Modelling of multiphase solid-liquid laminar flow with non-Newtonian carrier in pipelines using a layered model

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Abstract. This paper discusses two-phase laminar flows of mixtures of solids in non-Newtonian carrier consisting of spherical particles in a 50-mm pipe at the Czech Technical University in Prague. Special attention is paid to the frictional head loss and thickness of transport layers. For the prediction of various transport characteristics, a layered model is used. Suitability of the layered model to predict the transport characteristics is investigated. Prediction results are compared to own experimental data.

1 Introduction

Many slurries of industrial interest, such as mixtures of thickened tailing, long distance hauling of coal, minerals, ore and solid commodities or material processing, are mixtures of larger particles carried by a medium that exhibits non-Newtonian behaviour. Due to the high viscosity of the non-Newtonian carrier fluid, laminar flow regime may occur during transport industrial processes. As there is no other mechanism apart from particle collisions, slurries are generally stratified, with sliding bed formed by the coarser particles.

It has been well demonstrated in literature, that coarse particles, which would be under stationary conditions fully suspended in the carrier fluid due to the fact that submerged weight of the particles would not be enough to balance out the yield stress of the carrier fluid, still form the sliding bed during flow conditions, as the local viscosity of the sheared carrier fluid drops, allowing the coarse particles to settle down to the pipe invert (Cooke 2002, Pullum and Graham 2000). [1, 2]

This paper focuses on laminar steady-state flows of mixtures formed by non-Newtonian carrier fluid of Herschel-Bulkley rheological type with coarse particles tending to form stratified flow regime in a horizontal pipeline.

Mixtures of Carbopol Ultrez 10 Polymer with virtually monodisperse glass beads were used to obtain an experimental database containing runs with various rheological properties and various concentrations of solids.

Based on experimental measurements authors evaluate mechanisms that affect the height of the sliding bed layer and present their own empirical equation for layer height prediction. Own experimental measurements are also used to evaluate the laminar two-layer flow model described by Matoušek 2015 [3]. Predicted pressure gradients are compared to measured pressure gradients and discussed.

2 Experimental modelling and set up

In order to evaluate the mechanisms that affect the height of the sliding bed layer, 7 test series with total number of 50 data points in laminar flow regime were carried out in 50 mm pipe loop at the Czech Technical University in Prague. As an analogue to the slurries of industrial interest glass beads (TK15) were used to simulate coarse particles in carrier fluid represented by aqueous solutions of Carbopol polymer (CBP).

A typical experimental run involved measuring of delivered concentration of particles, pressure drop for a set of flow rate, visual observation of the particle flow patterns and set of rheological measurements using carrier fluid samples from before and after experimental run.

2. 1 Carrier fluid

Carbopol Ultrez 10 polymer is a acidic powder of particle size ranging from 2 to 7 microns, which aftrer dispersion in water forms a non-Newtonian visco-plastic fluid of the Herschel-Bulkley (HB) rheological type behaviour. In order to evaluate the viscosity effect on particle flow patterns, diverse solutions of CBP were used for every test run. Rheological properties for test runs are shows below in Table 1. The density of CBP solutions were takes as $\rho_f = 1000\ \mathrm{kg.m^{-3}}$. The diverse rheological properties of the carrier fluid were achieved by changing the mass concentration of CBP powder in water.

The rheology of the carrier fluid was measured in the rotational viscometer (Haake VT550 with a standard sensor) before and after the test series along with the fluid temperature to make sure that the rheological properties of the carrier fluid remained unchanged during the test series.

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Table 1. Rheological properties of CBP solutions.

Test run no.	τ_y	K	n
1	3.76	2.03	0.48
2	1.55	1.16	0.52
3	1.40	0.98	0.55
4	0.42	0.85	0.52
5	0.35	0.61	0.56
6	1.73	0.69	0.59
7	0.18	0.60	0.57

2.2 Particles used in experiments

Coarse glass beads (TK15) were used as an analogue to the coarse particles in thickened tailings handled in the mineral industry. The particles of TK15 are practically mono-disperse with d_{50} = 1,55 mm and the density ρ_s = 2488 kg.m^{-3}.

