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Inclusive $b \to \{u, c\}\, \ell^- \bar{\nu}_\ell$ modes: Polarized/unpolarized Λ_b

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Abstract. The increasing number of flavour anomalies motivates the investigation of new processes where tensions similar to the observed ones may emerge. It is necessary to identify observables sensitive to physics beyond the Standard Model. The analysis which follows concerns the inclusive semileptonic decays of polarized beauty baryons, computed through the Heavy Quark Expansion at $O(1/m_b^3)$ and at the leading order in α_s. New Physics interactions have been taken into account, extending the Standard Model $b \to U\,\ell\,\bar{\nu}_\ell$ low-energy Hamiltonian, where $U = \{c, u\}$ and $\ell = \{e, \mu, \tau\}$, including the full set of $D = 6$ operators with left-handed neutrinos. Among the possible observables one can consider, the ones depending on the spin of the decaying baryon are very appealing and can be considered for physics programmes of future facilities, such as FCC-ee.

1 Introduction

Deviations in a number of observables w.r.t. the Standard Model (SM) predictions, the *flavour anomalies*, have been recently detected. They represent a motivation for searching signals of New Physics (NP). Anomalies have been observed in tree-level and loop-induced decays of B, B_s and B_c mesons [1, 2]. Particularly interesting, hints of lepton flavour universality (LFU) violations have been collected. It is necessary to investigate other heavy hadron decay processes to get a full comprehension of LFU, for this reason inclusive semileptonic beauty baryon decays have been analyzed [3] treating the nonperturbative effects of strong interactions in a systematic way, exploiting an expansion in the inverse heavy quark mass [4, 5]. The calculation of the inclusive semileptonic decay width $H_b \to X_{c,u}\,\ell^-\,\bar{\nu}_\ell$ of a polarized heavy hadron H_b will be described below, expanding the hadronic matrix elements at $O(1/m_b^3)$ in the heavy quark expansion (HQE), at leading order in α_s. Observables have been computed for the transition $\Lambda_b \to X_{c,u}\,\ell^-\,\bar{\nu}_\ell$ for non vanishing charged lepton mass and considering the NP generalization of the SM in the effective Hamiltonian, including all the $D = 6$ semileptonic operators with left-handed neutrinos [6–12]. A recent improvement has been achieved in [13] where α_s corrections has been computed for the inclusive $B \to X_c\,\tau\,\bar{\nu}_\tau$ decay width and leptonic invariant mass spectrum up to $O(1/m_b^3)$ in the HQE.

At LHC the Λ_b is produced unpolarized [14–17] because the b quark is produced through strong interactions, a sizeable longitudinal Λ_b polarization is expected for b quarks produced in Z and t quark decays [18–20].

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2 Generalized effective Hamiltonian

Let us consider a beauty hadron H_b with spin s. The inclusive semileptonic decays $H_b(p,s) \to X_U(p_X)\,\ell^-(p_\ell)\,\bar\nu_\ell(p_\nu)$, induced by the quark transition $b \to U$, with $U = \{c, u\}$, are described by the general low-energy Hamiltonian which extends the SM one:

$$\mathcal{H}_{\rm eff}^{b\to U\,\ell\,\bar\nu} = \frac{G_F\,V_{Ub}}{\sqrt{2}}\left[(1+\epsilon_V^\ell)\,O_{\rm SM} + \epsilon_S^\ell\,O_S + \epsilon_P^\ell\,O_P + \epsilon_T^\ell\,O_T + \epsilon_R^\ell\,O_R\right] + \text{h.c.}\,, \tag{1}$$

that comprises the Fermi constant G_F, the CKM matrix element V_{Ub} and the full set of $D = 6$ semileptonic $b \to U$ operators with left-handed neutrinos: $O_{\rm SM,R} = [\bar u\,\gamma_\mu\,(1 \mp \gamma_5)\,b]\,[\bar\ell\,\gamma^\mu\,(1-\gamma_5)\,\nu_\ell]$, $O_S = [\bar u\,b]\,[\bar\ell\,(1-\gamma_5)\nu_\ell]$, $O_P = [\bar u\,\gamma_5\,b]\,[\bar\ell\,(1-\gamma_5)\nu_\ell]$ and $O_T = [\bar u\,\sigma_{\mu\nu}\,(1-\gamma_5)\,b]\,[\bar\ell\,\sigma^{\mu\nu}\,(1-\gamma_5)\,\nu_\ell]$. $\epsilon^\ell_{V,S,P,T,R}$ are complex lepton-flavour dependent couplings. In the case of $\epsilon_i^\ell = 0$ one recovers the SM case. We keep $m_\ell \neq 0$ for all leptons $\ell = \{e, \mu, \tau\}$.

