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On the possible origin of a vapour cone occurring during the transonic flight in moist air

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Abstract. In aerodynamics as in other sciences, there are some phenomena which are widely explained in a way that under a closer look turns out to be based on wrong assumptions and thus not very convincing. One of these is the effect referred to as vapour cone or shock collar which is the cloud of condensed water of the conical shape that forms around objects travelling at transonic speeds through the moist air. This paper aims to shed light on some basic principles that might stand behind this effect and based on the simple physical model provide possible explanation and correct some of the common misconceptions which are repeated ad nauseam not only on the internet sites of aerodynamics enthusiasts but also in some textbooks.

1 Introduction

Certainly, many people have been amazed by a similar photo as is shown in Figure 1 of the low flying F/A-18 Hornet or other fast aircraft surrounded by a strange-shaped cloud called *vapour cone* or *shock collar*. Such cloud is a product of an aircraft travelling at transonic speed in moist and warm air. The air which is in the relative motion with respect to an aircraft expands and eventually reach the supersonic speed - the local supersonic region closed by the strong shock wave is formed downstream of which the relative flow velocity is subsonic again. This paper aims to explain under which conditions this specific cloud might appear and why it disappears at higher supersonic speeds.

2 Atmospheric moist air properties

Homogeneous atmospheric air could be considered as an ideal gas. The atmospheric air is a mixture of dry air and humidity. All humidity in the homogeneous moist air is present in the form of water vapour. Based on the thermodynamics of moist air it is possible to determine the parameters on the saturation line beyond which the water condensate and forms water droplets (mist) or deposits to solid particles (snow, frost, rime, etc). Curves of saturated moist air for different values of specific humidity x in the $p - t$ diagram are shown in Figure 2 [1]. In this diagram the quality of the moist air expressed in terms of specific humidity is apparent for an arbitrary point - see Figure 3 and Table 1.

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Fig. 1. F/A-18 Hornet jet fighter travelling at transonic speed in moist air (U.S. Navy photo by Ensign John Gay).

3 Simplified aerodynamic model

To describe processes in the flow field around a flying aircraft the one-dimensional model of the compressible fluid flow, representing the stream tube in the three-dimensional flow field is used. The energy balance equation for the referred model includes only two forms of energy - enthalpy and kinetic energy. Assuming ideal gas model the energy equation can be written as

$$c_p T + \frac{v^2}{2} = c_p T_0 = const., \tag{1}$$

Table 1. Quality of the selected states of the moist air for three different values of the specific humidity x (see Figure 3).

State	$x = 0.001\ \frac{kg_V}{kg_{DA}}$	$x = 0.01\ \frac{kg_V}{kg_{DA}}$	$x = 0.1\ \frac{kg_V}{kg_{DA}}$
A	Heterogeneous: saturated moist air containing solid phase of H_2O	Heterogeneous: saturated moist air containing solid phase of H_2O	Heterogeneous: saturated moist air containing solid phase of H_2O
B	Homogeneous: unsaturated moist air	Heterogeneous: saturated moist air containing solid phase of H_2O	Heterogeneous: saturated moist air containing solid phase of H_2O
C	Homogeneous: unsaturated moist air	Heterogeneous: saturated moist air containing liquid phase of H_2O	Heterogeneous: saturated moist air containing liquid phase of H_2O
D	Homogeneous: unsaturated moist air	Homogeneous: unsaturated moist air	Heterogeneous: saturated moist air containing liquid phase of H_2O
E	Homogeneous: unsaturated moist air	Homogeneous: unsaturated moist air	Homogeneous: unsaturated moist air

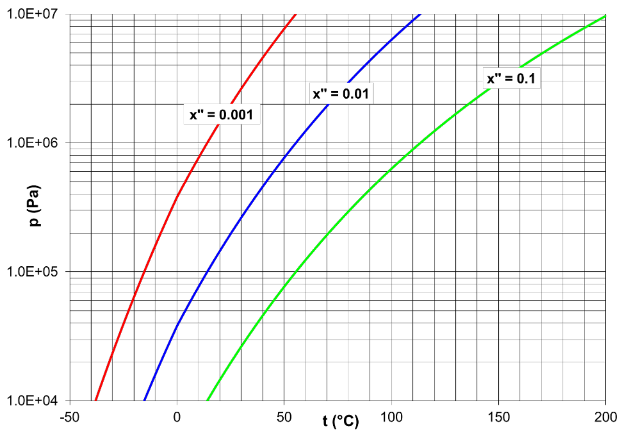

Fig. 2. Saturation lines in $p - t$ diagram for the moist air and different values of specific humidity x [1].