2.3 Experimental rig

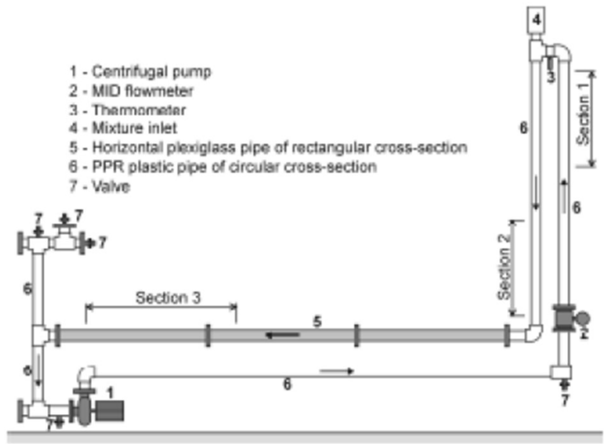

Fig. 1. The experimental rig.

The test series were carried out in Water Engineering Laboratory of the Czech Technical University in Prague (CTU). A pipe loop (Fig. 1) was used to study the slurry flow behaviour. The loop is composed of pieces of a PE pipe (I.D. 51.4 mm, blank pipe in Fig. 1) and a piece of transparent acrylic pipe (I.D. 50.0 mm, grey pipe in Fig. 1). Total length of the loop is 22.96 m and its volume is 45.08 l. The length of the horizontal section is 6.20 m. The pump EBARA 3M 40-200/7,5 kW is driven by an electric motor with a variable frequency converter TECO GD100-011G-4 11 kW. Pump parameters are: power 7,5 kW, impeller diameter 200 mm, maximum flow 11.67 l.s^{-1}, total head from 58 m to 44 m (valid for water for maximum flow). [3]

Differential pressures are measured over vertical Sections 1, 2 (1.3 m long) and the horizontal Section 3 (1 m long) using the differential pressure transducers Fischer Rosemount DP1151 (Sections 1 and 2) and the transducer Siemens Sitrans P DSIII (Section 3). An electromagnetic flow meter Krohne Optiflux 5000 was used to measure the flow rate in the vertical pipe mounted to the discharge outlet of the centrifugal pump. The temperature of the flowing medium was measured in the vertical invert pipe.

2.4 Visual observation of flow patterns

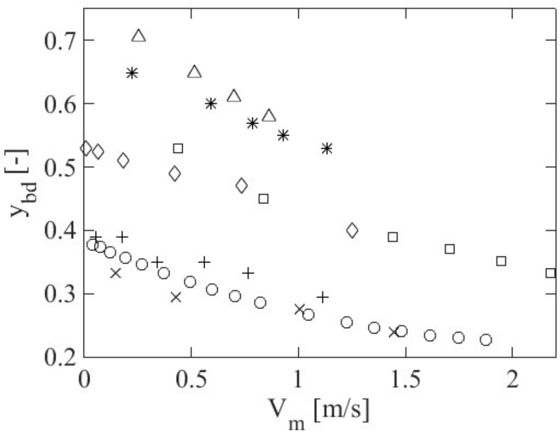

Fig. 2. Measured values of the sliding bed layer thickness y_{bd}. Legend: x – Run no.1; o – Run no.2; + – Run no.3; – Run no.4; □ – Run no.5; * – Run no.6; Δ – Run no.7.

Special attention was paid to the visual observation, as it provided the only means to determine the particle flow patterns at given mean velocity of the laminar flow.

At very low velocities, the thickness of the sliding bed could be determined very accurately. With increasing velocity however, a layer of rolling particles on top of the fully stratified sliding bed started to developed, which with further increase of the mean velocity transformed into shear layer of colliding particles, making the observation of the interfaces between layers much more difficult. The observation of the particle flow patterns ended either at the point of the transition into turbulent flow regime or when a human eye could no longer recognize the sliding bed layer, meaning that only collision shear layer remained under laminar flow conditions.

