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Weak decays of the double-beauty tetraquark

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Abstract. The spectroscopic parameters and decay channels of the axial-vector tetraquark $T^{\mathrm{AV}}_{b:\overline{s}}$ built of heavy diquark bb and light antidiquark $\overline{us}$ are explored using QCD sum rule method. Because this state is stable against strong and electromagnetic decays, we consider its semileptonic and nonleptonic decay modes. Partial widths of weak processes $T^{\mathrm{AV}}_{b:\overline{s}} \to \mathcal{Z}^0_{b:\overline{s}} l\overline{\nu}_l$ and $T^{\mathrm{AV}}_{b:\overline{s}} \to \mathcal{Z}^0_{b:\overline{s}} M$ are calculated using weak form factors $G_i(q^2)$ computed by QCD three-point sum rules: $\mathcal{Z}^0_{b:\overline{s}}$ is the scalar tetraquark with quark content $bc\overline{us}$ and M is a vector meson. The full width and mean lifetime of the tetraquark $T^{\mathrm{AV}}_{b:\overline{s}}$ obtained in this work shed light on features of double-beauty tetraquarks and are useful for their experimental investigations.

1 Introduction

Recently interest in double-charm and double-beauty tetraquarks was renewed after the discovery of the doubly charmed baryon $\Xi^{++}_{cc} = ccu$ by the LHCb Collaboration [1]. Thus, information on mass of this baryon was used in Ref. [2] to estimate the masses of the tetraquarks $T^{-}_{bb;\overline{u}\overline{d}}$ and $T^{+}_{cc;\overline{u}\overline{d}}$ in the context of a phenomenological model. The obtained prediction for $m = (10389 \pm 12)$ MeV confirms that the isoscalar state $T^{-}_{bb;\overline{u}\overline{d}}$ with spin-parity $J^P = 1^+$ is stable against strong and electromagnetic decays, whereas the tetraquark $T^{+}_{cc;\overline{u}\overline{d}}$ lies above the open charm threshold $D^0 D^{*+}$ and can decay to these mesons.

In the framework of the QCD sum rule method the spectroscopic parameters and weak decays of the double-beauty axial-vector tetraquark $T^{-}_{bb;\overline{u}\overline{d}}$ were calculated in Ref. [3]. The mass of the state $T^{-}_{bb;\overline{u}\overline{d}}$ found in this work is $m = (10035 \pm 260)$ MeV, which demonstrates its stability against the strong and electromagnetic decays. The full width and mean lifetime of $T^{-}_{bb;\overline{u}\overline{d}}$ were evaluated in Refs. [3, 4] using its various semileptonic and nonleptonic decay channels. The obtained results, $\Gamma_{\mathrm{full}} = (7.72 \pm 1.23) \times 10^{-8}$ MeV and $\tau = 8.53^{+1.57}_{-1.18}$ fs, are useful for the experimental investigation of the double-beauty tetraquark $T^{-}_{bb;\overline{u}\overline{d}}$.

In this work, we calculate partial widths of the semileptonic $T^{\mathrm{AV}}_{b:\overline{s}} \to \mathcal{Z}^0_{b:\overline{s}} l\overline{\nu}_l$ and nonleptonic weak decays $T^{\mathrm{AV}}_{b:\overline{s}} \to \mathcal{Z}^0_{b:\overline{s}} M$ of the axial-vector tetraquark $T^{\mathrm{AV}}_{b:\overline{s}}$ with quark content $bb\overline{us}$, where $\mathcal{Z}^0_{b:\overline{s}}$ is the scalar tetraquark $[bc][\overline{us}]$ built of a color-antitriplet diquark and triplet antidiquark, and M is one of the vector mesons ρ^-, $K^*(892)$, $D^*(2010)^-$, and D^{*-}_s. To this end, we calculate the weak form factors $G_i(q^2)$ from the three-point sum rules at momentum transfers q^2 where this method is applicable [5–8]. We determine model functions $\mathcal{G}_i(q^2)$ and find the partial widths of aforementioned weak processes. The full width and mean lifetime of $T^{\mathrm{AV}}_{b:\overline{s}}$ are obtained as well.

