

Implications of Heavy Quark Spin Symmetry and NRQCD on $B_c \rightarrow J/\psi, \eta_c$ form factors

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Abstract. We discuss the analysis of $B_c \rightarrow J/\psi, \eta_c$ form factors, based on the heavy quark spin symmetry and nonrelativistic QCD (NRQCD) performed in Ref. [1]. We perform an expansion on the inverse of the heavy quarks' masses using power counting rules of NRQCD up to the next-to-leading order. This allows us to classify the heavy quark spin symmetry-breaking terms and express the form factors in terms of universal functions in a selected kinematical range. Using as input the lattice QCD results for the $B_c \rightarrow J/\psi$ matrix element of the SM operator, we obtain information on other form factors.

1 Introduction

Over the last years several anomalies have been detected in the heavy flavour sector, most notably in the semileptonic decays involving the $b \rightarrow c\ell\bar{\nu}$ and $b \rightarrow s\ell^+\ell^-$ transitions [2, 3]. If these deviations originated in New Physics (NP) effects, one should expect to systematically see this effect on all of the modes induced by these quark transitions. In the case of $b \rightarrow c\ell\bar{\nu}$, deviations have been measured in $B \rightarrow D^{(*)}\ell\bar{\nu}$ decays [2] and other semileptonic decays involving the same transition are being extensively studied right now. In particular, $B_c \rightarrow J/\psi, \eta_c$ decays are important to study this transition. One of the main problems for the interpretation of the measurements is the hadronic uncertainties that affect these modes. While, in the case of $B \rightarrow D^{(*)}\ell\bar{\nu}$ several studies have been performed to understand better hadronic contributions, including Lattice QCD, Sum Rules and Heavy Quark Effective Theory (HQET) methods, the situation is less apparent on the side $B_c \rightarrow J/\psi, \eta_c$ decays, where only a limited amount of information is currently available [4–7].

It has been observed [4] that the semileptonic B_c form factors can be expressed in terms of universal functions in selected kinematical regions, based on the heavy quark spin symmetry for large heavy quark masses leading to several analyses of B_c decays [8–10]. The relations to the universal functions of $B_c \rightarrow J/\psi, \eta_c$ decays have been provided at the leading order [4] in the heavy quark mass expansion. The relations obtained in such a systematic expansion have two applications. They can be used to test the form factors obtained by different methods, for a quantitative assessment of their theoretical uncertainty. Moreover, information can be gained on form factors that have not been computed yet, which are needed for analyses based on the generalized low-energy Hamiltonian. When comparing these relations to recent lattice QCD results [7] violations of the order of 50% are found for the vector form factors, suggesting the need for a better understanding of next-to-leading order effects.

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In this contribution, we describe the analysis at the next-to-leading order in the expansion performed in Ref. [1], where we established further relations among the form factors and a set of universal functions based on the heavy quark spin symmetry and the power counting rules of nonrelativistic QCD (NRQCD). In Section 2 we introduce the relevant form factors, to then we give an outline of the calculation in Section 3. The details can be found in Ref. [1]. In Section 4 we present the main result and discuss its implications.

2 Form Factors for $B_c \rightarrow J/\psi, \eta_c$

We focus on the B_c and $J/\psi, \eta_c$ matrix elements relevant for semileptonic $B_c \rightarrow J/\psi(\eta_c)\ell\bar{\nu}$ decays, which involve the vector (axial) $\bar{Q}'\gamma_\mu(\gamma_5)Q$, (pseudo)scalar $\bar{Q}'(\gamma_5)Q$, and (pseudo)tensor $\bar{Q}'\sigma_{\mu\nu}(\gamma_5)Q$ currents. Due to the nature of QCD these matrix elements cannot be computed perturbatively and thus need to be parametrized in terms of form factors. We choose the following parametrization due to its ability to simplify the expressions that are obtained in the following section.

