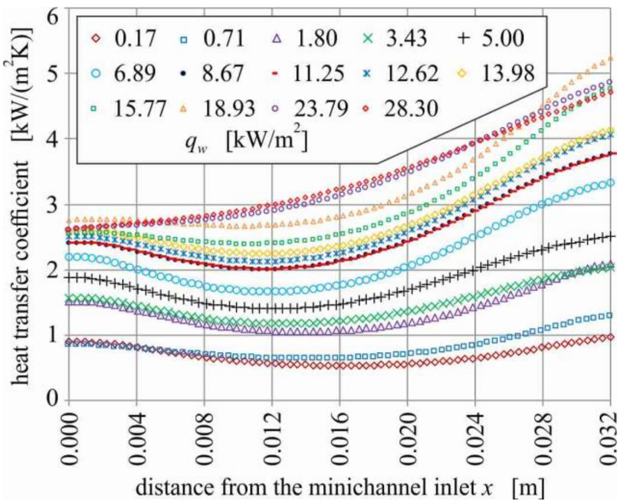

Fig. 5. The heat transfer coefficient vs. the distance from the minichannel inlet.

6 Conclusions

The two-dimensional mathematical model of flow boiling heat transfer in micro- and minichannels was proposed. Calculations with the use of Trefftz functions were based on the data from own experiments. The mathematical model described well the considered phenomenon of heat transfer in flow boiling in a single minichannel.

The Trefftz method allowed to effectively calculate two-dimensional temperature distributions in the channel's heated wall and in the flowing refrigerant. Obtained solutions satisfy the governing partial differential equation exactly.

The proposed numerical method was applied to DHCP and IHCP and the calculations are not overly complicated – the Trefftz functions are, in this case, polynomials. The Trefftz method is not limited by the number or the type of boundary conditions: temperature-related, flow-related, discrete or continuous. To verify the proposed mathematical model data from experiments were applied in calculations. The central minichannel of the group of parallel minichannels in a minichannel heat sink was selected for consideration. The heated element for Fluorinert FC-72 flowing along minichannels was a thin foil. The temperature of its outer side was measured using infrared thermography. The resulting graphs presented thermograms of measured temperature on outer surface of the heater, temperature distributions of fluid temperature and local values of the heat transfer coefficient. It was noticed that the heat transfer coefficient increased with the distance from the minichannel inlet for each heat flux value and achieved values up to 5.22 kW/(m^2K).

Acknowledgments

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