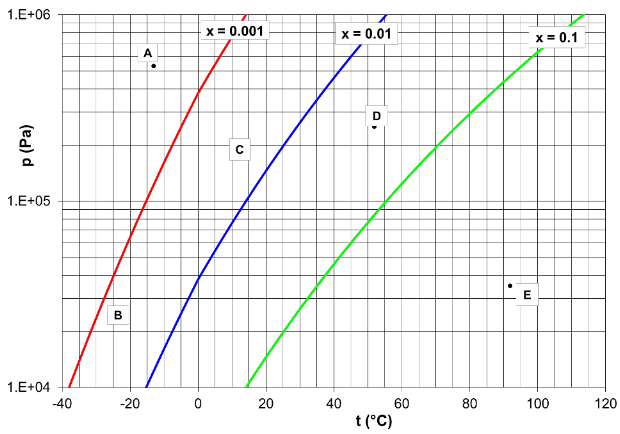

Fig. 3. Saturation lines in $p - t$ diagram for the moist air and different values of specific humidity x. Different states are marked and its quality is described in Table 1.

where c_p (J·kg^{-1}·K^{-1}) is the specific heat capacity at constant pressure, T (K) temperature, T_0 (K) stagnation temperature and v (m·s^{-1}) velocity. It is evident from equation (1) that for higher flow velocities v the temperature T decreases and vice versa.

4 Calculation of the moist air parameters using simplified model

Video records of such events as can be seen in Figure 1 suggests that the favourable conditions for observations usually takes place in a warm maritime climate and when the wind is calm. Parameters of the moist air under these conditions might be as follows: atmospheric pressure p_a = 0.1 MPa, temperature $t_a = 30\,°C$ and relative humidity $\varphi = 70\%$ i.e. specific humidity $x = 0.0019\,kg_V/kg_{DA}$, speed of sound $a = 350.7$ m·s^{-1}, specific heat capacity at constant pressure $c_p = 1020.5$ J·kg^{-1}·K^{-1} and the ratio of specific heats $\gamma = 1.397$.

Assuming an isentropic process, the moist air would saturate at the pressure approximately $p_a = 92.1$ kPa and the temperature $t = 23\,°C$ (see Figure 4).

For a given Mach number M, the speed of the aircraft is

$$v = Ma, \tag{2}$$

difference of the stagnation and atmospheric temperature

$$\Delta t = \frac{v^2}{2c_p}, \tag{3}$$

and thus the stagnation temperature

$$t_0 = t_a + \Delta t. \tag{4}$$

For an idealised model of isentropic fluid flow the stagnation pressure is given by equation

$$p_0 = p_a \left(\frac{T_0}{T_a}\right)^{\frac{\gamma}{\gamma-1}}. \tag{5}$$

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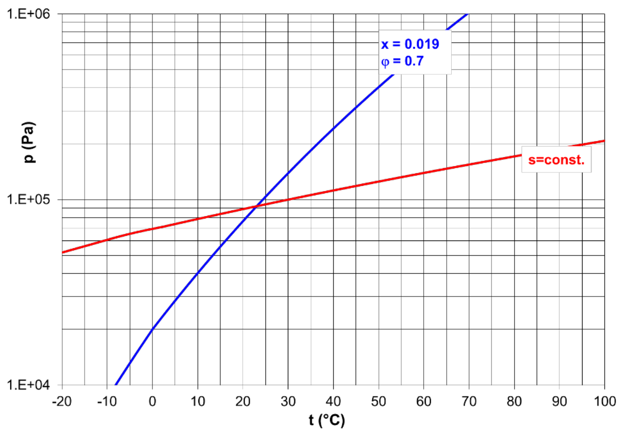

Fig. 4. $p - t$ phase diagram of the moist air with the specific humidity $x = 0.001\,\text{kg}_\text{V}/\text{kg}_\text{DA}$ and the isentrope (s = const.) crossing the point of the state $t_a = 30$ °C , $p_a = 0.1$ MPa and $\varphi =$ 70%.