3 Inclusive decay width

Any $D = 6$ operator in Eq. (1) can be written as the product of two currents, $O = J_M^{(i)}\,L^{M(i)}$, where $J_M^{(i)}$ and $L^{M(i)}$ are respectively the hadronic and the leptonic currents. M generically denotes the Lorentz indices of the currents. The H_b inclusive semileptonic differential decay width can be written as

$$\mathrm{d}\Gamma = \underbrace{\mathrm{d}\Sigma}_{\text{phace space}}\;\frac{G_F^2\,|V_{Ub}|^2}{4\,M_H}\sum_{i,j} g_i^*\,g_j\;\underbrace{(W^{ij})_{MN}}_{\text{hadronic tensor}}\;\underbrace{(L^{ij})^{MN}}_{\text{leptonic tensor}}\,, \tag{2}$$

with the phase-space element $d\Sigma = (2\,\pi)^4\,d^4q\,\delta^4(q-p_\ell-p_\nu)\,[dp_\ell]\,[dp_\nu]$. The notation $[dp] = \frac{d^3p}{(2\,\pi)^3\,2\,p^0}$ has been used. $g_V = 1+\epsilon_V^\ell$ and $g_{S,P,T,R} = \epsilon^\ell_{S,P,T,R}$ are the Wilson coefficients.
Using the optical theorem, the hadronic tensor $(W^{ij})_{MN} = \frac{1}{\pi}\,\mathrm{Im}\left[(T^{ij})_{MN}\right]$ is given in terms of the forward amplitude depicted in figure 1

$$(T^{ij})_{MN} = i\int d^4x\,e^{-i\,q\cdot x}\,\langle H_b(p,s)|\mathbb{T}\left\{J_M^{(i)\dagger}(x)\,J_N^{(j)}(0)\right\}|H_b(p,s)\rangle\;. \tag{3}$$

The hadron momentum $p = m_b\,v + k$ is written in terms of the 4-velocity v, the heavy quark

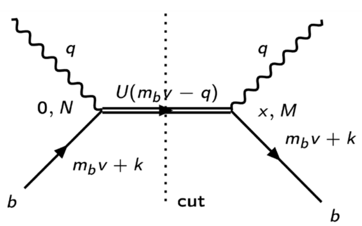

Figure 1. Discontinuity of the forward amplitude across the cut of the semileptonic process.

mass m_b and the residual momentum k, with $|k| \sim O(\Lambda_{\rm QCD})$. The QCD quark field $b(x)$ is redefined as $b(x) = e^{-i\,m_b\,v\cdot x}\,b_v(x)$, with $b_v(x)$ still QCD quark field satisfying the EoM $b_v(x) = \left(P_+ + \frac{i\,\not{D}}{2\,m_b}\right)\,b_v(x)$, where $P_\pm = \frac{1\pm\not{v}}{2}$ are the velocity projectors.

Introducing $p_X = m_b\,v + k - q$, the Eq. (3) becomes

$$(T^{ij})_{MN} = \langle H_b(v,s)|\bar b_v(0)\,\overline\Gamma_M^{(i)}\,\frac{1}{\not{p}_X - m_U}\,\Gamma_N^{(j)}\,b_v(0)|H_b(v,s)\rangle\;, \tag{4}$$

and considering $|k| \sim O(\Lambda_{\rm QCD})$ we have

$$(T^{ij})_{MN} = \sum_{n=0}^{+\infty} \langle H_b(v,s)|\bar{b}_v(0)\,\overline{\Gamma}^{(i)}_M\,(\not p_U + m_U)\,(i\not D\,(\not p_U + m_U))^n\,\Gamma^{(j)}_N\,b_v(0)|H_b(v,s)\rangle\;\frac{(-1)^n}{\Delta_0^{n+1}}\;, \tag{5}$$

where $\overline{\Gamma}^{(i)}_M = \gamma^0\,\Gamma^{(i)\dagger}_M\,\gamma^0$, $p_U = p_X - k$ and $\Delta_0 = p_U^2 - m_U^2$.

