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Impact of clustering inside compact tetraquarks

Hagop Sazdjian[1,*]

[1]Université Paris-Saclay, CNRS/IN2P3, IJCLab, 91405 Orsay, France

Abstract. Due to the reducibility of tetraquark operators into mesonic clusters, a strong interplay exists in tetraquarks between compact and molecular structures. This issue is studied within an effective field theory approach, where the compact tetraquark is treated as an elementary particle. Under the influence of the coupling to the mesonic clusters, an initially formed compact tetraquark bound state is deformed towards a new structure of the molecular type, having the attributes of a shallow bound state.

1 Introduction

There has been growing experimental evidence, during the last two decades, about the existence of exotic hadrons, also called multiquark states, containing more valence quarks than ordinary mesons ($\overline{q}q$) and baryons (qqq). Prototypes are tetraquarks with valence quark structure $\bar{q}\bar{q}qq$, pentaquarks, with structure $\overline{q}qqqq$, hexaquarks, with structure $qqqqqq$ or $\bar{q}\bar{q}\bar{q}qqq$ [1–15].

However, contrary to ordinary hadrons, multiquark states are not color-irreducible, in the sense that they can be decomposed along a finite number of combinations of ordinary mesonic or baryonic clusters [16, 17]. Schematically, ignoring here flavor and spin indices, a tetraquark interpolating operator, ($\bar{q}\bar{q}qq$), which is globally color invariant, can be decomposed by means of Fierz rearrangements into a form where clusters of mesonic operators have emerged:

$$(\bar{q}\bar{q}qq) = \sum(\bar{q}q)(\bar{q}q), \tag{1}$$

where the parentheses signify color invariance of the included operator. Similar decompositions can be done with the pentaquark and hexaquark interpolating operators:

$$(\overline{q}qqqq) = \sum(\overline{q}q)(qqq), \tag{2}$$

$$(qqqqqq) = \sum(qqq)(qqq), \qquad (\bar{q}\bar{q}\bar{q}qqq) = \sum(\bar{q}\bar{q}\bar{q})(qqq) + \sum(\bar{q}q)(\bar{q}q)(\bar{q}q). \tag{3}$$

Hadronic clusters, being color-singlets, mutually interact by means of short-range forces, like meson-exchanges or contacts. They would then form loosely bound states, in similarity with atomic molecules. These are called *hadronic molecules* or *molecular states* [18–20].

In contrast, multiquark states, formed directly from confining interactions acting on all quarks, would form *compact bound states*, called *compact multiquark states* [21–24].

Multiquark states can thus be formed by two different mechanisms, each leading to a different structure. The issue is to find, by theoretical justification, also guided by experimental data, the most faithful description.

*e-mail: sazdjian@ijclab.in2p3.fr

In the following, we concentrate on tetraquark states, ignoring details coming from quark flavors and spins, which do not play a fundamental role.

2 Energy balance

A first hint is provided by the study of the energy balance of the system [25]. This is most easily done in the *static limit* of the theory, with very heavy quarks, fixed at spatial positions. The system would choose configurations with *minimal energy*.

The problem is analytically solved in the strong coupling limit of lattice theory [26] and confirmed by direct lattice numerical calculations [27–32]. In the strong coupling limit, the potential energy is concentrated on the *Wilson lines*, which are the path-ordered gluon field phase factors, with constant linear energy density.

Figure 1 represents schematic geometric configurations of the two types of structure. In Fig. 1a, one has a compact tetraquark, formed by confining interactions, through diquark–antidiquark global interaction. In Fig. 1b, one has two meson clusters.

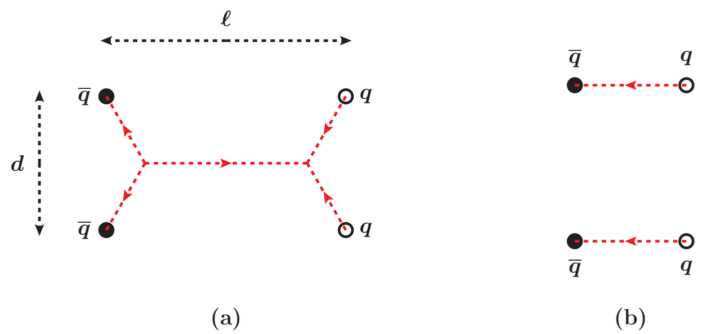

Figure 1. (a) Compact tetraquark, formed by confining interactions, through diquark–antidiquark global interaction. (b) Two meson clusters.

The compact tetraquark formation is energetically favored if the diquark interdistance d is much smaller than the quark-antiquark interdistance ℓ:

$$d \ll \ell. \tag{4}$$

This shows that both structures have nonzero probabilities to be formed. However, quarks have finite masses and move in space. Even if a compact tetraquark has been formed, there is a sizable probability that the quarks, during their motion, reach the two-meson-cluster configuration, in which case the system dislocates or decays.