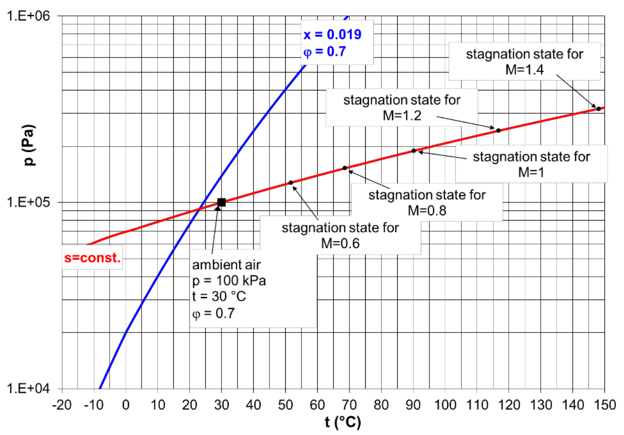

Fig. 5. $p - t$ phase diagram of the moist air with the specific humidity $x = 0.0019\,\text{kg}_\text{V}/\text{kg}_\text{DA}$ and the isentrope (s = const.) crossing the point of the state $t_a = 30\,°\text{C}$, $p_a = 0.1\,\text{MPa}$ and $\varphi = 70\%$ with the marked stagnation states for the selected values of the aircraft Mach number.

Figure 5 pictures the phase diagram of the moist air with marked stagnation states corresponding to different Mach numbers for atmospheric conditions ($p_a = 0.1$ MPa, $t_a = 30\,°\text{C}$, $\varphi = 70\%$).

Table 2 shows for various Mach numbers of the aircraft the Mach numbers of the flow for which the air in the stream tube would become saturated under the assumption of the isentropic process.

In Figure 6 we can see the dependence of the temperature of the moist air upon the Mach number for the case as described in Table 2. The temperature of the atmospheric air as well as the dew point for pressure and relative humidity considered are indicated.

5 Flow regimes

In the case of the isentropic flow past an aircraft (or an aerofoil) the air might possibly reach the state in which the air is saturated or even over saturated as it expands (accelerates) along the body and its conveniently shaped surface features such as canopy and wings. During the subsequent deceleration the air eventually returns back to unsaturated state.

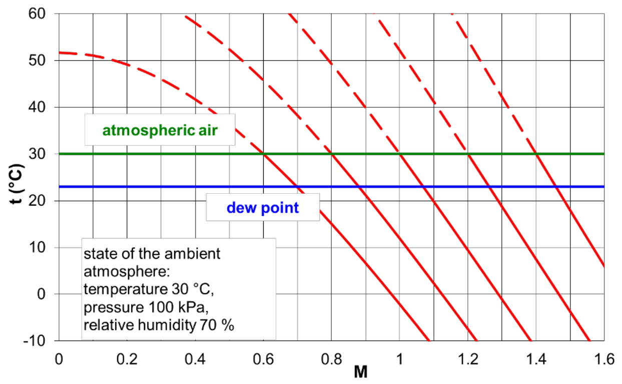

Fig. 6. Dependence of the moist air temperature t upon the aircraft Mach number M assuming isentropic flow conditions (red lines).

The flow past an aircraft can be isentropic only if the velocity anywhere in the flow field is subsonic. This occurs when the aircraft Mach number is lower than critical Mach number $M_{\text{crit.}}$, which depends on the exact shape of an aircraft and is typically in the range $M_{\text{crit.}} \in (0.8; 0.9)$.

