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Pseudoscalars and the η' in the holographic soft-wall model

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Abstract. Pseudoscalar mesons and glueballs are studied in a model based on the AdS/QCD correspondence. Glueball masses are obtained in the pure-gauge case, while the mixing between pseudoscalar glueballs and singlet pseudoscalar mesons produces the expected mass of the η' meson. The topological susceptibility is computed as a function of the quark mass. The model reproduces partial conservation of axial current, the anomaly equation, and the Witten-Veneziano relation.

1 Introduction

The η' meson has a mass of 958 MeV, too high for it to be a pseudo-Goldstone boson. Indeed, while $SU(2)_A$ symmetry is spontaneously and explicitly broken generating an octet of pseudo-Goldstone bosons, $U(1)_A$ is anomalous. We describe the $U(1)_A$ anomaly and reproduce the η' mass in a bottom-up holographic model by considering coupled singlet pseudoscalar mesons and glueballs.

2 Model and results

In this work, the η' mass and the topological susceptibility are computed in the soft-wall model [1], one of the bottom-up approaches that can be followed when applying the AdS/CFT correspondence to QCD. This is motivated by the fact that in AdS/CFT the nonperturbative regime of the gauge theory corresponds to the supergravity limit of the dual string theory, characterized by small coupling and a large radius of the spacetime. In a bottom-up approach, a $5d$ effective field theory is properly constructed in a anti-de Sitter (AdS) space with metric

$$ds^2 = \frac{R^2}{z^2}(dt^2 - d\bar{x}^2 - dz^2)\,, \tag{1}$$

with R the AdS curvature radius and $z \geqslant 0$ the additional bulk coordinate; $z = 0$ corresponds to the AdS boundary. The AdS/CFT correspondence links a gravity theory to a conformal gauge theory, but QCD is only asymptotically conformal, so in the soft-wall model conformal invariance is broken by inserting a background dilaton field $e^{-\phi}$ in the action, behaving as $e^{-c^2z^2}$, where c sets the mass scale of the model. The $5d$ theory contains the fields dual to the QCD operators under study, whose masses are fixed by the conformal dimension of their dual

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operator according to the relation $m_5^2 R^2 = (\Delta - p)(\Delta + p - 4)$; their boundary values (i.e. their values at $z = 0$) are the sources of the corresponding QCD operators. A crucial feature of AdS/CFT is that the partition function of the $5d$ model is equal to the generating functional of the correlators in the gauge theory. The former has a simple form in the supergravity limit, given by the exponential of the on-shell action. This means that the correspondence provides a straightforward way to compute correlation functions of a strongly-coupled gauge theory, simply deriving the on-shell action of the bulk theory with respect to the boundary values of the fields. To this aim, it is useful to write a field in Fourier space as $\phi(q,z) = K(q,z)\phi_0(q)$, where $\phi_0(q)$ is the source of the operator and $K(q,z)$ is a the so-called bulk-to-boundary propagator.

The soft-wall model was introduced in [1] to study chiral symmetry breaking, so the involved fields and the Lagrangian are well known. The fields dual to the left and right currents are the vector fields $A^M_{L/R}(x,z) = A^{M,A}_{L/R}(x,z)T^A$, where T^A ($A = 0, ..., n_f^2 - 1$) are the generators of $U(n_f)$, with $T^0 = \frac{1}{2n_f} I_{n_f}$ and $\mathrm{Tr}(T^a T^b) = \delta^{ab}/2$ for $a, b = 1, ..., n_f^2 - 1$. Their masses vanish, so they are $5d$ gauge fields. Indeed, the global chiral symmetry of QCD is promoted to a local symmentry in the dual $5d$ theory. The axial field is then defined as $A = (A_L - A_R)/2$. We choose the gauge $A_5 = 0$, while the other components of the axial field can be written as a sum $A^A_\mu = A^A_{\perp\,\mu} + \partial_\mu \varphi^A$ of a transverse part, describing axial-vector mesons, and a longitudinal part, contributing to the descritpion of pseudoscalar mesons. The $5d$ field dual to the QCD $\bar{q}_R q_L$ operator is a scalar field that has a vacuum expectation value (vev) $X_0(z)$, and a phase η^A describing pseudoscalar mesons: $X(x,z) = e^{i\eta^A(x,z)T^A} X_0(z) e^{i\eta^A(x,z)T^A}$. We consider a model with $n_f = 3$ identical quarks, so X_0 is proportional to the identity matrix: $X_0(z) = \sqrt{2}\, v_q(z)\, I_{n_f}$. We also assume [2]:

