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Quarter wavelength resonator in experiment and simulation

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Abstract. Quarter wavelength resonator provides a mean to create a pressure drop at a specified frequency. Design of hollow cylinder connected radially to a pipe was used and different lengths of this tuned resonator were measured. Correlation with computational approach using finite element method analysis (FEM) with ANSYS acoustics capabilities was achieved. Analytical formula for a length of a quarter wavelength resonator is compared to an experimental measurement data and simulation.

1 Introduction

Pressure oscillations, noise propagation and vibrations in piping systems are problems that will require for a foreseeable future effort of engineers so the machinery will not suffer from unneeded strain. These problems might be more or less complicated to solve, depending on a source of these pulsations. Self-excited vibrations are difficult kind of these problems since the change in a system might only lead to a change of frequency. [1]

Some of the problematic vibrations can be silenced using mufflers and resonators.[2] Resonance of acoustics cavities was studied for example in [3] in relation to various length and cross-section ratios.

This contribution aim is description of dynamic properties of short quarter wave resonators specifically resonators of geometry of a hollow tube installed on another tube so the findings could be further used in practice for example on pipeline systems, pneumatic, hydraulics, gas lines and so on. In process there is also motivation to validate simulation abilities for such purpose.

2 Experiment

Experiment was set up to measure preselected lengths of resonator tube. Basic description of a measurement stand was following: Two meter long tube was split in half and the quarter wave resonator was connected in this opening. The opening was constant for all measured lengths of resonator. Speaker was chosen to excite the air in a tube and to measure the response the set of four microphones was positioned along the length of tube. Main dimensions of this experimental quarter wave resonator system are listed in Tab. 1 and depicted in Fig. 3.

Motivation for a choice of tube length was to have size for at least three wavelengths along the length of tube on a studied frequency. Clearly this self imposed rule limits the lower boundary of studied frequencies. Reasoning behind dimension of resonator was in a way similar since the length of resonator is inversely proportional to its natural frequency. The shortest length of resonator is double of a opening into a tube. Other lengths were arbitrary chosen to cover spectrum of these short quarter wave resonators. Opening size makes it wide enough to not to create a resistance for a acoustic wave. Chosen diameter of a resonance cavity provides resonator cross-section close to a half of a tube cross-section.

Three main variations were measured. One was tube without resonator, for the purpose of reference and result processing. Two other consisted of change of opposite end of tube it was either closed or opened. Specified quarter wave resonator length were tested in a spectrum of frequencies. Process was to excite a specific frequency, mea-

Table 1. Summary of dimensions.

Parameter	Symbol [Unit]	Value
overall length	L_{tube} [m]	2.012
tube inner diameter	D_i [mm]	53.6
cavity outer diameter	D_{ro} [mm]	76.6
cavity inner diameter	D_{ri} [mm]	63
resonator opening	w [mm]	12
wall thickness	s [mm]	6.7
resonator lengths	L [mm]	24; 29; 35; 44; 54; 74; 94

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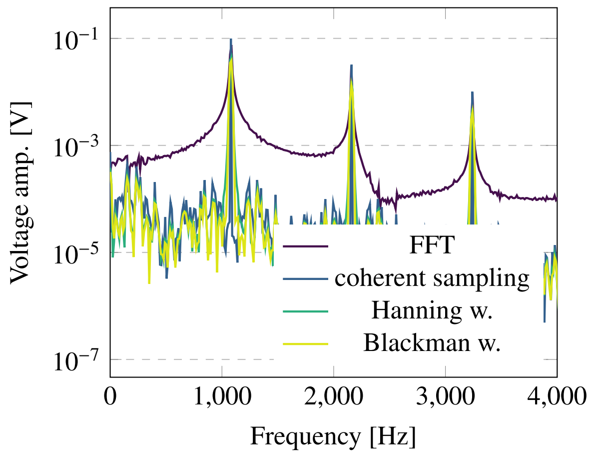

Fig. 1. Processed data.