When the speed of an aircraft is above the critical Mach number $M_{\text{crit.}}$ but lower than unity, the region where the flow exceeds sonic speed emerges together with the shock-wave at the rear part of this region. Such flow regimes are referred to as *transonic*. If an aircraft further accelerates the supersonic region will grow in size and the rear shock will travel towards the trailing edge. The shock waves in transonic flow fields might be considered as normal shock waves in one dimensional model of stream tube used in this work. When the flow passes the shock wave the pressure and temperature increases suddenly and the Mach number drops. Downstream the shock wave the flow is isentropic again.

If the speed exceeds certain Mach number over unity, the attached shock wave forms at the leading edge causing decrease in the flow velocity, however, the speed downstream the shock wave might be still supersonic. In this case the flow along the aerofoil is isentropic and expands until the exit shock wave knocks down the speed to the value upstream the aerofoil. Entire flow field is supersonic in this case.

6 Change of flow parameters across the shock wave

The change of pressure and temperature across the shock wave depends on the Mach number at which the shock wave occurs. In the case of normal shock wave the following relations apply

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1} M_1^2 - \frac{\gamma-1}{\gamma+1}, \tag{6}$$

Table 2. Table view of the results shown in Figure 6 including Mach number at dew point for different values of aircraft Mach number M.

$M\,(-)$ aircraft Mach number	0.6	0.8	1	1.2	1.4
$v\,(\mathrm{m{\cdot}s^{-1}})$ aircraft speed	210.4	280.5	350.7	420.8	491.0
$\Delta t\,(^\circ\mathrm{C})$ difference between total temperature and ambient atmospheric temperature	21.69	38.56	60.25	86.76	118.09
$t_0\,(^\circ\mathrm{C})$ stagnation temperature for given aircraft Mach number and atmospheric temperature	51.69	68.56	90.25	116.76	148.09
$p_0\,(\mathrm{Pa})$ stagnation pressure for given aircraft Mach number and atmospheric pressure	127501.4	152342.2	189141.5	242268.5	317910.2
$M_{\mathrm{DP}}\,(-)$ Mach number at dew point	0.698	0.880	1.069	1.262	1.458

$$\frac{T_2}{T_1} = \frac{\left(1 + \frac{\gamma-1}{2}M_1^2\right)\left(\frac{2\gamma}{\gamma-1}M_1^2 - 1\right)}{\frac{(\gamma+1)^2}{2(\gamma-1)}M_1^2}. \tag{7}$$

Table 3 shows the flow parameters upstream and downstream of the normal shock wave for two selected supersonic values of aircraft Mach number. Figure 7 depicts the states downstream the normal shock wave in the $p - t$ phase diagram of the moist air. Corresponding stagnation pressure and temperature are also shown.

It is obvious from the foregoing paragraphs that for sufficiently high relative humidity of the air the favourable conditions for condensations might take place even for subsonic flow. However, due to the shock free flow, the condensation is not suddenly ended by the shock wave.

In the case of transonic flow when the aircraft Mach number is above critical Mach number $M_{\mathrm{crit.}}$ but below unity, the condensation most likely takes place in the regions where the expansion is high enough to exceeds the dew point temperature. That is in the supersonic regions which are terminated by the shock waves. Across the shock waves the conditions suddenly change and prevent further condensation.

When the aircraft Mach number is above unity, the shock wave forms upstream the body. Across the shock wave the temperature and pressure rise and the relative humidity drops (see Figure 7). This causes a significant shift of the state downstream the shock wave further from the dew point temperature and thus from the conditions favourable for condensation. It is the reason why the vapour cone is much less likely to appear while the aircraft speed is supersonic.