$$v_q(z) = \frac{m_q}{R}\, z + \frac{\sigma}{R}\, z^3\,, \tag{2}$$

where m_q is the quark mass, and σ is an independent coefficient related to the chiral condensate: $\langle \bar{q}q \rangle = -\frac{N_c}{2\pi^2}\sigma$. Finally, the field $Y(x,z) = Y_0(z)e^{2ia(x,z)}$ is a scalar dual to the square of the gluon field strength. The phase $a(x,z)$ is dual to the $\frac{\alpha_s}{8\pi} G_{\mu\nu}\tilde{G}^{\mu\nu}$ operator describing the pseudoscalar glueballs, and it mixes with the singlet pseudoscalar meson fields. The vev is given by:

$$Y_0(z) = \frac{y_0}{R} + \frac{2\, y_1}{R\, c^4}(e^{c^2 z^2}(-1 + c^2 z^2) + 1)\,, \tag{3}$$

a form obtained solving the equation of motion assuming no potential term. It contains two additional parameters, y_0 and y_1. We assume $a(x,z) = a_P(x,z) + V_a(z)a_f(x,z)$, where a_P is invariant under the $U(1)_A$ symmetry, while a_f transforms under $U(1)_A$ by a shift; $V_a(z) = e^{-v_q(z)^2}$ is a potential term which vanishes in the pure-gauge limit [3].

The Lagrangian describing these fields is:

$$\mathcal{L} = \frac{1}{k}\, \sqrt{g}\, e^{-\phi} \left[\mathrm{Tr}\left\{ -\frac{1}{4g_5^2}(F_L^2 + F_R^2) + |DX|^2 - m_X^2 |X|^2 \right\} + \frac{1}{2}\mathcal{K}_a \right], \tag{4}$$

$$\mathcal{K}_a = |\partial_M Y_0(z) + 2iY_0(\partial_M a(x,z) - \eta^0(x,z)\partial_M V_a(z) - A^0_M(x,z)V_a(z))|^2\,. \tag{5}$$

The first term describes the octet of pseudoscalar mesons [1], while the second deals with the Y field and is invariant under $U(1)_A$ transformations:

$$\eta^0 \to \eta^0 - \alpha \tag{6}$$

$$\varphi^0 \to \varphi^0 - \alpha \tag{7}$$

$$a \to a - V_a \alpha\,. \tag{8}$$

The term in Eq. (5) has also been obtained from the Wess-Zumino action in another bottom-up model [4]. After expanding the Lagrangian, we obtain the following expression:

$$\begin{aligned}\mathcal{L} &= \frac{R}{k}e^{-\phi}\Bigg[\frac{1}{4n_f g_5^2 z}(\partial_z\partial_\nu\varphi^0)^2+\frac{1}{2g_5^2 z}(\partial_z\partial^\nu\varphi^8)^2-\frac{2R^2\,v_q^2}{n_f z^3}(\partial_z\eta^0)^2+\\ &+\frac{2R^2\,v_q^2}{n_f z^3}(\partial_\nu\eta^0-\partial_\nu\varphi^0)^2-\frac{4\,R^2\,v_q^2}{z^3}(\partial_z\eta^8)^2+\frac{4\,R^2\,v_q^2}{z^3}(\partial_\nu\eta^8-\partial_\nu\varphi^8)^2+\\ &-\frac{2R^2}{z^3}Y_0^2(\partial_z a-\eta^0\partial_z V_a)^2+\frac{2R^2}{z^3}Y_0^2(\partial_\nu a-V_a\partial_\nu\varphi^0)^2\Bigg],\end{aligned} \tag{9}$$

containing the known terms for η_8 and ϕ_8, which describe the octet of pseudoscalar mesons, plus new terms involving a, η_0 and ϕ_0, describing the singlet pseudoscalar mesons. Notice that the a field is only coupled to singlet pseudoscalar mesons, since we are considering identical quarks.