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2 Weak Decays

The mass of the tetraquark $T^{\rm AV}_{b:\overline{s}}$ $m_{\rm AV}=10215$ MeV is 480/477 MeV below the thresholds 10695/10692 MeV for its strong decays to mesons $B^{-}B_s^{*}$ and $B^{*-}\overline{B}{}^0_s$, respectively, and 431 MeV below the threshold 10646 MeV for electromagnetic decay $B^{-}\overline{B}{}^0_s\gamma$ [9]. This means that $T^{\rm AV}_{b:\overline{s}}$ is stable against strong and electromagnetic decays and transforms to ordinary mesons only through weak processes. Therefore, to evaluate the full width and lifetime of $T^{\rm AV}_{b:\overline{s}}$, we analyze the semileptonic and nonleptonic weak decays $T^{\rm AV}_{b:\overline{s}}\to\mathcal{Z}^0_{b:\overline{s}}l\overline{\nu}_l$ and $T^{\rm AV}_{b:\overline{s}}\to\mathcal{Z}^0_{b:\overline{s}}M$.

2.1 Semileptonic decays $T^{\rm AV}_{b:\overline{s}}\to\mathcal{Z}^0_{b:\overline{s}}l\overline{\nu}_l$

Weak decays of $T^{\rm AV}_{b:\overline{s}}$ can be generated by subprocesses $b\to W^{-}c$ and $b\to W^{-}u$. The processes triggered by the transition $b\to W^{-}c$ are dominant relative to $b\to W^{-}u$ ones: the latter decays are suppressed relative to the dominant modes by a factor $|V_{ub}|^2/|V_{cb}|^2\simeq 0.01$, with $V_{q_1q_2}$ being the Cabibbo-Khobayasi-Maskawa (CKM) matrix elements. In the present work, we restrict ourselves by analysis of the dominant weak decays of $T^{\rm AV}_{b:\overline{s}}$.

The effective Hamiltonian that describes the subprocess $b\to W^{-}c$ at the tree-level is determined by the expression

$$\mathcal{H}^{\rm eff}=\frac{G_F}{\sqrt{2}}V_{cb}\overline{c}\gamma_\mu(1-\gamma_5)b\overline{l}\gamma^\mu(1-\gamma_5)\nu_l, \tag{1}$$

where G_F and V_{cb} are the Fermi coupling constant and CKM matrix element, respectively. The matrix element of $\mathcal{H}^{\rm eff}$ between the initial and final tetraquarks contains the leptonic and hadronic factors

$$\langle\mathcal{Z}^0_{b:\overline{s}}(p')|\mathcal{H}^{\rm eff}|T^{\rm AV}_{b:\overline{s}}(p)\rangle=L_\mu H^\mu. \tag{2}$$

We are interested in calculation of H^μ, because the leptonic part of the matrix element L_μ is universal for all semileptonic decays and does not contain information on tetraquarks. Then, H^μ is the matrix element of the current

$$J^{\rm tr}_\mu=\overline{c}\gamma_\mu(1-\gamma_5)b \tag{3}$$

between the initial and final particles. It can be modeled in terms of four weak form factors $G_i(q^2)$ which parametrize long-distance dynamics of the weak transformation [10, 11]

$$\begin{aligned}\langle\mathcal{Z}^0_{b:\overline{s}}(p')|J^{\rm tr}_\mu|T^{\rm AV}_{b:\overline{s}}(p)\rangle &= \widetilde{G}_0(q^2)\epsilon_\mu+\widetilde{G}_1(q^2)(\epsilon p')P_\mu \\ &+\widetilde{G}_2(q^2)(\epsilon p')q_\mu+i\widetilde{G}_3(q^2)\varepsilon_{\mu\nu\alpha\beta}\epsilon^\nu p^\alpha p'^\beta.\end{aligned} \tag{4}$$

The scaled functions $\widetilde{G}_i(q^2)$ are connected with the dimensionless form factors $G_i(q^2)$ by the equalities

$$\widetilde{G}_0(q^2)=\widetilde{m}G_0(q^2),\ \widetilde{G}_j(q^2)=\frac{G_j(q^2)}{\widetilde{m}},\ j=1,2,3. \tag{5}$$

Here, $\widetilde{m}=m_{\rm AV}+m_{\mathcal{Z}}$, p_μ and ϵ_μ are the momentum and polarization vector of the tetraquark $T^{\rm AV}_{b:\overline{s}}$, p' is the momentum of the scalar state $\mathcal{Z}^0_{b:\overline{s}}$, $P_\mu=p'_\mu+p_\mu$, and $q_\mu=p_\mu-p'_\mu$ is the momentum transferred to the leptons. q^2 varies within the limits $m_l^2\leq q^2\leq(m_{\rm AV}-m_{\mathcal{Z}})^2$, where m_l is the mass of a lepton l.