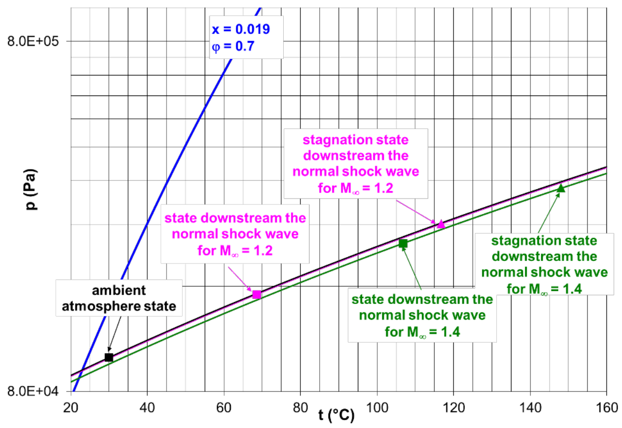

Fig. 7. $p - t$ phase diagram of the moist air with the specific humidity $x = 0.0019\,\mathrm{kg_V/kg_{DA}}$ and the isentrope (s = const.) crossing the point of the state $t_a = 30\,^\circ\mathrm{C}$, $p_a = 0.1$ MPa and $\varphi = 70\%$ with the marked state downstream the normal shock wave and its corresponding stagnation state for aircraft Mach numbers $M_\infty = 1.2$ and 1.4.

7 Amount of water condensed

The amount of condensed water depends upon the atmospheric conditions and the maximal Mach number achieved on the aircraft surface.

The maximal Mach number on the aircraft surface depends upon its exact shape. For the simplicity we might consider the flow of an inviscid fluid about an ellipsoid of revolution [2]. Thick line in Figure 8 shows an incremental velocity as referred in [2] for an ellipsoid of revolution with an aspect ratio $AR = 0.3$. The dashed line shows extrapolated values to higher velocities of the free stream.

For the case of ambient atmospheric conditions considered in foregoing paragraphs (atmospheric pressure $p_a = 0.1$ MPa, temperature $t_a = 30\,^\circ\mathrm{C}$ and relative humidity $\varphi = 70\%$) the maximal Mach number achievable

Table 3. Flow properties across the normal shock wave for two selected supersonic upstream Mach numbers.

M (–) Mach number	upstream shock-wave 1.2	downstream shock-wave 0.84	upstream shock-wave 1.4	downstream shock-wave 0.74
v (m·s^{-1}) speed	421	312	491	290
p (MPa) static pressure	0.1	0.152	0.1	0.212
t (°C) static temperature	30	68.59	30	106.79
p_0 (MPa) stagnation pressure	0.242	0.241	0.318	0.305
t_0 (°C) stagnation temperature	116.76	116.76	148.09	148.09

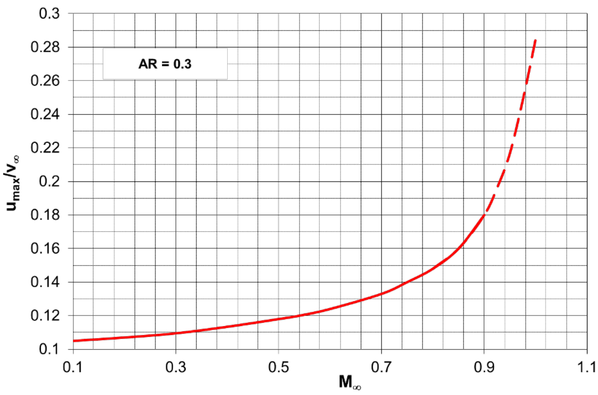

Fig. 8. Graph of the maximum incremental velocity as a function of free stream Mach number M_∞ on an ellipsoid of revolution of the aspect ratio $AR = 0.3$.

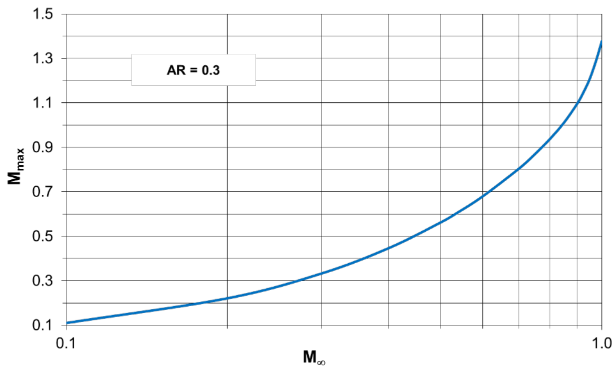

Fig. 9. Graph of the maximum Mach number M_{max} as a function of free stream Mach number M_∞ on an ellipsoid of revolution of the aspect ratio $AR = 0.3$.