Some of the parameters appearing in the Lagrangian are already known from previous studies in the soft-wall model: $R/k = N_c/16\pi^2$ and $g_5^2 = 3/4$ are fixed from scalar and vector meson two-point functions [5]; $c = 388$ MeV is obtained by fitting the ρ meson mass [1]. There are four unknown parameters, m_q, $\bar{q}q$, y_0, y_1, that we shall fix from phenomenology.

2.1 Pseudoscalar glueballs in pure gauge

Let us first consider the pure-gauge case, where the action only contains the a field describing pseudoscalar glueballs:

$$\mathcal{S}_{PG} = \frac{R}{k}\int d^5x\, e^{-\phi}\left[-\frac{2R^2}{z^3}Y_0^2(\partial_z a_P)^2+\frac{2R^2}{z^3}Y_0^2(\partial_\mu a_P)^2\right]. \tag{10}$$

The bulk-to-boundary propagator and the two-point function of the $G\tilde{G}$ operator can be computed analytically [2]. The two-point function has an infinite number of poles, the positions of which correspond to the masses of the pseudoscalar glueballs $m_n^2 = 4c^2(n+2)$, while the residues R_n are related to the decay constants: $R_n = m_n^4 f_n^2$. The lightest state has mass $m_{G\tilde{G},0} = 1.1$ GeV while the first radial excitation has mass $m_{G\tilde{G},1} = 1.34$ GeV, suggesting it could correspond to the $\eta(1405)$.
The first term in the high $Q^2 = -q^2$ expansion of the two-point function can be matched to QCD [6], thus fixing $y_0 = \frac{1}{\sqrt{N_c}}\frac{\alpha_s}{\pi}$ [2]. This shows that the Y field appears in the Lagrangian at a lower order in the large N_c expansion with respect to the other fields. Indeed, in the limit $N_c \to \infty$ the anomaly turns off, and the η' is degenerate with pions, while the degeneracy is lifted by $q\bar{q}$ annihilation diagrams, which are suppressed by $1/N_c$. Assuming $y_0 = \frac{1}{\sqrt{N_c}\pi}$, the decay constant of the ground state is 9.8 MeV.
The $q^2 \to 0$ limit of the two-point function gives the topological susceptibility [7]:

$$\chi_t = -\lim_{q^2\to 0}\Pi_{aa}(q^2). \tag{11}$$

In pure gauge we find $\chi_{PG} = \alpha_s y_1/\pi^3$. Assuming $\chi_{PG} \sim (191\text{ MeV})^4$, we can fix $y_1 = 0.041\text{ GeV}^4$.

2.2 Nonsinglet pseudoscalar mesons

Away from pure gauge, a tower of singlet and nonsinglet pseudoscalar mesons appears. The equations of motion for the fields describing each nonsinglet meson in the octet are:

$$\partial_z\left(\frac{v_q^2 e^{-\phi}}{z^3}\partial_z\eta^8\right)+q^2\frac{v_q^2 e^{-\phi}}{z^3}(\eta^8-\varphi^8)=0 \tag{12}$$

$$\partial_z \left(\frac{e^{-\phi}}{g_5^2 z} \partial_z \varphi^8 \right) + \frac{8v_q^2 e^{-\phi}}{z^3} (\eta^8 - \varphi^8) = 0\,. \tag{13}$$

A combination of the two equations gives a constraint equation:

$$\frac{q^2}{g_5^2 z} \partial_z \varphi^8 - \frac{8v_q^2}{z^3} \partial_z \eta^8 = 0\,. \tag{14}$$