Using the trace formalism [21], we can write the n-th term in the series as

$$\begin{aligned}&\langle H_b(v,s)|\bar{b}_v(0)\,\overline{\Gamma}^{(i)}_M\,(\not p_U + m_U)\;\underbrace{i\not D\,(\not p_U + m_U)\dots i\not D\,(\not p_U + m_U)}_{\text{n times}}\;\Gamma^{(j)}_N\,b_v(0)|H_b(v,s)\rangle = \\ &= \left[\overline{\Gamma}^{(i)}_M\,(\not p_U + m_U)\,\prod_{k=1}^{n}\left[\gamma_{\mu_k}\,(\not p_U + m_U)\right]\Gamma^{(j)}_N\right]_{ab}\;\underbrace{\langle H_b(v,s)|\bar{b}_v(0)\,iD^{\mu_1}\,\dots\,iD^{\mu_n}\,b_v(0)|H_b(v,s)\rangle_{ba}}_{(\mathcal{M}^{\mu_1\cdots\mu_n})_{ba}}\;.\end{aligned} \tag{6}$$

The expression in Eq. (6) involves the hadronic matrix elements $(\mathcal{M}^{\mu_1\cdots\mu_n})_{ba}$ where a and b are Dirac indices. These matrix elements can be written in terms of nonperturbative parameters, the number of which increases with the order of the expansion:

$$O(1/m_b^n)\dots\begin{cases} O(1/m_b^3)\begin{cases} O(1/m_b^2)\begin{cases} -2\,M_H\,\hat{\mu}_\pi^2 = \langle H_b|\bar{b}_v\,iD^\mu\,iD_\mu\,b_v|H_b\rangle \\ 2\,M_H\,\hat{\mu}_G^2 = \langle H_b|\bar{b}_v\,(-i\sigma_{\mu\nu})\,iD^\mu\,iD^\nu\,b_v|H_b\rangle \end{cases} \\ 2\,M_H\,\hat{\rho}_D^3 = \langle H_b|\bar{b}_v\,iD^\mu\,(iv\cdot D)\,iD_\mu\,b_v|H_b\rangle \\ 2\,M_H\,\hat{\rho}_{LS}^3 = \langle H_b|\bar{b}_v\,(-i\sigma_{\mu\nu})\,iD^\mu\,(iv\cdot D)\,iD^\nu\,b_v|H_b\rangle \end{cases} \\ \dots \end{cases}\;. \tag{7}$$

To compute $\mathcal{M}^{\mu_1\cdots\mu_n}$ one can follow the methods proposed in [21, 22].
In the case of baryons, the dependence on the spin s_μ in Eq. (6) must be kept into account as done up to $O(1/m_b^{-2})$ in [23]. In [3] $\mathcal{M}^{\mu_1\cdots\mu_n}$ have been derived at $O(1/m_b^{-3})$ for a polarized baryon, extending the previous results obtained in [10, 12, 23–25].

From the expressions of the matrix elements $\mathcal{M}^{\mu_1\cdots\mu_n}$ the hadronic tensor can be computed and expanded in Lorentz structures which depend on v, q and s. The results for the effective Hamiltonian Eq. (1) are collected in [3]. The four-fold differential decay rate for the $H_b(p,s) \to X_U(p_X)\,\ell^-(p_\ell)\,\bar{\nu}_\ell(p_\nu)$ transition reads

$$\frac{\mathrm{d}^4\Gamma}{\mathrm{d}E_\ell\,\mathrm{d}q^2\,\mathrm{d}q_0\,\mathrm{d}\cos\theta_P} = \frac{G_F^2\,|V_{Ub}|^2}{32\,(2\,\pi)^3\,M_H}\,\sum_{i,j} g_i^*\,g_j\,\frac{1}{\pi}\,\mathrm{Im}\left[(T^{ij})_{MN}\right]\,(L^{ij})^{MN}\;, \tag{8}$$

where $p_\ell = (E_\ell, \vec{p}_\ell)$, $q_0 = q\cdot v$ and θ_P is the angle between the two 3-vectors $\vec{p}_\ell$ and $\vec{s}$ in the H_b rest frame. Double and single distributions are obtained integrating Eq. (8) over the phase-space [26]. The full decay width can be obtained by performing all integrations and can be written as:

$$\Gamma(H_b \to X_U\,\ell^-\,\bar{\nu}_\ell) = \Gamma_b\,\sum_{i,j} g_i^*\,g_j\,\left[C_0^{(i,j)} + \frac{\hat{\mu}_\pi^2}{m_b^2}\,C^{(i,j)}_{\hat{\mu}_\pi^2} + \frac{\hat{\mu}_G^2}{m_b^2}\,C^{(i,j)}_{\hat{\mu}_G^2} + \frac{\hat{\rho}_D^3}{m_b^3}\,C^{(i,j)}_{\hat{\rho}_D^3} + \frac{\hat{\rho}_{LS}^3}{m_b^3}\,C^{(i,j)}_{\hat{\rho}_{LS}^3}\right]\,, \tag{9}$$

where $\Gamma_b = \frac{G_F^2\,|V_{Ub}|^2\,m_b^5}{192\,\pi^3}$ is the *partonic term* corresponding to a free quark decay. The indices i and j runs over the contributions of the various operators and of their interferences. The coefficients $C^{(i,j)}$ can be found in [3]. The OPE breaks down in the endpoint region of the

Table 1. Inclusive semileptonic Λ_b branching fractions in SM, obtained for the central values of the parameters.

$b \to c$		$b \to u$	
$\mathcal{B}(\Lambda_b \to X_c\, \mu\, \bar{\nu}_\mu)$	11.0×10^{-2}	$\mathcal{B}(\Lambda_b \to X_u\, \mu\, \bar{\nu}_\mu)$	11.65×10^{-4}
$\mathcal{B}(\Lambda_b \to X_c\, \tau\, \bar{\nu}_\tau)$	2.4×10^{-2}	$\mathcal{B}(\Lambda_b \to X_u\, \tau\, \bar{\nu}_\tau)$	2.75×10^{-4}

spectra, where singularities appear and which should be resummed in a H_b shape function, the convolution with such a function smears the spectra at the endpoint. The effects of the baryon shape function has not been considered here. Perturbative QCD corrections have also been not included: in the SM case they can be found in [27–32]. NLO QCD corrections have also been recently computed in [13, 33].

4 Results for $\Lambda_b \to X_{c,u}\, \ell\, \bar{\nu}_\ell$

In [3] several observables for the modes $\Lambda_b \to X_{c,u}\, \ell\, \bar{\nu}_\ell$ have been studied. Here we consider a few of them. Further ones, as well as the input parameters, can be found in [3]. For the couplings ϵ_i^ℓ in Eq. (1) we fix three NP benchmark points, set in [7, 34] for $U = c$, while for $U = u$ we use the ranges fixed in [35]. Using $G_F = 1.16637(1) \times 10^{-5}\,\mathrm{GeV}^{-2}$, $|V_{cb}| = 0.042$ and $|V_{ub}| = 0.0037$, together with $\tau_{\Lambda_b} = 1.471(9)\,\mathrm{ps}$ [36], we compute the inclusive Λ_b branching fraction for the two quark transitions and for final τ and μ lepton. The results in the SM, for the central values of the parameters and neglecting QCD corrections, are in table 1. For comparison, the available measurements are $\mathcal{B}(\Lambda_b \to \Lambda_c\, \ell^-\, \bar{\nu}_\ell + \text{anything}) = (10.9 \pm 2.2) \times 10^{-2}$, with $\ell = e, \mu$ and $\mathcal{B}(\Lambda_b \to p\, \mu^-\, \bar{\nu}_\mu) = (4.1 \pm 1.0) \times 10^{-4}$[36].

For inclusive semileptonic Λ_b decays it is interesting to consider a ratio analogous to $R(D^*)$ for B meson, to compare the τ and the muon mode using a quantity in which several theoretical uncertainties cancel:

$$R_{\Lambda_b}(X_U) = \frac{\Gamma(\Lambda_b \to X_U\, \tau\, \bar{\nu}_\tau)}{\Gamma(\Lambda_b \to X_U\, \mu\, \bar{\nu}_\mu)} \qquad \text{with} \qquad U = u, c\,. \tag{10}$$