$$\begin{aligned} \langle P(v')|\bar{Q}'\gamma_\mu Q|B_c(v)\rangle &= \sqrt{m_P m_{B_c}} [h_+(w)(v+v')_\mu + h_-(w)(v-v')_\mu] \\ \langle P(v')|\bar{Q}'Q|B_c(v)\rangle &= \sqrt{m_P m_{B_c}} h_S(w)(1+w) \\ \langle P(v')|\bar{Q}'\sigma_{\mu\nu}Q|B_c(v)\rangle &= -i\sqrt{m_P m_{B_c}} h_T(w)(v_\mu v'_\nu - v_\nu v'_\mu) \end{aligned} \quad (1)$$

$$\begin{aligned} \langle V(v', \epsilon)|\bar{Q}'\gamma_\mu Q|B_c(v)\rangle &= i\sqrt{m_V m_{B_c}} h_V(w)\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}v'^\alpha v'^\beta \\ \langle V(v', \epsilon)|\bar{Q}'\gamma_\mu\gamma_5 Q|B_c(v)\rangle &= \sqrt{m_V m_{B_c}} [h_{A_1}(w)(1+w)\epsilon_\mu^* - h_{A_2}(w)(\epsilon^* \cdot v)v_\mu - h_{A_3}(w)(\epsilon^* \cdot v)v'_\mu] \\ \langle V(v', \epsilon)|\bar{Q}'\gamma_5 Q|B_c(v)\rangle &= -\sqrt{m_V m_{B_c}} h_P(w)(\epsilon^* \cdot v) \\ \langle V(v', \epsilon)|\bar{Q}'\sigma_{\mu\nu}Q|B_c(v)\rangle &= -\sqrt{m_V m_{B_c}} \epsilon^{\mu\nu\alpha\beta} [h_{T_1}(w)\epsilon_\alpha^*(v+v')_\beta + h_{T_2}(w)\epsilon_\alpha^*(v-v')_\beta \\ &\quad + h_{T_3}(w)(\epsilon^* \cdot v)v_\alpha v'_\beta] \end{aligned} \quad (2)$$

where $P = \eta_c$, $V = J/\psi$, $v = \frac{p}{m_{B_c}}$, $v' = \frac{p'}{m_{P(V)}}$, $w = v \cdot v'$ and ϵ is the J/ψ polarization vector. We use $\epsilon^{0123} = +1$ and the relation $\sigma_{\mu\nu}\gamma_5 = \frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}$.

The form factors h_i can be related to a set of universal functions in a kinematical range close to $w = 1$ through the Heavy quark expansion. For hadrons comprising a single heavy quark, this has been done in Refs. [11–13]. The modifications for heavy quarkonium are discussed in the next section. The results for the vector and axial form factors computed in [7] will be exploited in our numerical analysis.

3 Expansion of the Heavy Quark Field and QCD Lagrangian

To construct the heavy quark expansion, the heavy quark QCD field $Q(x)$ with mass m_Q is written factorizing a fast oscillation mass term:

$$Q(x) = e^{-im_Q v \cdot x} \psi(x) = e^{-im_Q v \cdot x} (\psi_+(x) + \psi_-(x)) \quad (3)$$

with $\psi_\pm = P_\pm \psi(x)$ and $P_\pm = \frac{1 \pm \not{v}}{2}$. ψ_+ is the positive energy component of the field (we use the notation adopted in Ref. [14]). v is identified with the heavy meson (quarkonium) 4-velocity with $v^2 = 1$. The equation of motion allows us to relate ψ_- to ψ_+ and obtain an expansion on $1/m_Q$ for both $Q(x)$ and the QCD Lagrangian.