To study correlation functions it is convenient to use the matrix formalism [8], since the two fields are given by a linear combination of the sources in Fourier space:

$$\Phi = \begin{pmatrix} \varphi^8 \\ \eta^8 \end{pmatrix} = F\,\Phi_0\,, \tag{15}$$

where $\Phi_0(q^2) = (\varphi_0^8(q^2), -\eta_0^8(q^2))^T$ is the vector containing the sources of the two operators. Then, the two one-point functions are contained in a vector:

$$J^8 = \left. \frac{\partial \mathcal{S}_{os}}{\partial \Phi_0} \right|_{z \to \varepsilon} = \begin{pmatrix} \langle J_\varphi^8 \rangle \\ \langle J_\eta^8 \rangle \end{pmatrix}, \tag{16}$$

where $\langle J_\varphi^8 \rangle$ and $\langle J_\eta^8 \rangle$ are the one-point functions of the longitudinal axial current $(\partial_\mu \bar{\psi} \gamma_5 \gamma^\mu T^8 \psi)$ and the pseudoscalar current $(2m_q \bar{\psi} \gamma_5 T^8 \psi)$, respectively, in the nonsinglet sector. Using Eq. (14), we find

$$\langle J_\varphi^8 \rangle = \langle J_\eta^8 \rangle\,, \tag{17}$$

which establishes the partial conservation of axial current.
The two-point functions are collected in the matrix

$$\Pi^{88} = \left. \frac{\partial^2 \mathcal{S}_{os}}{\partial \Phi_0 \partial \Phi_0} \right|_{z \to \varepsilon} = \begin{pmatrix} \Pi_{\varphi\varphi}^{88} & \Pi_{\varphi\eta}^{88} \\ \Pi_{\eta\varphi}^{88} & \Pi_{\eta\eta}^{88} \end{pmatrix}. \tag{18}$$

We have checked that in the large $Q^2 = -q^2$ limit they behave as in QCD [2]. The pion decay constant in the chiral limit is:

$$f_\pi^2 = -\frac{R}{k} \frac{e^{-\phi}}{g_5^2 z} \partial_z F_{00} \Bigg|_{\substack{q^2=0 \\ z \to \varepsilon}} = (91.6 \text{ MeV})^2\,, \tag{19}$$

a value obtained after setting $|\langle q\bar{q} \rangle| = (0.281 \text{ GeV})^3$. The strange quark mass can be set requiring that the first pole of the two-point function is located at the mass of the η meson, obtaining $m_{\eta_8} = 548$ MeV if $m_q = m_s = 59.5$ MeV.

2.3 Singlet pseudoscalar mesons

Next, we have solved the three coupled equations of motion for the singlet states η_0, ϕ_0 and a:

$$\partial_z \left(\frac{v_q^2 e^{-\phi}}{n_f z^3} \partial_z \eta^0 \right) + q^2 \frac{v_q^2 e^{-\phi}}{n_f z^3} (\eta^0 - \varphi^0) + \frac{Y_0^2 e^{-\phi}}{z^3} (\partial_z V_a)(\partial_z a - \eta^0 \partial_z V_a) = 0 \tag{20}$$

$$\partial_z \left(\frac{e^{-\phi}}{2n_f g_5^2 z} \partial_z \varphi^0 \right) + \frac{4v_q^2 R^2 e^{-\phi}}{n_f z^3} (\eta^0 - \varphi^0) + \frac{4Y_0^2 R^2 e^{-\phi}}{z^3} V_a (a - \varphi^0 V_a) = 0 \tag{21}$$

$$\partial_z\left(\frac{Y_0^2 e^{-\phi}}{z^3}(\partial_z a - \eta^0\,\partial_z V_a)\right) + q^2\,\frac{Y_0^2 e^{-\phi}}{z^3}(a - \varphi^0 V_a) = 0\,. \tag{22}$$