Another ratio sensitive to LFU violating NP effects can be defined. It can be constructed from the distribution $\frac{d\Gamma(\Lambda_b \to X_U\, \ell\, \bar{\nu}_\ell)}{d\cos\theta_P} = A_\ell^U + B_\ell^U \cos\theta_P$. The dependence of $\frac{d\Gamma(\Lambda_b \to X_U\, \ell\, \bar{\nu}_\ell)}{d\cos\theta_P}$ on $\cos\theta_P$ is linear, and NP contributions modify both the slope and the intercept of the curve as displayed in figure 2. Similarly to the ratio defined in Eq. (10), we define the ratio of the slopes $R_S^U = B_\tau^U / B_\mu^U$ which has a definite value in SM, and can deviate due to NP as specified in table 2. A correlation between $R_{\Lambda_b}(X_U)$ and R_S^U can be constructed. As an example, for

Table 2. Ratios $R_{\Lambda_b}(X_u)$ and $R_{\Lambda_b}(X_c)$ (left) and ratios R_S^u and R_S^c (right) for SM and NP at the BP.

	$R_{\Lambda_b}(X_u)$	$R_{\Lambda_b}(X_c)$	R_S^u	R_S^c
SM	0.234	0.214	0.081	0.100
NP	0.238	0.240	0.091	0.074

$\Lambda_b \to X_c\, \tau\, \bar{\nu}_\tau$ with the effective Hamiltonian extended including a tensor operator, we vary the couplings $(\mathrm{Re}[\epsilon_T^\mu], \mathrm{Im}[\epsilon_T^\mu])$ and $(\mathrm{Re}[\epsilon_T^\tau], \mathrm{Im}[\epsilon_T^\tau])$ in the regions determined in [7]. The correlation plot in figure 3 shows that the (challenging) measurement of the two ratios would discriminate SM and NP.

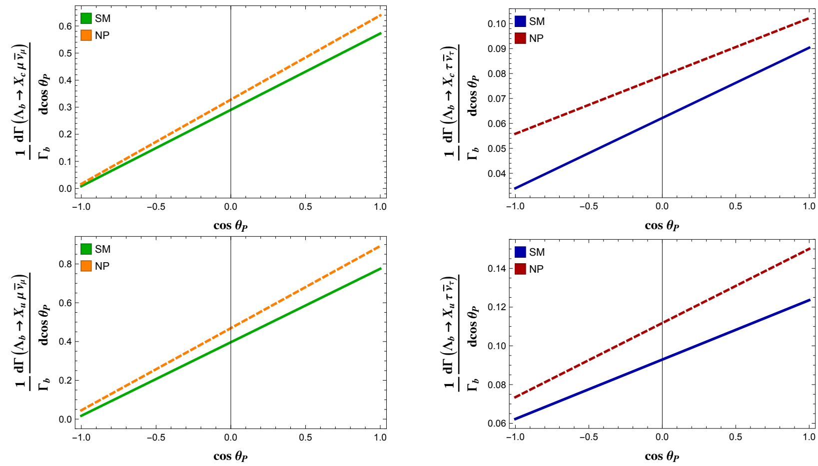

Figure 2. $\frac{1}{\Gamma_b}\frac{d\Gamma}{d\cos\theta_P}$ distribution for $\Lambda_b \to X_U\,\ell\,\bar{\nu}_\ell$, with $U = c$ (top row), $U = u$ (bottom row), $\ell = \mu$ (left column) and $\ell = \tau$ (right column). The solid line is the SM result, the dashed line the NP result at the benchmark point.

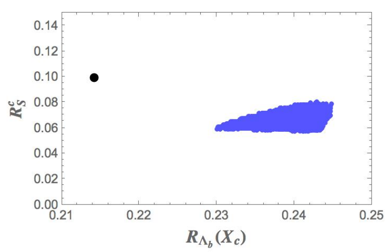

Figure 3. Correlation between $R_{\Lambda_b}(X_c)$ and the ratio R_S^c of the slopes of the $\frac{d\Gamma}{d\cos\theta_P}$ distribution. The dot corresponds to SM, the broad region to NP with the effective couplings varied as specified in the text.

5 Conclusions

The calculation of the fully differential inclusive semileptonic decay width of a polarized heavy hadron at $\mathcal{O}(1/m_b^3)$ in the HQE, at leading order in α_s and for non vanishing charged lepton mass has been described. NP generalization of the SM has been considered in the effective Hamiltonian in Eq.(1) including all the $D = 6$ semileptonic operators with left-handed neutrinos. The correlation plot in figure 3 shows that the ratios $R_{\Lambda_b}(X_c)$ and R_S^c can assume different values in the SM and when NP is included. Although this measurement is experimentally challenging, it represents a suitable observable to test violation of LFU.

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