

Study of fibre movement in a bifurcation

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Abstract. The nature of fibre motion is much more complex than the motion of spherical particles. A complex model must be used for tracking of the fibre carried by the flow. The model must take into account the orientation of the fibre against the flow and solve the consequent rotation of the fibre. Description of such model can be found in this article. It is then used for simulation and analysis of fibre movement in a single planar bifurcation.

1 Introduction

The prediction of fibre orientation during flow through bifurcating channels such as human airways is required for accurate modelling of the fate of inhaled fibres. However, although there were several studies published in the past years (e.g. [1]), there is still no widely available model applicable to realistic inhalation and whole lungs. Obviously, the motion of fibres is much more complex compared to spherical particles, as fibres rotate around their own axis and around the centre of mass.

This article presents a model which accounts for the translational and rotational motion of fibre experiencing a drag force in a bifurcation channel. The attention is paid to the influence of the velocity gradient on the rotation of the fibre.

2 Model of fibre movement

In this study, fibres were tracked by the Lagrangian approach. Superposition principle was applied for the description of fibre motion. The motion of fibre is considered to consist of translation and rotation around its centre. Equations are provided for both translation and rotation parts of the motion. These equations are based on Newton's second law.

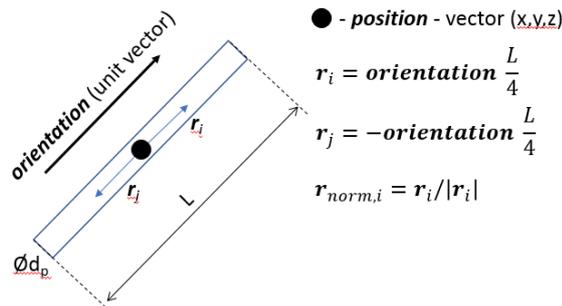


Fig. 1. Fibre description

For the construction of the equations of motion, an approach similar to that presented by Lo Iacono et al. [2] has been applied. Several modifications to the original equations were implemented in this study.

The fibre is divided into discrete segments. In this study, only two segments were defined. The centre of the fibre was characterized by a position vector, while its orientation was given by a unit vector. Vectors r_i and r_j are connections of fibre centre with centres of each segment – see figure 1.

This study focuses on only one force acting on the fibre which is the drag force from the surrounding fluid. At each segment, the drag force F_i is decomposed into two components – the force S_i parallel with fibre's axis and force D_i perpendicular to fibre's axis. The schematic decomposition of the drag force can be seen in figure 2.

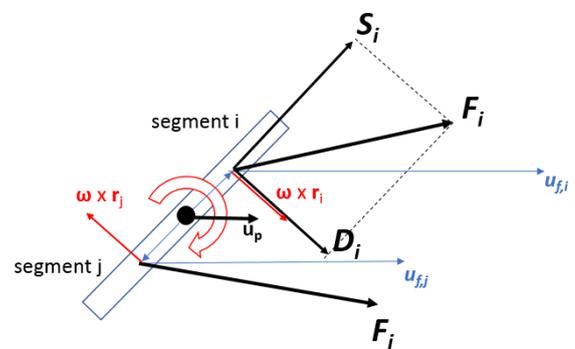


Fig. 2. Forces acting on the fibre during movement in the fluid

The relative velocity of the fibre to the velocity of the surrounding fluid is required for evaluation of the forces S_i and D_i . The velocity of segment i is the sum of fibre translation velocity u_p and a contribution from rotation $\omega \times r_i$, where ω is the angular velocity of fibre around its centre. Inclusion of term $\omega \times r_i$ is the main

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difference between Lo Iacono et al. [2] and our study. The relative velocity is defined as:

$$\mathbf{u}_{rel,i} = \mathbf{u}_{f,i} - (\mathbf{u}_p + \boldsymbol{\omega} \times \mathbf{r}_i), \quad (1)$$

where $\mathbf{u}_{f,i}$ is the fluid velocity at the centre of the segment i . The parallel relative velocity is obtained as a projection of the relative velocity to axis direction defined as:

$$\mathbf{u}_{\parallel,rel,i} = \mathbf{r}_{norm,i}(\mathbf{u}_{rel,i} \cdot \mathbf{r}_{norm,i}) \quad (2)$$

and perpendicular relative velocity is obtained in a way that the sum of parallel and perpendicular component must result in relative velocity:

$$\mathbf{u}_{\perp,rel,i} = \mathbf{u}_{rel,i} - \mathbf{u}_{\parallel,i}. \quad (3)$$