These equations can be properly combined and integrated to get a constraint equation:

$$\frac{4v_q^2 R^2}{n_f z^3}\partial_z\eta^0 - \frac{q^2}{2n_f g_5^2 z}\partial_z\varphi^0 + \frac{4Y_0^2 R^2}{z^3}V_a(\partial_z a - \eta^0\partial_z V_a) = 0\,. \tag{23}$$

In the matrix formalism, the three fields are aggregated in a vector

$$\Psi = \begin{pmatrix} \varphi^0 \\ \eta^0 \\ a \end{pmatrix} = H\,\Psi_0\,, \tag{24}$$

where $\Psi_0(q^2) = (\varphi_0^0(q^2), -\eta_0^0(q^2), a_0(q^2))^T$ is the vector containing the sources of the three operators.
The masses of the singlet states can be obtained from the poles of the two-point functions:

$$\Pi^{00} = \left.\frac{\partial^2 S_{os}}{\partial\Psi_0\partial\Psi_0}\right|_{z\to\varepsilon} = \begin{pmatrix} \Pi^{00}_{\varphi\varphi} & \Pi^{00}_{\varphi\eta} & \Pi_{\varphi a} \\ \Pi^{00}_{\eta\varphi} & \Pi^{00}_{\eta\eta} & \Pi_{\eta a} \\ \Pi_{a\varphi} & \Pi_{a\eta} & \Pi_{aa} \end{pmatrix}. \tag{25}$$

The lightest state, which we indentify with the η' meson, has mass 958 MeV, while in the chiral limit its mass is 903 MeV. In Fig. 1 we show the values of the squared mass of the lightest state and of the first radial excitation as a function of the quark mass, together with the result for the lighest state in the octet.

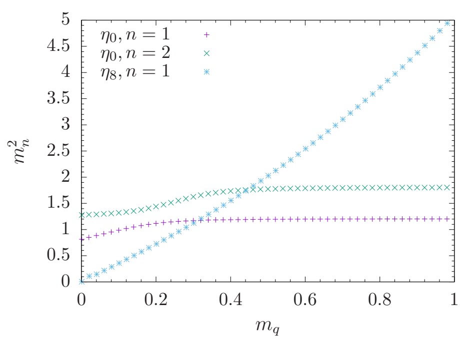

Figure 1. Squared mass (GeV^2) of the indicated states versus the quark mass (GeV).

The one-point functions are:

$$J^0 = \left.\frac{\partial S_{os}}{\partial\Psi_0}\right|_{z\to\varepsilon} = \begin{pmatrix} \langle J^0_\varphi\rangle \\ \langle J^0_\eta\rangle \\ \langle J^0_a\rangle \end{pmatrix} = \begin{pmatrix} -\frac{R}{k}\frac{e^{-\phi}q^2}{4n_f g_5^2 z}\varphi_0'\Big|_{z\to\varepsilon} \\ -\frac{R}{k}\frac{e^{-\phi}2v_q^2R^2}{n_f z^3}\eta_0'\Big|_{z\to\varepsilon} \\ \frac{R}{k}\frac{e^{-\phi}2Y_0^2R^2}{z^3}a'\Big|_{z\to\varepsilon} \end{pmatrix}, \tag{26}$$

where $\langle J_\varphi^0 \rangle$ and $\langle J_\eta^0 \rangle$ are the one-point functions of the longitudinal axial current $(\partial_\mu \bar\psi \gamma_5 \gamma^\mu T^0 \psi)$ and of the pseudoscalar current $(2m_q \bar\psi \gamma_5 T^0 \psi)$ in the singlet sector, respectively, while $\langle J_a^0 \rangle$ is the one-point function of the topological charge density $(\langle \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde G^{\mu\nu} \rangle)$. The constraint equation (23) establishes a relation between the three one-point functions:

$$-\langle J_\eta^0 \rangle + \langle J_\varphi^0 \rangle + \langle J_a^0 \rangle = 0\,, \tag{27}$$