The sum rules for the form factors $G_i(q^2)$ can be obtained by analyzing the three-point correlation function

$$\Pi_{\mu\nu}(p,p')=i^2\int d^4xd^4ye^{i(p'y-px)}\langle 0|\mathcal{T}\{J_{\mathcal{Z}}(y)J^{\rm tr}_\nu(0)J^\dagger_\mu(x)\}|0\rangle, \tag{6}$$

with $J_\mu(x)$ and $J_{\mathcal{Z}}(y)$ being the interpolating currents for the tetraquarks $T^{\rm AV}_{b:\overline{s}}$ and $\mathcal{Z}^0_{b:\overline{s}}$, respectively. In accordance with standard prescriptions, we express $\Pi_{\mu\nu}(p,p')$ using the masses and couplings of the tetraquarks and determine the physical side of the sum rules $\Pi^{\rm Phys}_{\mu\nu}(p,p')$. The function $\Pi^{\rm Phys}_{\mu\nu}(p,p')$ can be written down in the form

$$\Pi^{\rm Phys}_{\mu\nu}(p,p') = \frac{\langle 0|J_{\mathcal{Z}}|\mathcal{Z}^0_{b:\overline{s}}(p')\rangle\langle\mathcal{Z}^0_{b:\overline{s}}(p')|J^{\rm tr}_\nu|T^{\rm AV}_{b:\overline{s}}(p,\epsilon)\rangle}{(p^2-m^2_{\rm AV})(p'^2-m^2_{\mathcal{Z}})} \times\langle T^{\rm AV}_{b:\overline{s}}(p,\epsilon)|J^{\dagger}_\mu|0\rangle+\cdots, \quad (7)$$

where contribution of the ground-state particles is presented explicitly, whereas effects due to excited and continuum states are denoted by dots.

The phenomenological side of the sum rules can be simplified by substituting into Eq. (7) expressions of the matrix elements. To this aim, we apply Eq. (4) and introduce the matrix elements of the tetraquarks $T^{\rm AV}_{b:\overline{s}}$ and $\mathcal{Z}^0_{b:\overline{s}}$,

$$\langle 0|J_\mu|T^{\rm AV}_{b:\overline{s}}(p)\rangle = m_{\rm AV}f_{\rm AV}\epsilon_\mu, \quad (8)$$

and

$$\langle 0|J_{\mathcal{Z}}|\mathcal{Z}^0_{b:\overline{s}}(p')\rangle = f_{\mathcal{Z}}m_{\mathcal{Z}}, \quad (9)$$

respectively. Then it is not difficult to find that

$$\Pi^{\rm Phys}_{\mu\nu}(p,p') = \frac{f_{\rm AV}m_{\rm AV}f_{\mathcal{Z}}m_{\mathcal{Z}}}{(p^2-m^2_{\rm AV})(p'^2-m^2_{\mathcal{Z}})}\left\{\widetilde{G}_0(q^2)\left(-g_{\mu\nu}+\frac{p_\mu p_\nu}{m^2_{\rm AV}}\right)+\left[\widetilde{G}_1(q^2)P_\mu +\widetilde{G}_2(q^2)q_\mu\right]\left(-p'_\nu+\frac{m^2_{\rm AV}+m^2_{\mathcal{Z}}-q^2}{2m^2_{\rm AV}}p_\nu\right) -i\widetilde{G}_3(q^2)\varepsilon_{\mu\nu\alpha\beta}p^\alpha p'^\beta\right\}+\cdots. \quad (10)$$