Camera (NIKON D5100) was used to capture the motion of the particles and millimetre scale, which was fixed on the transparent pipe was used to calculate the actual thickness of the sliding bed layer. It should be noted, that the thickness of the shear layer is not negligible and with increasing velocity, it can even become dominant in the cross section, while the flow is still under laminar flow regime. In Fig. 2 measured values of the sliding bed layer thickness y_{bd} are plotted against mean velocity of the flow V_m.

3 Analysis of sliding bed layer thickness

Thickness of the bed layer along the pipe invert is an important parameter for mathematical modelling of multi-phase slurry flow and for predicting parameters such as hydraulic gradient I_m or mean delivered concentration of solids C_{vd}.

Fig. 2 shows somewhat linear trend in dissipation of the sliding bed layer thickness, that seems not to be affected by the viscosity of the carrier fluid, as the runs no. 1-3 have very different rheological properties of the Carbopol solution, yet the trend seems to be fairly identical. However, the dissolution of the layer thickness seems to be affected by the volumetric concentration of solids in the cross section of the pipeline, as the dissipation of the layer thickness seems to be steeper when the initial thickness of the sliding bed layer is higher.

In order to predict the thickness of the sliding bed layer, following empirical equation 2 was derived based on the assumption, that the layer thickness dissolution in laminar flow regime of Herschel-Bulkley type of non-Newtonian carrier liquid is affected only by the initial bed thickness.

$$y_{bd} = \frac{y_b}{D} \tag{1}$$

Where D is diameter of pipe.

$$y_{bd} = \frac{y_{bd0}}{k} . V_m + y_{bd0} \tag{2}$$

Where k = -4.7 is the found constant and y_{bd0} is initial thickness of the bed layer, which is given by the volumetric concentration of particles in the rig.

Fig. 3 shows experimental data fitted with the predictions by equation 2. The agreement between the experimental data and predictions is very good, even though some of the experimental data runs could be fitted more accurately with 3-parametric curve. Authors however decided to use the linear fitting, as the nonlinearity of the bed thickness dissipation can be caused by the visual observation inaccuracy.

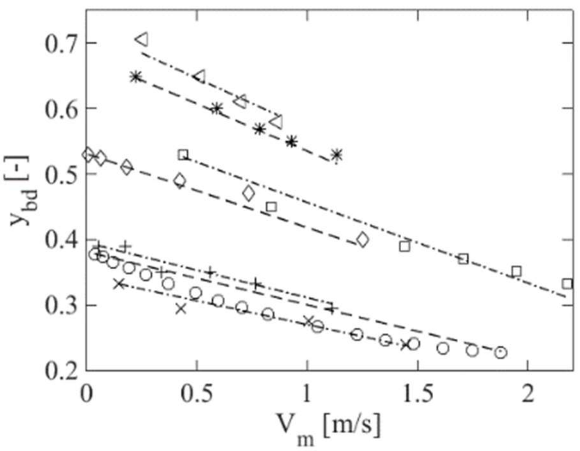

Fig. 3. Sliding bed layer thickness expressed as a function of initial thickness of the bed and mean velocity of the flow.

4. Predictive modelling using two-layer model

Various versions of layered model approaches are available in the literature, when Newtonian fluid is considered as a carrier. However, only a few non-Newtonian carrier based models have been discussed so far. Pullum et al. [4] presented a simple generic two-layer model for laminar and turbulent flow of power-law and visco-plastic carriers based on concept developed by Wilson [5]. This model uses empirical parameters and formulae based on comprehensive experimental database. Later Rojas and Saez [6] introduced and experimentally verified their version of two-layer model. This is a two-layer model to predict laminar and turbulent flows of mixture non-Newtonian fluid (Casson fluid) and dense and fine particles.