a Ward identity representing the anomaly equation in this formalism.
From Π_{aa}, the two-point function of the $G\tilde G$ operator, we can compute the topological susceptibility as in Eq. (11). Expanding the equations of motion in the chiral limit for large N_c [2], at lowest order the constraint equation reproduces the Witten-Veneziano relation $\chi_{PG} = \frac{m_{\eta'}^2 f_\pi^2}{2n_f}$. At $q^2 = 0$ the equations of motion can be integrated and the following relation holds [2]:

$$\frac{1}{\chi_t} = \frac{1}{\chi_{PG}} + \frac{1}{\chi_f} \tag{28}$$

with

$$\frac{1}{\chi_f} = \frac{k}{R} n_f \int_0^\infty dz\, \frac{e^{\phi(z)} z^3}{4} \left(\frac{V_a(z)}{v_q(z) R} \right)^2 , \tag{29}$$

so the full (inverse) topological susceptibility gets a contribution from the pure-gauge one and another one accounting for finite quark masses. The numerical result is represented by the black curve in Fig. 2. As $m_q \to \infty$ it approaches the pure-gauge value, while for small m_q it depends linearly on the quark mass, in agreement with chiral perturbation theory, which predicts $\chi_t \xrightarrow[m_q \to 0]{} \chi_f \sim \frac{\langle \bar q q \rangle}{n_f} m_q$ [9], shown by the blue dashed line in Fig. 2. χ_t vanishes at $m_q = 0$, as expected. The relation

$$\frac{1}{\chi_t} = \frac{1}{\chi_{PG}} + \frac{n_f}{m_q |\langle \bar q q \rangle|} \tag{30}$$

represented by the orange curve in Fig. 2, was proposed in [10][11]. Finally, in Fig. 3 we

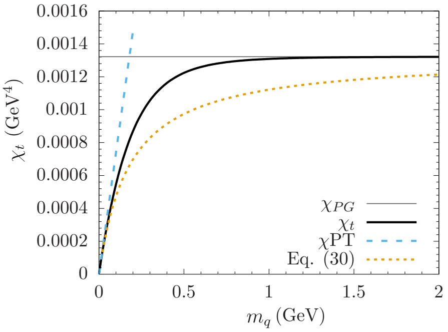

Figure 2. Topological susceptibility (GeV^4) vs quark mass (GeV) in this model (black curve), in pure gauge (horizontal grey line), from Eq. (30) (dotted orange line); expected topological susceptibility at small quark mass (dashed cyan line).

have compared our results for $n_f = 2$ with some lattice data [12][13][14]. It is not possible to get a quantitative comparison since the various points have been obtained with different values of the chiral condensate, producing different initial slopes.

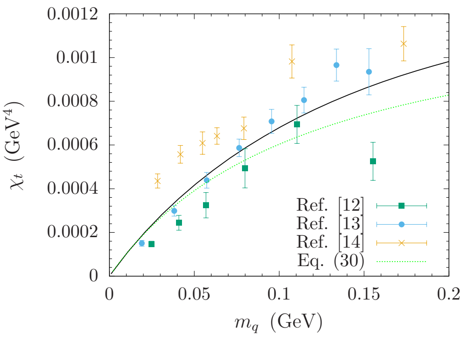

Figure 3. Topological susceptibility computed in this model (solid black line), and from Eq. (30) (dashed green line) at $n_f = 2$ and $|\langle \bar{q}q \rangle| = (0.281 \text{ GeV})^3$, compared with lattice data.

3 Conclusions

The masses of singlet and nonsinglet pseudoscalar mesons have been computed in the soft-wall holographic model of QCD. The mixing among the fields dual to the axial current, pseudoscalar current and the $G\tilde{G}$ operator can explain the large mass of the η'. Partial conservation of axial current, anomaly equation and Witten-Veneziano relation have been derived from the constraint equations (14) and (23), obtained by a combination of the equations of motion of the involved fields. The topological susceptibility χ_t has been computed for any value of the quark mass. The correction χ_f to the pure-gauge value depends linearly on the quark mass for small values of m_q, as in chiral perturbation theory, while it diverges as m_q^4 at high m_q.

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