The function $\Pi^{\rm OPE}_{\mu\nu}(p,p')$ constitutes the second side of the sum rules and in terms of quark-gluon degrees of freedom has the following form:

$$\Pi^{\rm OPE}_{\mu\nu}(p,p') = \int d^4xd^4ye^{i(p'y-px)}\left\{{\rm Tr}\left[\gamma_5\widetilde{S}^{ba'}_s(x-y) \times\gamma_5 S^{a'b}_u(x-y)\right]\left({\rm Tr}\left[\gamma_\mu\widetilde{S}^{aa'}_b(y-x)\gamma_5 S^{bi}_c(y)\gamma_\nu(1-\gamma_5) \times S^{ib'}_b(-x)\right]+{\rm Tr}\left[\gamma_\mu\widetilde{S}^{ia'}_b(-x)(1-\gamma_5)\gamma_\nu\widetilde{S}^{bi}_c(y)\gamma_5 \times S^{ab'}_b(y-x)\right]\right)-{\rm Tr}\left[\gamma_5\widetilde{S}^{b'a}_s(x-y)\gamma_5 S^{a'b}_u(x-y)\right] \times\left({\rm Tr}\left[\gamma_\mu\widetilde{S}^{aa'}_b(y-x)\gamma_5 S^{bi}_c(y)\gamma_\nu(1-\gamma_5)S^{ib'}_b(-x)\right] +{\rm Tr}\left[\gamma_\mu\widetilde{S}^{ia'}_b(-x)(1-\gamma_5)\gamma_\nu\widetilde{S}^{bi}_c(y)\gamma_5 S^{ab'}_b(y-x)\right]\right)\right\}, \quad (11)$$

where $S_{Q(q)}(x)$ are quark propagators and

$$\widetilde{S}_{Q(q)}(x) = CS_{Q(q)}(x)C \quad (12)$$

with C being the charge conjugation matrix.

To extract the sum rules for the form factors $G_i(q^2)$, we equate invariant amplitudes corresponding to the same Lorentz structures in $\Pi^{\rm Phys}_{\mu\nu}(p,p')$ and $\Pi^{\rm OPE}_{\mu\nu}(p,p')$, perform a double

Borel transformation over the variables p'^2 and p^2 to suppress contributions of the higher excited and continuum states, and perform continuum subtraction. For instance, to extract the sum rule for $\widetilde{G}_0(q^2)$, we use the structure $g_{\mu\nu}$, whereas for $\widetilde{G}_3(q^2)$, the term $\sim \varepsilon_{\mu\nu\alpha\beta}p^{\alpha}p'^{\beta}$ is employed. The sum rules for the scaled form factors $\widetilde{G}_i(q^2)$ can be written in a single formula

$$\widetilde{G}_i(\mathbf{M}^2, \mathbf{s}_0, q^2) = \frac{1}{f_{\mathrm{AV}} m_{\mathrm{AV}} f_{\mathcal{Z}} m_{\mathcal{Z}}} \int_{\mathcal{M}^2}^{s_0} ds e^{(m_{\mathrm{AV}}^2 - s)/M_1^2} \int_{\widetilde{\mathcal{M}}^2}^{s_0'} ds' \rho_i(s, s') e^{(m_{\mathcal{Z}}^2 - s')/M_2^2}, \tag{13}$$

where $\rho_i(s, s')$ are spectral densities calculated as the imaginary parts of the corresponding terms in $\Pi_{\mu\nu}^{\mathrm{OPE}}(p, p')$. They include the perturbative and nonperturbative contributions, and are calculated with dimension-six accuracy. In Eq. (13), $\mathbf{M}^2 = (M_1^2, M_2^2)$ are the Borel parameters, and $\mathbf{s}_0 = (s_0,\ s_0')$ are the continuum threshold parameters that separate the main contribution to the sum rules from the continuum effects. The pair of parameters $(M_1^2,\ s_0)$ corresponds to a channel of the initial tetraquark $T_{b:\overline{s}}^{\mathrm{AV}}$, whereas $(M_2^2,\ s_0')$ describe the final-state particle $\mathcal{Z}_{b:\overline{s}}^0$.

The form factors $\widetilde{G}_i(q^2)$ determine the differential decay rate $d\Gamma/dq^2$ of the semileptonic decay $T_{b:\overline{s}}^{\mathrm{AV}} \to \mathcal{Z}_{b:\overline{s}}^0 \bar{l}\nu_l$, the explicit expression of which can be found in Ref. [3]. The sum rules give reliable results for $G_i(q^2)$ in the region $m_l^2 \leq q^2 \leq 8\ \mathrm{GeV}^2$, which is not enough to find the partial width of the processes under consideration. To calculate the width of the semileptonic decay, $d\Gamma/dq^2$ must be integrated over q^2 in the boundaries $m_l^2 \leq q^2 \leq (m_{\mathrm{AV}} - m_{\mathcal{Z}})^2$, i.e., in the limits $m_l^2 \leq q^2 \leq 11.87\ \mathrm{GeV}^2$. This region is wider than the one where the sum rules lead to reliable predictions. This problem can be solved by introducing extrapolating (fit) functions $\mathcal{G}_i(q^2)$, which at the momentum transfers q^2 accessible for the sum rule computations coincide with $G_i(q^2)$, but have forms suitable to carry out integrations over q^2.