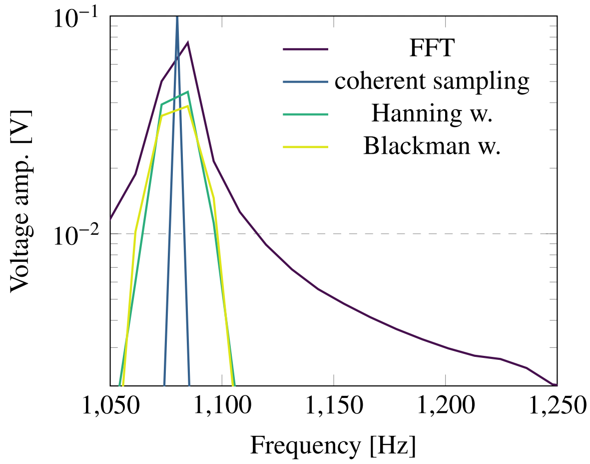

Fig. 2. Processed data, peak value detail.

sure and move to next one. This was repeated for all respective lengths of resonator and end of tube variations.

Excitation was equal through whole measurement and the loudspeaker was generating sinusoidal wave on an appropriate frequency. Frequency was upped by 20 Hz between steps and measured interval was from 100 to 8000 Hz. Sampling frequency of 108 kHz was used for a two second measuring time. Unspecified pause to system stabilization proceeded this data acquisition.

2.1 Data processing

Format of gathered data was time sampled amplitude of sensor voltage. Fast Fourier transform is needed to get amplitude-frequency characteristics from sensor recording. There is number of ways to further process data to get spectral space data readings with high level of accuracy and significance.[4] On data sets captured from aforementioned measurement there were tested four approaches to evaluate their suitability. Raw data from FFT, Hanning window, Blackman window and coherent sampling.

IEEE norm 1241 states formula (1) to be used for coherent sampling, where f_i is frequency of interest, f_s is sampling frequency, M is record length and J is integer relatively prime to M. Relative prime is defined as relation between two numbers with greatest common division of one. [4]

$$f_i = \frac{J}{M} \cdot f_s \tag{1}$$

In Fig. 1 comparison between four methods of data processing is presented. Two parameters of adequacy of method is considered. Spectral leakage and signal loss occurs and depending on further data use the appropriate method choice differs. Both mentioned effects on data are visible in Fig. 2. Clearly the coherent sampling method provides the least loss of signal and its frequency bin division provides lowest spectral leakage in comparison to the rest of tested methods. This reasoning justifies the choice of coherent sampling as suitable tool to process measured data.

Experimentally obtained data were compared to analytical formula (2) for selected resonator lengths. This is depicted in Fig. 4. Second and third shortest resonator lengths show bit of a difference to analytical value, but overall the measurement data fit very nicely with theory.

$$f = \frac{a}{4 \cdot L} \tag{2}$$

3 Simulation

To simulate quarter wave resonator FEM ANSYS package with acoustic capabilities was chosen. First difference to experiment was use of Harmonic analysis, so the simulation was performed in a frequency domain. This approach is valid since the simulated effect of resonator is response to a frequency at which it is excited by a loudspeaker.

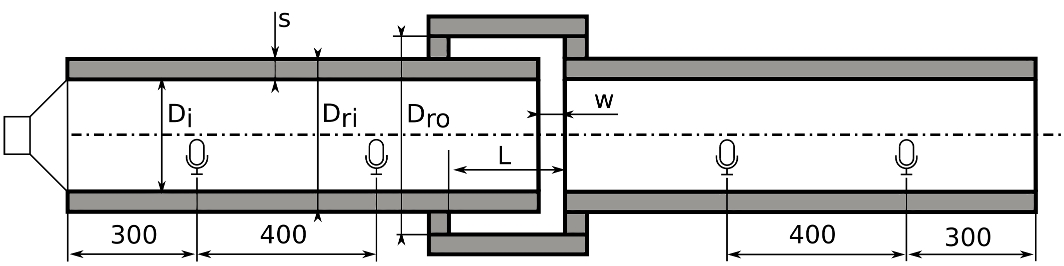

Fig. 3. Measurement apparatus schematics with positions of microphones installed in tube (dimensions are not up to scale to better illustrate geometry.