on the ellipsoid surface can be read from the graph in Figure 8. The graph of the maximal Mach number with respect to free stream Mach number is pictured in Figure 9. To each value of maximal Mach number the corresponding value of moist air temperature and pressure can be attributed and thus also the value of maximal specific humidity $x''_{M_{max}}$. The difference between the maximal specific humidity $x''_{M_{max}}$ which is in the region of maximal velocity around the object and the specific humidity of the ambient air x'' (in our case $x'' = 0.019\,\mathrm{kg_V/kg_{DA}}$) determines the amount of water which can condensate $x_{cond.}$ for given conditions of the ambient moist air.

If the Mach number of an aircraft is above unity $M > 1$, the shock wave forms in front of it. Assuming the normal shock wave, the Mach number downstream the shock wave lower than unity can be calculated. Then the flow past the object might be isentropic again and the maximal Mach number can be determined (Figure 9) similarly as in the case described above, however, the decrease in the stagnation pressure across the shock wave must be taken into account. Results of calculations are summarized in Table 4 and Figure 10.

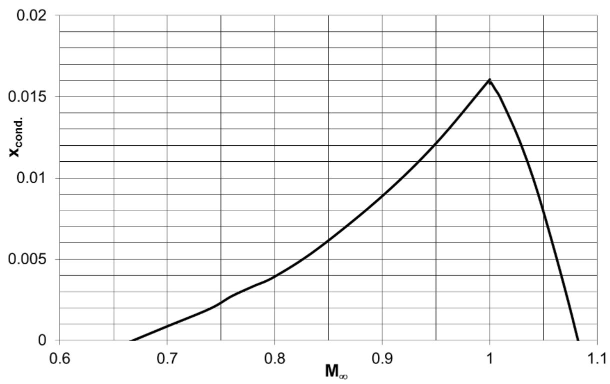

Fig. 10. Graph of the maximum possible amount of water that might condensate in the moist air $x_{cond.}$ as a function of free stream Mach number M_∞ for the case of the flow past an ellipsoid of revolution of the aspect ratio $AR = 0.3$.

Table 4. The following table summarizes the results of calculations for a simple case of flow past ellipsoid of revolution in the moist air.

M (−) aircraft Mach number	0.6	0.8	1	1.2	1.4
t_0 (°C) stagnation temperature for given aircraft Mach number and atmospheric temperature	51.69	68.56	90.25	116.76	148.09
p_0 (Pa) stagnation pressure for given aircraft Mach number and atmospheric pressure	127501.4	152342.2	189141.5	242268.5	317910.2
p_{02} (Pa) stagnation pressure for given aircraft Mach number and atmospheric pressure	N/A	N/A	189141.5	240522.5	304605.1
M_2 (−) Mach number downstream the normal shock wave	N/A	N/A	1	0.842	0.740
M_{max} (−) maximal Mach number around the ellipsoid of revolution	0.680	0.935	1.370	1.0	0.855
p_{min} (Pa) pressure in the location of M_{max}	93599.25	86738.08	62049.63	127165.4	189062.8
t_{min} (°C) temperature in the location of M_{max}	24.35	17.98	−8.48	52.12	94.65
x'' maximum possible specific humidity in the location of M_{max}	0.020922	0.015128	0.002991	0.075076	0.491829
$x_{cond.}$ amount of water that might condensate in the location of M_{max}	N/A	0.00393	0.016067	N/A	N/A

8 Conclusions

The objective of this paper is to describe and explain the phenomenon of the phase change of the atmospheric moist air in the vicinity of an object (most frequently a jet fighter) travelling at the transonic speed. The parameters used for the calculations were estimated using the model of the inviscid flow around an ellipsoid of revolution. It was shown that the emergence of the *vapour cone* is limited for the cases of transonic speeds. For supersonic speeds the vapour cloud lapses.

It is necessary to state that the transonic flow is still a relevant research subject and in combination with multiphase flow it represents deeply uncharted territory.

The phenomenon of the phase transition of the expanding moist air takes place for example in turbines with the air as a working fluid o r d uring t he t ests i n high-speed wind tunnels [3].

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