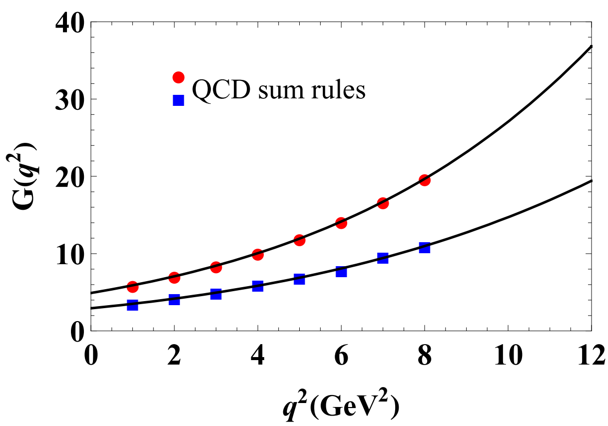

Figure 1. Sum rule results for the form factors $G_0(q^2)$ (red circles) and $G_1(q^2)$ (blue squares). The solid curves are fit functions $\mathcal{G}_0(q^2)$ and $\mathcal{G}_1(q^2)$.

To this end, we use the functions

$$\mathcal{G}_i(q^2) = \mathcal{G}_0^i \exp\left[g_1^i \frac{q^2}{m_{\mathrm{AV}}^2} + g_2^i \left(\frac{q^2}{m_{\mathrm{AV}}^2}\right)^2\right], \tag{14}$$

where parameters $\mathcal{G}_0^i$, g_1^i, and g_2^i are fitted to satisfy the sum rules' predictions. The parameters of the functions $\mathcal{G}_i(q^2)$ extracted from numerical analysis are collected in Table 1.

Table 1. Parameters of the extrapolating functions $\mathcal{G}_i(q^2)$.

$\mathcal{G}_i(q^2)$	$\mathcal{G}_0^i$	g_1^i	g_2^i
$\mathcal{G}_0(q^2)$	4.91	19.29	−15.34
$\mathcal{G}_1(q^2)$	2.94	18.73	−20.09
$\mathcal{G}_2(q^2)$	−22.67	20.50	−22.95
$\mathcal{G}_3(q^2)$	−21.14	20.77	−23.62

In Fig. 1, we show fit functions $\mathcal{G}_i(q^2)$ which are in nice agreement with relevant sum rule predictions.

The Fermi constant, CKM matrix elements, and masses of leptons in numerical computations are set equal to

$$G_F = 1.16637 \times 10^{-5}\ \mathrm{GeV}^{-2},\ |V_{cb}| = (42.2 \pm 0.08) \times 10^{-3}, \tag{15}$$

$m_e = 0.511$ MeV, $m_\mu = 105.658$ MeV, and $m_\tau = (1776.82\ \pm 0.16)$ MeV [12].

2.2 Nonleptonic decays $T_{b:\overline{s}}^{\mathrm{AV}} \to \mathcal{Z}_{b:\overline{s}}^0 \rho^-(K^*(892),\ D^*(2010)^-,\ D_s^{*-})$

In this section, we analyze the nonleptonic weak decays $T_{b:\overline{s}}^{\mathrm{AV}} \to \mathcal{Z}_{b:\overline{s}}^0 \rho^-(K^*(892),\ D^*(2010)^-,\ D_s^{*-})$ of the tetraquark $T_{b:\overline{s}}^{\mathrm{AV}}$ in the framework of the QCD factorization approach, which allows us to calculate partial widths of these processes. This method was applied to study nonleptonic decays of conventional mesons [13, 14], but can be also used for investigating decays of tetraquarks. Thus, the nonleptonic decays of scalar exotic mesons $T_{b:\overline{s}}^-$, $T_{b:\overline{s}}^-$, Z_{bc}^0, and $T_{bs;\overline{u}\overline{d}}^-$ were explored using this approach in Refs. [15–18], respectively.