$$Q(x) = e^{-im_Q v \cdot x} \left(1 + \frac{i\vec{D}_\perp}{2m_Q} + \frac{(-iv \cdot D)}{2m_Q} \frac{i\vec{D}_\perp}{2m_Q} + \dots \right) \psi_+(x), \quad (4)$$

$$\mathcal{L}_{QCD} = \bar{\psi}_+(x) \left(iv \cdot D + \frac{(iD_\perp)^2}{2m_Q} + \frac{g}{4m_Q} \sigma \cdot G_\perp + \frac{i\vec{D}_\perp}{2m_Q} \frac{(-iv \cdot D)}{2m_Q} (i\vec{D}_\perp) + \dots \right) \psi_+(x). \quad (5)$$

In this expression $G_{\perp\mu\nu} = (g_{\mu\alpha} - v_\mu v_\alpha)(g_{\nu\beta} - v_\nu v_\beta)G^{\alpha\beta}$ and $D_{\perp\mu} = (g_{\mu\alpha} - v_\mu v_\alpha)D^\alpha$. In the rest frame $G_{\perp\mu\nu} = G_{ij}$ for $\mu = i = 1, 2, 3$ and $\nu = j = 1, 2, 3$, while the other components vanish.

The power counting is delicate since there are two small parameters. On one side, the inverse of the heavy quark mass (HQET), and on another side the 3-velocity in the hadron rest frame (NRQCD). The power counting within NRQCD takes the following form:

$$D_t \sim \tilde{v}^2, \quad D_\perp \sim \tilde{v}, \quad \psi_+ \sim \tilde{v}^{3/2}, \quad E_i = G_{0i} \sim \tilde{v}^3 \quad \text{and} \quad B_i = \frac{1}{2} \epsilon_{ijk} G^{jk} \sim \tilde{v}^4$$

where $\tilde{v} = |\vec{v}| \ll 1$ is the relative heavy quark 3-velocity in the hadron rest frame. In this work, we consider a mixed approach, where we perform the expansion considering terms up to NLO in \tilde{v} and NNLO in $1/m_Q$ in our expansion. NLO terms in \tilde{v} involving three derivatives ($\mathcal{O}(1/m_Q^3)$) are excluded, we assume that they provide numerically suppressed effects.

The first and second terms in Eq. (5) are $\mathcal{O}(\tilde{v}^2)$ and provide the leading order Lagrangian \mathcal{L}_0 while the third and fourth terms are $\mathcal{O}(\tilde{v}^4)$ and give the NLO Lagrangian \mathcal{L}_1 .

3.1 Trace formalism

The B_c and $J/\psi, \eta_c$ matrix elements of the various terms can be expressed using the trace formalism by exploiting the spin-symmetry present in the NRQCD expansion. In this formalism, the lowest-lying S-wave $\bar{b}c$ and $\bar{c}c$ bound states are described by 4×4 matrices

$$H^{c\bar{b}}(v) = \frac{1 + \not{v}}{2} [B_c^{*\mu} \gamma_\mu - B_c \gamma_5] \frac{1 - \not{v}}{2} \quad H^{c\bar{c}}(v') = \frac{1 + \not{v}'}{2} [\Psi^{*\mu} \gamma_\mu - \eta_c \gamma_5] \frac{1 - \not{v}'}{2} \quad (6)$$

satisfying the relations $\not{v}H(v) = H(v) = -H(v)\not{v}$ and $H(v')\not{v}' = H(v') = -\not{v}'H(v')$. $B_c^{*\mu}$, B_c and $\Psi^{*\mu}$, η_c annihilate vector and pseudoscalar $\bar{b}c$ and $\bar{c}c$ mesons of velocity v and v' , respectively. This formalism can be extended to other charmonia states as discussed in Ref. [15].