The parallel component of the force \mathbf{S}_i is modelled as the friction force acting on a flat plate of equivalent surface area to the cylinder:

$$\mathbf{S}_i = \frac{2.6}{Re_l^{1/2}} \frac{1}{2} \rho |\mathbf{u}_{\parallel,rel,i}| \mathbf{u}_{\parallel,rel,i}, \quad (4)$$

where

$$Re_l = \rho L |\mathbf{u}_{rel,i}| / \mu. \quad (5)$$

Perpendicular force component \mathbf{D}_i is computed according to the classical drag expression for a cylinder of infinite length:

$$\mathbf{D}_i = \frac{1}{2} C_D \rho |\mathbf{u}_{\perp,rel,i}| \mathbf{u}_{\perp,rel,i} d_p |\mathbf{r}_i|, \quad (6)$$

where C_D is the drag coefficient as measured by Munson and Wieleberger-Lamb [3].

The overall force acting on segment i is then the sum of parallel and perpendicular component:

$$\mathbf{F}_i = \mathbf{D}_i + \mathbf{S}_i. \quad (7)$$

Only the perpendicular force component \mathbf{D}_i generates a torque:

$$\mathbf{T}_i = \mathbf{r}_i \times \mathbf{D}_i \quad (8)$$

With knowledge of force acting on a fibre, the equations of motion can be formed. Second Newton's law of motion is applied for computation of the change of translation velocity of fibre:

$$m_p \frac{d\mathbf{u}_p}{dt} = \mathbf{F}. \quad (9)$$

\mathbf{F} is the sum of forces over all segments. The change of angular velocity is expressed as:

$$I_p \frac{d\boldsymbol{\omega}}{dt} = \mathbf{T} \quad (10)$$

where I_p is the moment of inertia of the cylinder rotating about a perpendicular axis going through its centre of mass.

The solution of the system formed by equations (9) and (10) will give the trajectory of the fibre and orientation of the fibre along its trajectory. The two equations are linked together through the orientation of the fibre. The force acting on the fibre in equation (9) is strongly dependent on the orientation of the fibre.

3 Test case

Performance and abilities of the introduced model of fibre motion were tested on case of simple planar bifurcation. In the previous work, the fibre motion was studied in a simple channel with a parabolic velocity profile. The flow in bifurcation is more complex and the velocity field cannot be described as a mathematical expression. Therefore, the velocity field was calculated in ANSYS Fluent and velocity at fibre position is interpolated from the calculated field. The calculation of fibre trajectory and orientation was done in MATLAB.

The bifurcation consists of an inlet channel with a height of $2H = 0.0056$ m. Centre of the coordinate system is in the middle of the channel height at the beginning of the channel. After 8.5 mm, the channels split into two branches under angle of 17.2° . The length of the outer edge of the sub-channel is 9.8 mm and width of the sub-channel is 4.5 mm. The geometry of the bifurcation studied in this work can be seen in figure 3. The flow direction coincides with direction of the x-axis. At the inlet, parabolic velocity profile is prescribed with a centre-line velocity of 1.35 m/s (bulk-velocity 0.675 m/s). The density of the fluid was considered to be 1 kg/m^3 , the dynamic viscosity of $1.8\text{e-}5 \text{ Pa.s}$. Fibres are from material with a density of 2500 kg/m^3 . Other important parameters that specify the fibre are its length and diameter. These parameters are varying in this study. Fibres were released into the channel with zero initial velocity and angular velocity in all presented cases.

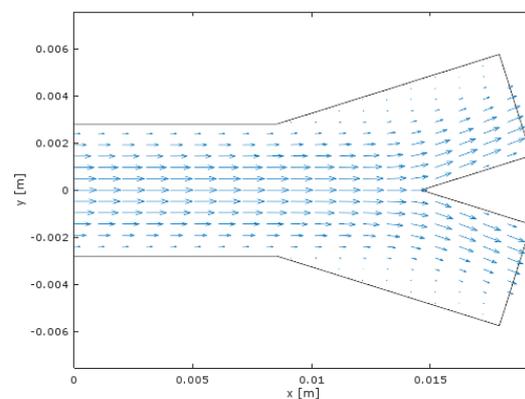


Fig. 3. The geometry of the bifurcation

4 Results

First, fibres with a length of $10 \mu\text{m}$ and diameter of $1 \mu\text{m}$ were tracked. Trajectory and orientation of fibre with various initial positions are depicted in figure 4.