At the quark level, the effective Hamiltonian for this decay is given by the expression

$$\mathcal{H}_{\mathrm{n.-lep}}^{\mathrm{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left[c_1(\mu) Q_1 + c_2(\mu) Q_2\right], \tag{16}$$

where

$$Q_1 = \left(\overline{d}_i u_i\right)_{\mathrm{V-A}} \left(\overline{c}_j b_j\right)_{\mathrm{V-A}},\ Q_2 = \left(\overline{d}_i u_j\right)_{\mathrm{V-A}} \left(\overline{c}_j b_i\right)_{\mathrm{V-A}}. \tag{17}$$

In Eq. (17) i and j are the color indices, and notation $(\overline{q}_1 q_2)_{\mathrm{V-A}}$ means

$$(\overline{q}_1 q_2)_{\mathrm{V-A}} = \overline{q}_1 \gamma_\mu (1 - \gamma_5) q_2. \tag{18}$$

The short-distance Wilson coefficients $c_1(\mu)$ and $c_2(\mu)$ are given at the factorization scale μ.

The amplitude of the decay $T_{b:\overline{s}}^{\mathrm{AV}} \to \mathcal{Z}_{b:\overline{s}}^0 \rho^-$ is determined by the expression

$$\mathcal{A} = \frac{G_F}{\sqrt{2}} V_{bc} V_{ud}^* a(\mu) \langle \rho^-(q) | \left(\overline{d}_i u_i\right)_{\mathrm{V-A}} |0\rangle \, \langle \mathcal{Z}_{b:\overline{s}}^0(p') | \left(\overline{c}_j b_j\right)_{\mathrm{V-A}} | T_{b:\overline{s}}^{\mathrm{AV}}(p)\rangle, \tag{19}$$

where

$$a(\mu) = c_1(\mu) + \frac{1}{N_c} c_2(\mu), \tag{20}$$

with $N_c = 3$ being the number of quark colors. The matrix element $\langle \rho^-(q) | \left(\overline{d}_i u_i\right)_{\mathrm{V-A}} |0\rangle$ in $\mathcal{A}$ can be given as follows

$$\langle \rho^-(q) | \left(\overline{d}_i u_i\right)_{\mathrm{V-A}} |0\rangle = f_\rho m_\rho \epsilon_\mu^*(q). \tag{21}$$

Then, it is evident that

$$\mathcal{A} = i\frac{G_F}{\sqrt{2}} f_\rho V_{bc} V_{ud}^{*} a(\mu) \Big[\widetilde{G}_0(q^2) \epsilon_\mu(p) \epsilon^{*\mu}(q) + 2\widetilde{G}_1(q^2)(p'\epsilon(p))(p'\epsilon^*(q)) + i\widetilde{G}_3(q^2) \varepsilon_{\mu\nu\alpha\beta} \epsilon^{*\mu}(q) \epsilon^\nu(p) p^\alpha p'^\beta \Big]. \quad (22)$$

A similar analysis can be carried out for the decay modes $T_{b:\overline{s}}^{\rm AV} \to \mathcal{Z}_{b:\overline{s}}^0 K^*(892)[D^*(2010)^-, D_s^{*-}]$ as well. We replace in relevant expressions m_ρ and f_ρ by spectroscopic parameters of the $K^*(892)$, $D^*(2010)^-$, and D_s^{*-} mesons, and substitute $V_{ud} \to V_{us}$, V_{cd}, and V_{cs}.

The width of the nonleptonic decay $T_{b:\overline{s}}^{\rm AV} \to \mathcal{Z}_{b:\overline{s}}^0 \rho^-$ can be evaluated using the expression

$$\Gamma = \frac{|\mathcal{A}|^2}{24\pi m_{\rm AV}^2} \lambda\left(m_{\rm AV}, m_{\mathcal{Z}}, m_\rho\right), \quad (23)$$

where

$$\lambda(a,b,c) = \frac{1}{2a}\left[a^4 + b^4 + c^4 - 2\left(a^2b^2 + a^2c^2 + b^2c^2\right)\right]^{1/2}.$$

The key component in Eq. (23), i.e., $|\mathcal{A}|^2$ has a simple form

$$|\mathcal{A}|^2 = \sum_{j=0,1,2} H_j \widetilde{G}_j^2 + H_3 \widetilde{G}_0 \widetilde{G}_1, \quad (24)$$

where the explicit expressions of H_j are given in Ref. [9]

In Eq. (24), the weak form factors $\widetilde{G}_j$ are real functions of q^2, and their values are fixed at $q^2 = m_M^2$ where M is one of the vector mesons ρ^-, $K^*(892)$, $D^*(2010)^-$, and D_s^{*-}.