In this study two-layer model developed by Matoušek et al. was used, as in authors previous work [8], this model approach was used to calculate the value of deposition limit velocity (the mean velocity of the mixture at which the solid particles at the bottom of a pipeline start to form a stationary deposit) with reasonably accurate results.

Inputs of the predictive model are rheological parameters of carrier fluid τ_y – yield stress, K – consistency index, n – flow index, density of fluid – ρ_f and basic characteristic of particles – ρ_s, diameter of the particles – d, bed concentration – c_b, coefficient of mechanical friction between sliding bed and pipe wall μ and thickness of the sliding bed layer y_{bd}. In this study value of μ was set to the value of 0.38, based on the experimental investigation by Matoušek [3], bed concentration was taken as equal to 0.54 and the values of y_{bd} were predicted using the equation 2, where y_{bd0} was derived from the volumetric concentration of solids in the experimental rig.

The model predicts hydraulic gradient – I_m, delivered concentration of grains – C_{vd}.

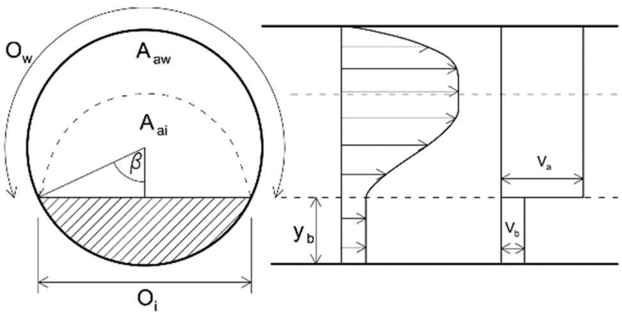

Fig. 4. Schematic of the two-layer model approach.

5. Prediction of the hydraulic gradient using two-layer model

In order to test the accuracy of the Matoušek [3] two layered model, experimental database consisting of 7 test runs with total number of 45 data points were used. 4 of these test runs were used to derive the equation 2, 3 new test runs were added to verify the equation 2 (see Table 2 for test run properties).

Table 2. Rheological properties of CBP solutions used for layered model predictions.

Test run no.	τ_y	K	n	C_{vi}
1	3.76	2.03	0.48	0.08
2	1.55	1.16	0.52	0.08
3	1.40	0.98	0.55	0.1
7	0.18	0.60	0.57	0.27
8	2.99	3.18	0.45	0.11
9	2.82	2.23	0.45	0.26
10	5.06	3.74	0.46	0.07

The results indicate, that the model predictions show good agreement with experimental data, with the highest error no more than 15%. The results also indicate, that for higher velocities near the threshold between laminar and turbulent flow regime the model calculations tend to underpredict the trend shown by experimental measurements (Fig. 5, 6). This can be caused by several reasons. Firstly, the deviation can be caused by a change in rheological parameters due to the rising temperature of the mixture, which was observed during the measurements. Secondly, the model takes in account only the contribution of the sliding bed layer to the resulting value of hydraulic gradient. Experiments however showed occurrence of two layers - sliding bed (bed layer) and a shear layer above it, which gets more dominant with higher flow velocities.

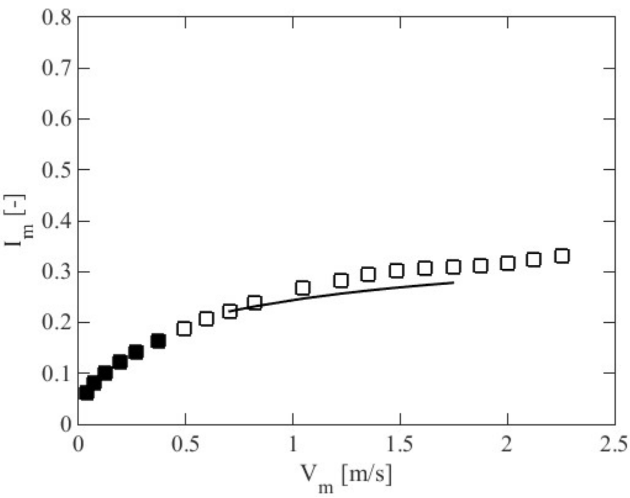

Fig. 5. Figure of hydraulic gradient predictions for run no. 2. Legend: filled squares – measured point in stationary bed, squares – measured points in sliding bed, continuous line – prediction.