An interesting and nontrivial case is when the compact tetraquark mass lies below the two-meson threshold, which prevents dislocation. Nevertheless, there will be, through fluctuations, constant transitions to the two-meson virtual states. This will have the tendancy to expand the compact system into a more loosely bound system, close to a molecular-type state. This is a dynamical mechanism, whose precise outcome necessitates the resolution of the four-body bound state problem, in the presence of the confining forces. For the time being, this problem has not yet been satisfactorily solved. One is obliged, to analyze the problem with sufficient accuracy, to have recourse to approximation schemes. We shall consider below the framework provided by the effective field theory approach. A detailed account of this work can be found in [33].

3 Effective field theory approach

According to the energy balance analysis, there is always a probability that a compact tetraquark be formed from the action of the confining forces (actually an infinite tower of such states). We assume that in first approximation, because of the compactness of the state, the latter can be assimilated to a pointlike object and treated as an elementary particle.

We are mainly interested in the case where the mass of the latter particle lies below the lowest possible two-meson threshold. This might be the case when the compact tetraquark is the ground state of the corresponding spectrum.

The two mesons are designated by M_1 and M_2, with masses m_1 and m_2, respectively. The bare mass of the tetraquark is m_{t0}.

Quark flavors and spins will be ignored, as not playing a fundamental role here.

Because of the initial general structure of the tetraquark, containing two meson clusters, the pointlike tetraquark has necessarily a coupling to the mesons M_1 and M_2. The coupling is assumed scalar and is designated by g'; the latter is dimensionless and is factored by the mass term $(m_1 + m_2)$.

The coupling g' generates, through meson loops, radiative corrections inside the tetraquark propagator, modifying the parameters of the bare propagator. In particular, the bare mass m_{t0} gets changed into a physical mass m_t. This is graphically represented in Fig. 2.

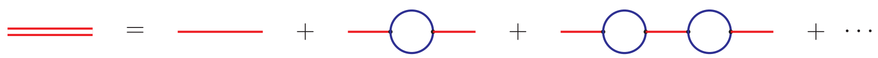

Figure 2. The full tetraquark propagator, including the radiative corrections, coming from the coupling to the two meson clusters.

Mutual interactions of mesons are neglected (in first approximation); they play a less important role than the direct tetraquark-two-meson coupling.

The full tetraquark propagator becomes

$$D_t(s) = \frac{i}{s - m_{t0}^2 + i(m_1 + m_2)^2 g'^2 J(s)}, \tag{5}$$

where s stands for p^2 and J is the standard loop function. The divergence of J is absorbed by the bare mass term, yielding a renormalized mass m_{t1}:

$$m_{t1}^2 = m_{t0}^2 - i(m_1 + m_2)^2 g'^2 J^{\mathrm{div}}. \tag{6}$$

The renormalized tetraquark propagator is now

$$D_t(s) = \frac{i}{s - m_{t1}^2 + i(m_1 + m_2)^2 g'^2 J^{\mathrm{r}}(s)}, \tag{7}$$

where J^{r} is the renormalized finite part of J. Notice that g' does not undergo any renormalization. The mass term m_{t1} does not yet represent the physical mass of the tetraquark. The latter is determined from the pole position of the propagator.

We stick here to the case of heavy quarks and heavy mesons, treating the problem in its nonrelativistic limit, referred to the two-meson threshold. The nonrelativistic energy E is introduced through the standard definition

$$\sqrt{s} = (m_1 + m_2) + E. \tag{8}$$

The nonrelativistic energy corresponding to the renormalized mass m_{t1} of the tetraquark is defined similarly:

$$E_{t1} = m_{t1} - (m_1 + m_2), \qquad E_{t1} < 0. \tag{9}$$

Since we are considering the case of stable tetraquarks (under the strong interactions), the mass of the tetraquark is expected to lie below the two-meson threshold. The physical nonrelativistic energy of the tetraquark will be designated by E_t and is also expected to be negative.

To simplify notations, we shall use henceforth the reduced dimensionless energy variables e through the definitions

$$e \equiv \frac{E}{2m_r}, \quad e_{t1} \equiv \frac{E_{t1}}{2m_r}, \quad e_t \equiv \frac{E_t}{2m_r}, \quad m_r = \frac{m_1 m_2}{(m_1 + m_2)}. \tag{10}$$

The quantity $-e_t$ represents the nonrelativistic binding energy of the tetraquark by reference to the two-meson threshold, although it is different from the binding energy defined from the confining forces by reference to the quark masses.

One finds for the nonrelativistic energy e_t of the tetraquark the equation

$$-e_t + e_{t1} + \frac{g'^2}{16\pi}\sqrt{-e_t} = 0, \tag{11}$$

whose solution is

$$\sqrt{-e_t} = \frac{1}{2}\Big[-\frac{g'^2}{16\pi} + \sqrt{\Big(\frac{g'^2}{16\pi}\Big)^2 - 4e_{t1}}\Big]. \tag{12}$$

The binding energy $-e_t$ is a decreasing function of $g'^2/(16\pi)$ and comes out smaller than $-e_{t1}$,