3.2 Expanding the weak current (Local corrections)

Keeping terms up to $\mathcal{O}(\tilde{v}^3)$ and $\mathcal{O}(1/m_Q^2)$, the current expansion can be written as

$$\bar{Q}'(x)\Gamma Q(x) = J_0 + \left(\frac{J_{1,0}}{2m_Q} + \frac{J_{0,1}}{2m_Q} \right) + \left(-\frac{J_{2,0}}{4m_Q^2} - \frac{J_{0,2}}{4m_Q^2} + \frac{J_{1,1}}{4m_Q m_Q} \right) \quad (7)$$

$$\begin{aligned} J_0 &= \bar{\psi}'_+ \Gamma \psi_+ & J_{0,1} &= \bar{\psi}'_+ \left(-i\overleftarrow{D}'_\perp \right) \Gamma \psi_+ & J_{0,2} &= \bar{\psi}'_+ i\overleftarrow{D}'_\perp \left(iv' \cdot \overleftarrow{D}' \right) \Gamma \psi_+ \\ J_{1,0} &= \bar{\psi}'_+ \Gamma i\overrightarrow{D}_\perp \psi_+ & J_{2,0} &= \bar{\psi}'_+ \Gamma \left(iv \cdot \overrightarrow{D} \right) i\overrightarrow{D}_\perp \psi_+ & J_{1,1} &= \bar{\psi}'_+ \left(-i\overleftarrow{D}'_\perp \right) \Gamma \left(i\overrightarrow{D}_\perp \right) \psi_+ \end{aligned} \quad (8)$$

Using the trace formalism, we parametrize the matrix elements of the various terms in the expansion. For instance, the matrix element of J_0 and $J_{1,0}$ are parametrized as

$$\langle M'(v') | J_0 | M(v) \rangle = -\Delta(w) \text{Tr} \left[\overleftarrow{H}'(v') \Gamma H(v) \right] \quad (9)$$

$$\langle M'(v') | \bar{\psi}'_+ \Gamma^\alpha iD_\alpha \psi_+ | M(v) \rangle = -\text{Tr} \left[\Delta_\alpha(v, v') \overleftarrow{H}'(v') \Gamma^\alpha H(v) \right] \quad (10)$$

and involve the form factor $\Delta(w)$ and the function $\Delta_\alpha(v, v')$ respectively

$$\Delta_\alpha(v, v') = \Delta_+(w)(v + v')_\alpha + \Delta_-(w)(v - v')_\alpha - \Delta_3(w)\gamma_\alpha \quad (11)$$

The remaining terms, including the higher derivative terms, are parametrized analogously [1].

3.3 Expanding the states (Non-local corrections)

In addition to the corrections obtained by the expansion of the weak currents, we must consider the corrections to the states given by the corrections to the Lagrangian in Eq. (5). They can be written as

$$\begin{aligned} \langle M'(v') | i \int d^4x T[J_0(0)\mathcal{L}_1(x)] | M(v) \rangle = & -\frac{1}{2m_Q^2} \chi_1(w) \text{Tr} \left[\bar{H}'(v') \Gamma H(v) \right] \\ & - \frac{1}{4m_Q} \text{Tr} \left[\chi_{2\mu\nu}(w) \bar{H}'(v') \Gamma P_+ \left(-\frac{i}{2} \right) \sigma^{\mu\nu} H(v) \right] \end{aligned} \quad (12)$$

for the quark field and analogously for the antiquark field. The $\chi_i(w)$ functions are arbitrary functions, in the case of $\chi_{2\mu\nu}(w)$ we parametrize it as

$$\chi_{2\mu\nu} = \chi_2^A i\sigma_{\mu\nu} + \chi_2^B (v_\mu \gamma_\nu - v_\nu \gamma_\mu) + \chi_2^C (v'_\mu \gamma_\nu - v'_\nu \gamma_\mu) \quad (13)$$

3.4 Relations between universal functions

The x -dependence of the matrix element $\mathcal{M}_0(x) = \langle M'(v') | \bar{\psi}'_+(x) \Gamma \psi_+(x) | M(v) \rangle$ can be obtained exploiting the dependence in the effective theory [1, 16, 17] obtaining

$$\mathcal{M}_0(x) = e^{-i\phi \cdot x} \mathcal{M}_0(0) \quad (14)$$

with $\phi = \tilde{\Lambda}v - \tilde{\Lambda}'v'$. For $B_c \rightarrow J/\psi(\eta_c)$, the binding energies $\tilde{\Lambda}^{(\prime)}$ are given by $\tilde{\Lambda} = m_{B_c} - m_b - m_c$ and $\tilde{\Lambda}' = m_{J/\psi(\eta_c)} - 2m_c$.