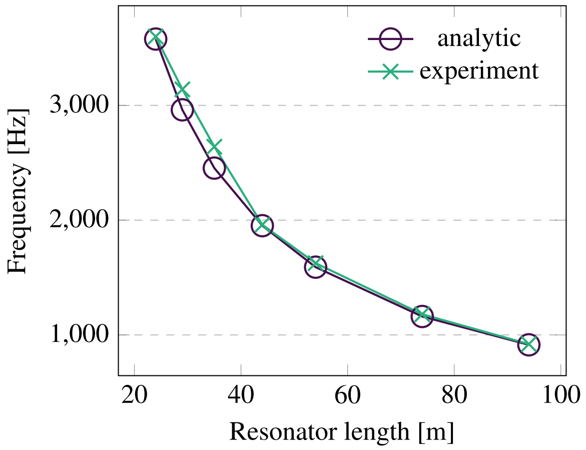

Fig. 4. Measured frequencies and analytically obtained values.

Geometry of simulated domain matches the experimental setup (as depicted in Fig. 1) but it is limited to internal volume. Tube walls are thick compared to diameter and the effect of resonator is dominant process order of magnitude stronger than effects of wall deformations and vibrations. So the structural part was not simulated and the computational domain consisted of acoustic body only.

Tube ends get replaced by a boundary conditions, harmonic pressure excitation substitutes loudspeaker and on other end either rigid wall or non reflective radiation boundary condition (open ended case). Rest of domain surfaces have default boundary condition of rigid wall. Parameters of acoustic medium are chosen to match experimental environment. Speed of sound is defined using formula (3) with temperature being 20°C, this was also used to calculate natural frequencies for resonators in Fig. 4 using formula (2). Values of media properties are stated in a Tab. 2.

Table 2. Simulation parameter settings.

Parameter	Symbol [Unit]	Value
density	ρ [kg$\cdot$ m^{-3}]	1.225
sound speed	a [m $\cdot$s^{-1}]	343.5
viscosity	ν [Pa $\cdot$s]	$2 \cdot 10^{-4}$
bulk viscosity	ν_b [Pa $\cdot$s]	0.3

$$a = 331.6 \cdot \sqrt{1 + \frac{t}{273.15}} \tag{3}$$

Mesh size of a ten thousand quadratic HEX20 elements was used to meet recommended number of elements [5] (6 quadratic elements in acoustic domain per wavelength) and further mesh density increase did not change evaluated parameters.

In Fig. 5 correlation between experimental and simulated data is visible. Experimental values are represented only by a natural frequency for respective resonator length and simulation of quarter wave resonator is presented by a acoustic pressure damping interval across frequency range. There is also a visible downward trend of acoustic pressure response with ascending frequency. This is caused by a frequency independent bulk viscosity and the reference of simulated damping by a bulk viscosity in a tube without resonator can be seen as a red dotted curve. In this case loudspeaker was substituted by a acoustic pressure excitation with amplitude of 1 kPa.

4 Transfer matrix

Simulated case provides velocity and pressure field needed for determination of another dynamic property of quarter wave resonator. Transfer function, which can be used to model such system using Transfer Matrix Method (TMM). TMM in its simplest form consist of two state vectors and transfer matrix, that enables to a variable from one state vector to change its value to the corresponding value of respective variable in second state vector - formula (4).