3 Numerical Results

In this section we present the results of our numerical computations. In general, some of the input parameters used in the form factors $\widetilde{G}_i(\mathbf{M}^2, \mathbf{s}_0, q^2)$ should be defined before numerical analysis. All input information necessary for such analysis, i.e., the vacuum condensates of quark, gluon, and mixed operators, the spectroscopic parameters of the final-state mesons and tetraquarks $T_{b:\overline{s}}^{\rm AV}$ and $\mathcal{Z}_{b:\overline{s}}^0$, as well as the CKM matrix elements can be found in Ref. [9]. The Borel and continuum threshold parameters $\mathbf{M}^2$ and $\mathbf{s}_0$ are chosen to satisfy all constraints of sum rule analysis. Also we have to take into account that $\widetilde{G}_i(\mathbf{M}^2, \mathbf{s}_0, q^2)$ depends on the masses and couplings of the initial and final tetraquarks, which have been evaluated also in the context of the sum rule approach. We fix the auxiliary parameters (M_1^2, s_0) and (M_2^2, s_0') similar to the mass computations, because they meet standard restrictions of three-point sum rule calculations and do not produce additional uncertainties in the spectroscopic parameters of relevant tetraquarks.

Predictions obtained for the partial widths of the semileptonic $T_{b:\overline{s}}^{\rm AV} \to \mathcal{Z}_{b:\overline{s}}^0 l\overline{\nu}_l$ and nonleptonic $T_{b:\overline{s}}^{\rm AV} \to \mathcal{Z}_{b:\overline{s}}^0 \rho^-(K^*(892),\ D^*(2010)^-,\ D_s^{*-})$ decays are presented below

$$\begin{aligned}
\Gamma(T_{b:\overline{s}}^{\rm AV} &\to \mathcal{Z}_{b:\overline{s}}^0 e^-\overline{\nu}_e) = (5.34 \pm 1.43) \times 10^{-8}\ \mathrm{MeV},\\
\Gamma(T_{b:\overline{s}}^{\rm AV} &\to \mathcal{Z}_{b:\overline{s}}^0 \mu^-\overline{\nu}_\mu) = (5.32 \pm 1.41) \times 10^{-8}\ \mathrm{MeV},\\
\Gamma(T_{b:\overline{s}}^{\rm AV} &\to \mathcal{Z}_{b:\overline{s}}^0 \tau^-\overline{\nu}_\tau) = (2.15 \pm 0.54) \times 10^{-8}\ \mathrm{MeV},\\
\Gamma(T_{b:\overline{s}}^{\rm AV} &\to \mathcal{Z}_{b:\overline{s}}^0 \rho^-) = (3.47 \pm 0.92) \times 10^{-10}\ \mathrm{MeV},\\
\Gamma(T_{b:\overline{s}}^{\rm AV} &\to \mathcal{Z}_{b:\overline{s}}^0 K^*(892)) = (1.47 \pm 0.37) \times 10^{-11}\ \mathrm{MeV},\\
\Gamma(T_{b:\overline{s}}^{\rm AV} &\to \mathcal{Z}_{b:\overline{s}}^0 D^*(2010)^-) = (1.54 \pm 0.39) \times 10^{-11}\ \mathrm{MeV},\\
\Gamma(T_{b:\overline{s}}^{\rm AV} &\to \mathcal{Z}_{b:\overline{s}}^0 D_s^{*-}) = (4.97 \pm 1.32) \times 10^{-10}\ \mathrm{MeV}.
\end{aligned} \quad (25)$$

The partial widths of different weak decays from Eq. (25) can be used to estimate the full width and mean lifetime of the axial-vector tetraquark $T^{\rm AV}_{b:\overline{s}}$

$$\Gamma_{\rm full} = (12.9 \pm 2.1) \times 10^{-8}\ \mathrm{MeV},\ \tau = 5.1^{+0.99}_{-0.71} \times 10^{-15}\ \mathrm{s}. \tag{26}$$

These predictions can be employed to explore the double-heavy tetraquarks.

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