Exploiting the relation

$$i\partial_\alpha (\bar{\psi}'_+ \Gamma \psi_+) = \bar{\psi}'_+ (i\overleftarrow{D}_\alpha) \Gamma \psi_+ + \bar{\psi}'_+ \Gamma (i\overrightarrow{D}_\alpha) \psi_+ \quad (15)$$

the equations of motion and the x -dependence in Eq. (14) we can reduce the number of independent universal functions.

3.5 Form factors in terms of universal functions

Using the expansions above and relating some of the universal functions, one can obtain expressions for the form factors h_i in terms of universal functions. We show here an example for the $h_{A_2}(w)$ form factor.

$$\begin{aligned} h_{A_2} = & \frac{1}{m_c} \left(\frac{1}{1+w} [\phi_K(w) - \Delta(w)\tilde{\Lambda} - \Delta_3(w)] + \frac{1}{2} \tilde{\chi}_2^B(w) \right) + \frac{1}{m_c^2} \frac{1}{2(w+1)} (\tilde{\Lambda}w - \tilde{\Lambda}') [\phi_K(w) - \Delta(w)\tilde{\Lambda} - \Delta_3(w)] \\ & + \frac{1}{4m_c^2} \left[-(1+w)\psi_2^S(w) - (w-1)\psi_3^S(w) + \psi_4^S(w) + \psi_5^S(w) - 2w\psi_6^S(w) - \psi_1^A(w) + \psi_2^A(w) + \psi_4^A(w) \right] \\ & + \frac{1}{4m_b m_c} \left[-\psi_1^S(w) - (1+w)\psi_2^S(w) + (w+1)\psi_3^S(w) \right. \\ & \quad \left. + 3\psi_4^S(w) + 3\psi_5^S(w) - (1+w)\psi_1^A(w) + 3\psi_2^A(w) + \psi_3^A(w) + 3\psi_4^A(w) \right] \end{aligned} \quad (16)$$

The collection of these relations can be found in App. B of Ref.[1].

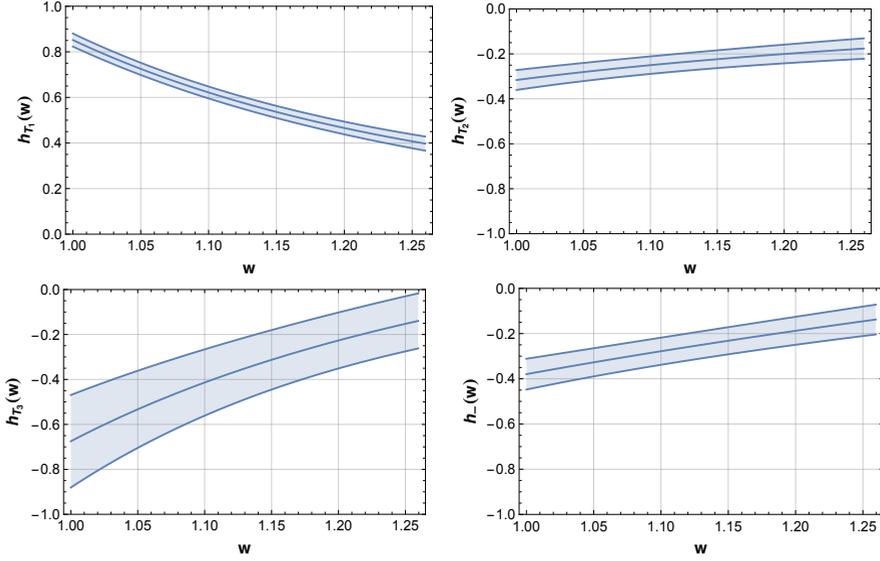


Figure 1. $B_c \rightarrow J/\psi(\eta_c)$ form factors obtained applying Eqs. (17) to (20) in the full kinematical range and using lattice QCD results from [7].