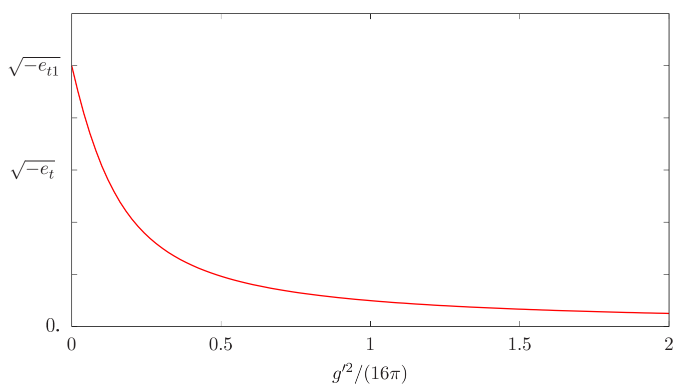

Figure 3. Variation of the square-root of the binding energy as a function of $g'^2/(16\pi)$.

reaching the value 0 when $g' \to \infty$. This is represented in Fig. 3.

We find a very rapid decrease of the binding energy. With ordinary values of $g'^2/(16\pi)$, of the order of 1, the binding energy decreases by a factor of 1/100. The state takes the appearance of a *shallow* bound state.

4 Compositeness

The comparison of the molecular and compact schemes is reminiscent of a general problem, already raised in the past in the case of the deuteron state, denoted under the term of *compositeness* [34].

Weinberg has shown that this question can receive, in the nonrelativistic limit, a precise and model-independent answer, by relating the probability of a state as being elementary (or compact) to observable quantities, represented by the scattering length and the effective range of the two constituents of the molecular scheme in the S-wave state of their scattering amplitude. Designating by Z this probability, one has the following relations for the scattering length a and the effective range r_e, adapted to the tetraquark problem:

$$a = \frac{2(1-Z)}{(2-Z)}R, \qquad r_e = -\frac{Z}{(1-Z)}R, \qquad R = (-2m_rE_t)^{-1/2}, \tag{13}$$

where R is the radius of the bound state, m_r the reduced mass of the two-meson system and E_t the tetraquark nonrelativistic energy.

Investigations about the compositeness criterion and its applicability to various tetraquark candidates, as well as to ordinary hadrons, can be found in [35–50].

The contribution of the tetraquark state, in the s-channel, to the two-meson elastic scattering amplitude is obtained by inserting the tetraquark propagator between two tetraquark-two-meson couplings (cf. Fig. 4).

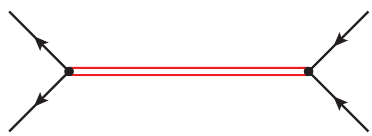

Figure 4. The tetraquark contribution, in the s-channel, to the two-meson elastic scattering amplitude.

From the scattering length and the effective range one obtains Z:

$$Z = \frac{\sqrt{-e_t}}{\sqrt{-e_t} + \frac{1}{2}\frac{g'^2}{16\pi}}. \tag{14}$$

When $Z = 1$, we have the case of a pure compact tetraquark, while for $Z = 0$, we have a pure molecular state. The variation of Z with respect to $g'^2/(16\pi)$ is represented in Fig. 5.

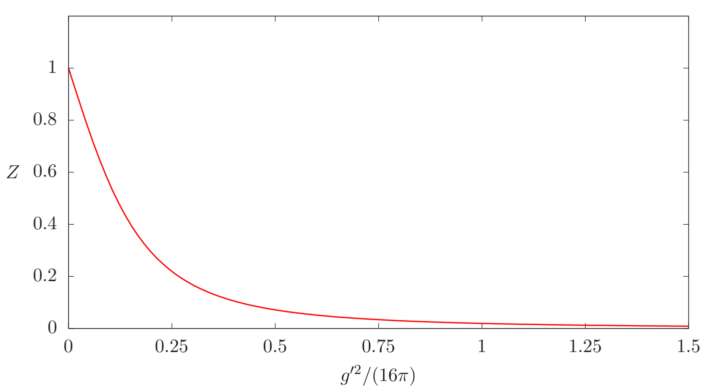

Figure 5. Variation of Z as a function of $g'^2/(16\pi)$.

Like the binding energy, Z is a rapidly decreasing function of $g'^2/(16\pi)$. For values of the latter of the order of 1, Z is very close to zero. This confirms the interpretation that the coupling of the compact tetraquark to its internal mesonic clusters qualitatively deforms its initial structure, bringing it into a form closer to a molecular-type state.

5 Conclusion

A compact tetraquark, formed from the confining forces acting between quarks and gluons, rapidly evolves, under the influence of the clustering phenomenon, towards a molecular-type state. The origin of the state is, however, of compact nature: nowhere, in the present model, did we consider direct interactions between mesons.

The experimental test for this phenomenon is provided by the measure of the elementariness coefficient Z. Pure molecular states are characterized by the value $Z = 0$. A value $Z \neq 0$, beyond uncertainties, reflects the existence of an original compact tetraquark. Many tetraquark candidates fall in this category.

Because of the shrinking of the tetraquark binding energy to values close to zero, shallowness of many bound states may receive a natural explanation from the above mechanism.

The present study can also be extended to the case of resonances and to the case when direct meson-meson interactions are incorporated.

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