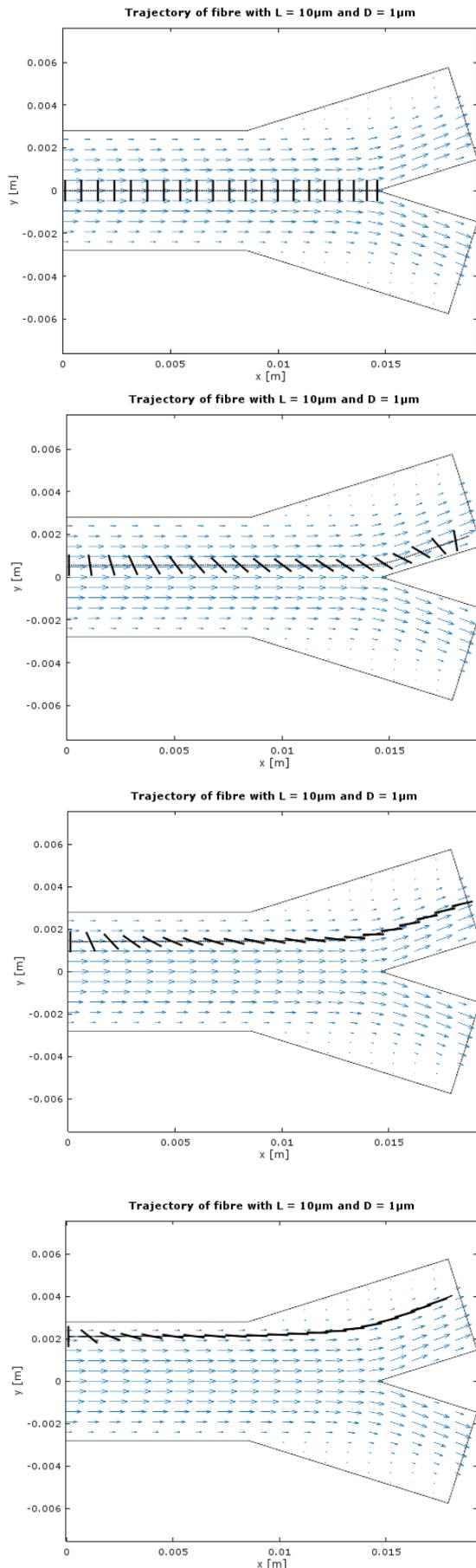


Fig. 4. Trajectory and orientation of fibre with initial positions of $y/H = 0$, $y/H = 0.2$, $y/H = 0.5$ and $y/H = 0.75$, respectively.

The initial fibre's orientation is traverse to the flow. Length of fibre in figures with trajectories does not reflect the real fibre length. Fibre length in the figure was enlarged for visualisation purposes only. The effect of velocity gradient on fibre orientation is clearly visible in figure 4. Fibre with an initial position at the centre of the channel experiences zero velocity difference at its segments and therefore no torque is generated. Fibre retains its orientation during its motion. Finally, the fibre deposits at the channel junction. Velocity gradient increases with decreasing distance from the wall of the initial position of fibre and this leads to increased torque. Fibre located further from channel axis tends to align with flow direction more and more rapidly. It takes only a small distance for the fibre located near to wall to orient into the flow direction.

All these facts are summarized in figure 5 where the x-components of orientation are plotted as a function of time. Initially, this component is zero because of initial fibre orientation in the wall-normal direction. Then it starts to go toward -1 which indicates that fibre is aligning to the flow direction. The only exception is fibre with an initial position of $y/H=0$. It retains the same orientation until it deposits. At the end of the inlet channel, the orientation vector of the fibres is almost $(-1, 0)$ and fibre is oriented parallel to the wall. The orientation is a unit vector. The negative sign in x-component of orientation is because the upper end of fibre in initial position is the end of the orientation vector.