4 Relations between Form Factors at $O(1/m_Q)$

We can take advantage of the relations discussed in the previous section to relate unknown form factors, with the ones that have already been computed on the lattice. Using the above expression at $O(1/m_Q)$ one can derive relations between the form factors for $B_c \rightarrow J/\psi$ (and similarly for $B_c \rightarrow \eta_c$) and the known vector and axial ones.

$$h_{T_1}(w) = \frac{1}{2} \left((1+w)h_{A_1}(w) - (w-1)h_V(w) \right) \quad (17)$$

$$h_{T_2}(w) = \frac{1+w}{2(m_b+3m_c)} \left((m_b-3m_c)h_{A_1}(w) + 2m_c(h_{A_2}(w) + h_{A_3}(w)) - (m_b-m_c)h_V(w) \right) \quad (18)$$

$$h_{T_3}(w) = h_{A_3}(w) - h_V(w) \quad (19)$$

$$h_-(w) = \frac{m_b-m_c}{2(m_b+3m_c)} (1+w) \left(3h_{A_1}(w) - h_{A_2}(w) - h_{A_3}(w) - 2h_V(w) \right) \quad (20)$$

$$h_T(w) = h_+(w) - \frac{m_b+m_c}{2(m_b+3m_c)} (1+w) \left(3h_{A_1}(w) - h_{A_2}(w) - h_{A_3}(w) - 2h_V(w) \right) \quad (21)$$

$$h_T(w) = h_S(w) - \frac{m_b+m_c}{(m_b+3m_c)} \left(3h_{A_1}(w) - h_{A_2}(w) - h_{A_3}(w) - 2h_V(w) \right). \quad (22)$$

Their extrapolations are displayed in Fig. 1. The value of the universal functions at $w = 1$ is not predicted, however from Eqs. (17), (21) and (22) the relations $h_{T_1}(w = 1) = h_{A_1}(w = 1)$ and $h_S(w = 1) = h_+(w = 1)$ are obtained.

As mentioned before, lattice QCD results [7] violate the results in [4] at leading order, however, due to the limited amount of form factors computed on the lattice, consistency checks including the NLO corrections cannot be performed as of now. Future results from lattice QCD for other $B_c \rightarrow J\psi$ currents or the $B_c \rightarrow \eta_c$ mode are fundamental to check the consistency between the formalism discussed here and lattice computations, leading to a

better understanding of all form factors, including the already computed $B_c \rightarrow J/\psi$ SM form factors.

5 Conclusions

Using the heavy quark expansion, the heavy quark spin symmetry and NRQCD power counting we have expressed the form factors parametrizing the matrix elements $\langle J/\psi(\eta_c) | \bar{c} \Gamma_i b | B_c \rangle$ in terms of universal functions near the zero-recoil point and we established relations among form factors in this kinematical range. Lattice QCD results for the matrix element of the Standard Model operator between B_c and J/ψ allow us to predict the tensor form factors and the h_- form factor for $B_c \rightarrow \eta_c$. Further information from lattice QCD is fundamental to further check the relations worked out here and the general coherence of both lattice results and the heavy quark expansion predictions. In the future, a better understanding can be obtained through the use of unitarity constraints and the dispersion matrix [18, 19] in order to constrain the form factor extrapolations out of the zero-recoil point $w \sim 1$.

Acknowledgments

I would like to thank my collaborators P. Colangelo, F. de Fazio, F. Lopalco and N. Losacco with whom this work was developed. This study has been carried out within the INFN project (Iniziativa Specifica) QFT-HEP.

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