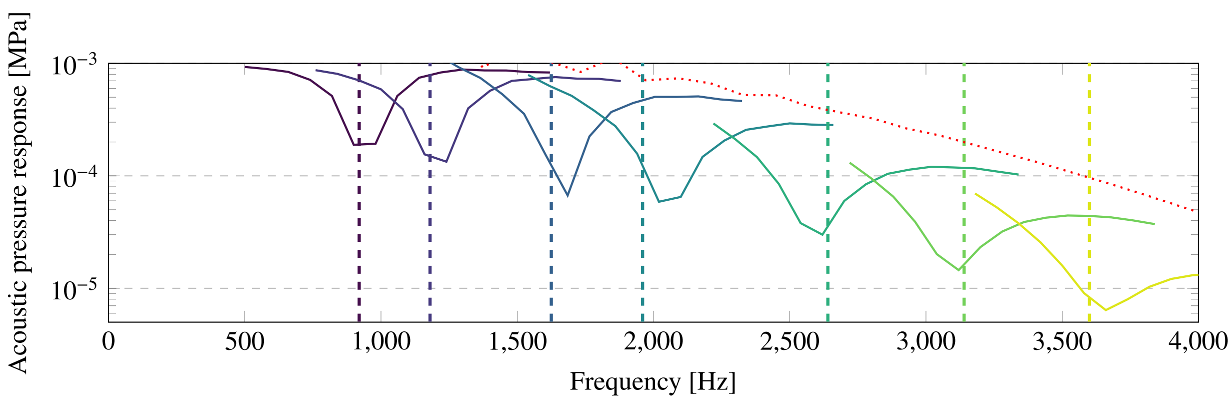

Fig. 5. Natural frequencies of resonators evaluated from experiment (dashed line) compared to computed acoustic pressure response to excitation at opposite end of tube (solid lines). Color pairs are to match same resonator lengths.

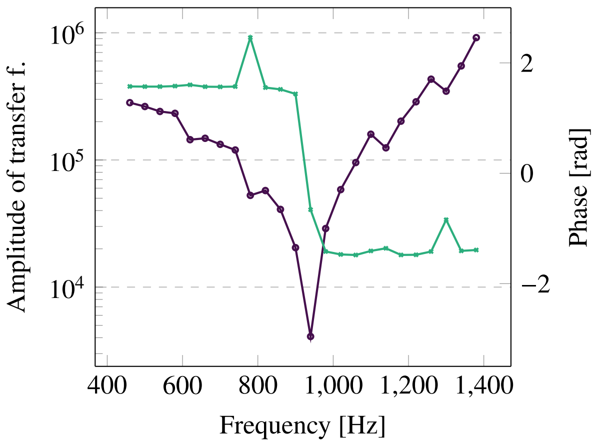

Fig. 6. Transfer function.

$$\begin{pmatrix} a \\ b \end{pmatrix}_{i+1} = \mathbf{P}_T \begin{pmatrix} a \\ b \end{pmatrix}_i \qquad (4)$$

Presented case can be written as state vector of flow and pressure and transfer matrix consisting of one variable - formula (5). This assumes pressure change of state vectors to be zero. For purpose of finding the transfer function, the state vectors are set at positions at cross-section of before and after the opening of the resonator into the tube. Assumption of zero pressure change is not completely true, but compared to velocity difference the pressure change is much smaller. This is in agreement with knowledge that in resonator there is node of velocity oscillation and anti-node of pressure at closed end and vice versa for open end - the opening into the tube. All variables in this equation are complex numbers.

$$\begin{pmatrix} Q \\ p \end{pmatrix}_{i+1} = \begin{pmatrix} 1 & X \\ 0 & 1 \end{pmatrix} \begin{pmatrix} Q \\ p \end{pmatrix}_i \qquad (5)$$

By using eq. 5 transfer function X can be solved. Inputs are flow and pressure field on specified surfaces at opening of resonator and then the amplitude and phase can visualised in chart with x axis being frequency. In Fig. 6 There is visible transfer function amplitude extreme at frequency that is unsurprisingly the natural frequency of simulated resonator. Phase change can be found at the same position as well.

5 Conclusion

In this contribution measurement of quarter wave resonator was studied and there was a focus on specific geometry of resonators. Dimension of length was also defining factor for this contribution, namely that respective resonators were chosen to be short. Chosen lengths of were measured and acquired data were processed using various techniques to obtain frequency and amplitude of acoustic waves. This testing concluded that coherent sampling provides best result in terms of spectral leakage and signal loss.

In next step the Finite Element Method simulation had shown that it is possible to model damping system of quarter wave resonator with a acceptable level of precision. Fields of acoustic pressure and flow allows to further study dynamics of this setup and so the transfer matrix method was used to quantify transfer function for a interval of measured frequencies. Considering the equation (5) the transfer function amplitude describes the effect of change between selected positions of defined state vectors. So the lowest value of transfer function matching the natural frequency of resonator is in agreement with a measurement where at this frequency the least energy of acoustic waves is transferred between the excited and opposite tube's end.

These findings are interesting with respect to a possible use of quarter wave resonator to damp and suppress acoustic waves, vibrations or to reduce noise. Advantages of such resonator are price, since it is only passive element and also very low pressure drop since it does not create obstruction for a fluid flow.

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