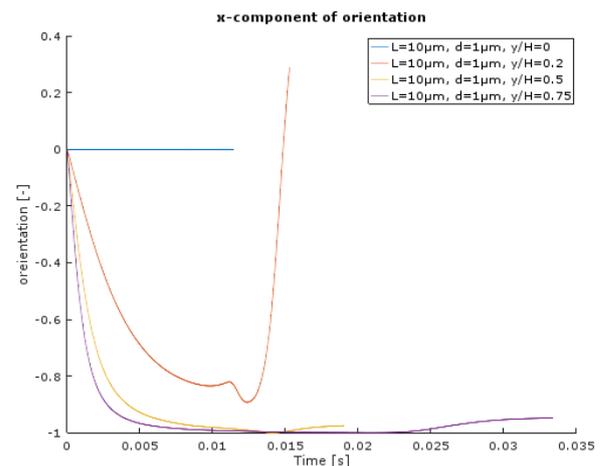


Fig. 5. The x-component of fibre orientation in time

The fibre with an initial position $y/H = 0.2$ is close to the channel centre where the velocity gradient is relatively low. The fibre gains orientation of the flow more slowly compared with other fibres closer to the wall. In the inlet channel, the fibre rotates counterclockwise, the torque is positive. When it reaches the region where the flow splits, the fibre enters boundary layer of inner bifurcation wall and experiences negative torque. The fibre starts to rotate clockwise. This is represented by the steep rise of x-component of the orientation from time 0.012 s. Fibres with the initial position of $y/H = 0.5$ and 0.75 behave similarly. Due to the strong velocity gradient, they orient

horizontally in the inlet channel and then parallel to the wall after the flow splits.

In figure 6, the x-velocity component is depicted. Fibres accelerate almost immediately to the flow velocity. Fibres with these dimensions have very low particle response time and react very quickly to changes in the velocity of the carrier phase.

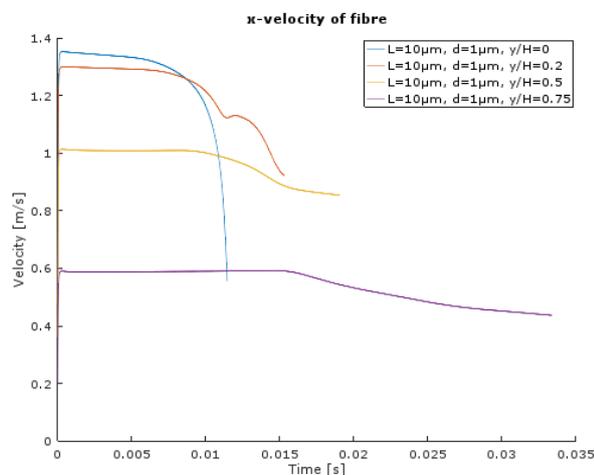


Fig. 6. The x-velocity component of fibre in time.

The x-components of the orientation of various fibres diameters are compared in figure 7. In the region of the inlet channel, all the fibres behave similarly. Changes in fibre diameter have only limited effect on the ability of the fibre to orient into flow direction. This can be explained using the equation (10) for angular velocity change. When the diameter is increased, then higher force D is acting on fibre segments and more torque is generated. On the other hand, an increase in diameter will increase the moment of inertia as well. So, the higher torque is partially compensated by higher moment of inertia of fibre and this results in similar behaviour of fibres. Higher differences in fibre behaviour can be observed when fibres get near the bifurcation. A fibre with larger diameter has higher rotation momentum and it takes more time to reverse the character of rotation. A fibre with a diameter of $5 \mu\text{m}$ leaves domain shortly after it changes the character of rotation.

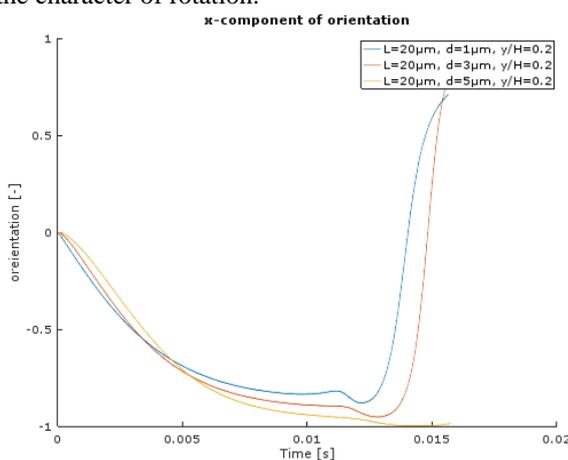


Fig. 7. The streamwise component of the orientation of fibre $L=20\mu\text{m}$ and $d_p=1, 3$ and $5\mu\text{m}$

5 Conclusion

The motion of fibres in a simple planar bifurcation was studied in this work. The character of the fibre motion is more complex than in the simple channel. Fibres with an initial position near the centre of the inlet channel undergo change of rotation direction.

It is important to mention that the results of this model were not confronted by measurements. Such measurements are now being evaluated at Faculty of Mechanical Engineering, BUT. After the confrontation, some modification will probably be necessary to the model.

The most important parameter that affects the fibre rotation in the flow is the velocity gradient at fibre positions. Length and diameter have a lower impact on the fibre motion.

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