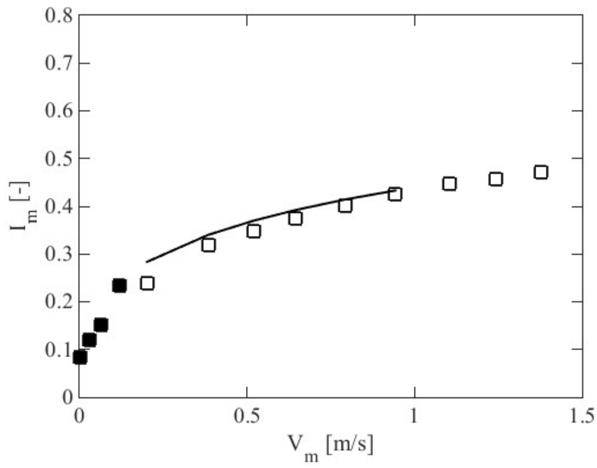

Fig. 6. Figure of hydraulic gradient predictions for run no. 8. Legend: filled squares – measured point in stationary bed, squares – measured points in sliding bed, continuous line – prediction.

Tendency of the model to underpredict the frictional losses for higher mean velocities can be also seen in a Fig. 7, where measured hydraulic gradient is plotted against predicted hydraulic gradient.

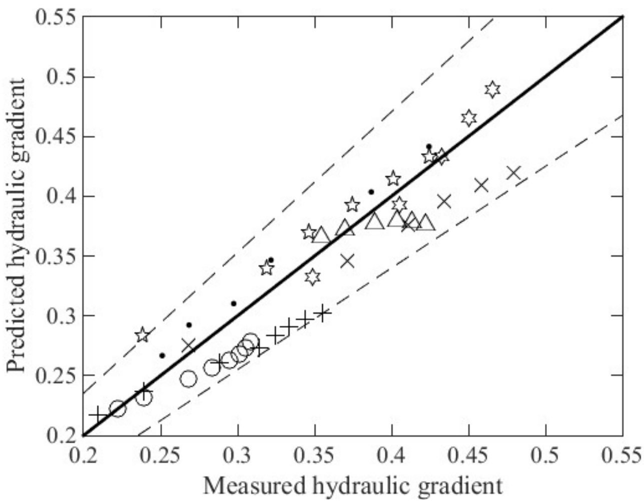

Fig. 7. Parity plot comparing observed and predicted hydraulic gradients. Legend: x – Run no.1; o – Run no.2; + – Run no.3; Δ – Run no.7; five-pointed star – Run no. 8; six-pointed star – Run no. 9; ▪ – Run no. 10; dashed line +- 15% quartile.

6 Conclusions

Experimental investigation of laminar steady-state flows of mixtures formed by non-Newtonian carrier fluid of Herschel-Bulkley rheological type with coarse particles tending to form stratified flow regime in a horizontal pipeline showed somewhat linear trend in dissolution of the sliding bed layer with increasing velocity. Based on experimental measurements, an empirical formulae predicting the thickness of the sliding bed in laminar flow was derived and used in two-layer predictive model to calculate frictional losses in horizontal pipeline.

The comparison of model predictions with experimental data showed good agreement with highest error equal to 15%. At higher mean velocities however the model predictions seem to underpredict the trend given by experimental measurements. These underpredictions may be caused by change in temperature of the carrier fluid and therefore slightly lesser viscosity in later stages of the experiments, but more likely the underpredictions show, that the contribution of the shear layer to the overall frictional losses cannot be neglected in calculations. The implementation of the shear layer contribution will be subject of the future work.

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