Application of model selection criteria for $K^+\Sigma^-$ photoproduction within an isobar approach

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Abstract. We study kaon photoproduction off a neutron target using an isobar model. Recently acquired polarization data from the CLAS collaboration are incorporated in our analysis, while a novel fitting procedure is employed that allows us to reduce the number of parameters used with a minimal cost on the information content.

1 Introduction

Reactions such as $\gamma n \rightarrow K^+\Sigma^-$, which is the focus of the current study, play an important role in our understanding of the baryon spectrum. However, due to the plethora of nucleon resonances in the energy region of interest (W $\approx 1.6 - 2.6$ GeV) we are faced not only with a large number of parameters to use for fitting, but most importantly, a huge number of possible combinations of resonances to include in our model. It should be stressed that by "model selection" we refer to the selection of an optimal subset out of a group of candidate resonances. The techniques presented in section 3 have been used recently in similar contexts [1–3] and are well-established statistical learning tools [4]. The results of the current investigation have been reported in [5] and the data used are from [6] and [7].

2 The isobar approach

Isobar models, due to their phenomenological nature, rely on a number of approximations [8, 9]: the fundamental fields are hadrons and their interactions are based on effective Lagrangians, while reaction amplitudes are constructed as sums of lowest-order Feynman diagrams. Higher-order effects are implicitly taken into account in the *effective* couplings, which are obtained by fits to experimental data. At this level of approximation, there are three types of diagrams, depending on the nature of the exchanged particle (nucleon, kaon or hyperon), each corresponding to the **s**, **t** and **u** Mandelstam variables. Another distinction, based on whether the exchanged particle is in its ground, or in a resonant state (N^*, K^*, Y^*), results in further classifying these diagrams as being of the Born or non-Born types, respectively. Of these six types of interactions, only the exchange of N^* or Δ^* (the **s**, non-Born diagram) produces resonant structures in the cross-section, with the rest of the diagrams contributing only to the "background".

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3 Regularized least-squares fitting and information criteria

The question of optimal model complexity is central in statistical modeling. Most commonly, it refers to the optimal number of parameters needed by a certain model to describe a set of data. Using more parameters than necessary may lead to overfitting the data, while using fewer will lead to underfitting. Overfitting is caused by fitting the noise in the sample, thus resulting in a model of very low predictive power. A common way to deal with this problem is through regularization, where the χ^2 error function, defined as

$$\chi^2 = \sum_{i=1}^{N} \frac{[d_i - p_i(w_1, \ldots, w_k)]^2}{(\sigma_{d_i}^{stat})^2}, \tag{1}$$

is modified by the addition of a penalty term $P(\lambda)$ containing the k parameters $\{w_1, w_2 \ldots w_k\}$, with respect to which the χ^2 function is to be minimized. We thus have a penalized χ_T^2, which reads

$$\chi_T^2 = \chi^2 + P(\lambda), \quad \text{with} \quad P(\lambda) = \lambda \sum_{j=1}^{k} |w_j|^q \tag{2}$$

The effect of the $P(\lambda)$ term in the error function is to prevent the parameters from taking large values that would allow them to overfit the data. The magnitude of the λ regularization parameter determines the extent of parameter shrinkage. The case of $q = 1$ is known as Least Absolute Shrinkage and Selection Operator (LASSO) regularization [4] and results in some of the parameters to be driven to zero, with the value of λ determining the degree of sparsity of the resulting model.

At this point, it is the use of information criteria (IC), namely the Akaike (AIC) [10] and Bayesian (BIC) [11], that decide on the optimal value of λ, as the one that minimizes the corresponding expressions: AIC $= 2k + \chi_T^2$ and BIC $= k \log(N) + \chi_T^2$, where k is the number of parameters of the model and N is the size of the data set. The result of this process can be seen in Fig. 1. As can be observed, all criteria give similar results that differ only by a scale factor.

4 Numerical results and discussion

In the present work we conducted two fits: one, denoted as "fit M", where we performed ordinary χ^2 minimization (using the Minuit package) on a set of resonances derived from previous studies [9, 12] and a second one denoted as "fit L", where we applied LASSO regularization in conjunction with the aforementioned information criteria that led to a sparser model.

The free parameters of our model, apart from the coupling constants, comprise cut-off values for hadron form factors, as well. Moreover, resonances with spin-1/2 contribute one parameter and resonances with spins-3/2 or 5/2 contribute two parameters each. Masses and widths of resonances were taken from [13]. Table 1, shows the reduction in the number of resonances and parameters as a result of our fitting procedure.

The photon beam asymmetry $\mathbf{\Sigma}$ is defined as $\mathbf{\Sigma} = (d\sigma^{\perp} - d\sigma^{\parallel})/(d\sigma^{\perp} + d\sigma^{\parallel})$, where $d\sigma^{\parallel}$ denotes the differential cross section when the incident photon beam is linearly polarized in the x-direction and $d\sigma^{\perp}$ the differential cross section when the photon beam is polarized in the y-direction.

Figures 2 and 3 (taken from [5]) show the results of the two fittings M and L as they compare to data on the $\mathbf{\Sigma}$ asymmetry. Another useful result that can be drawn from these

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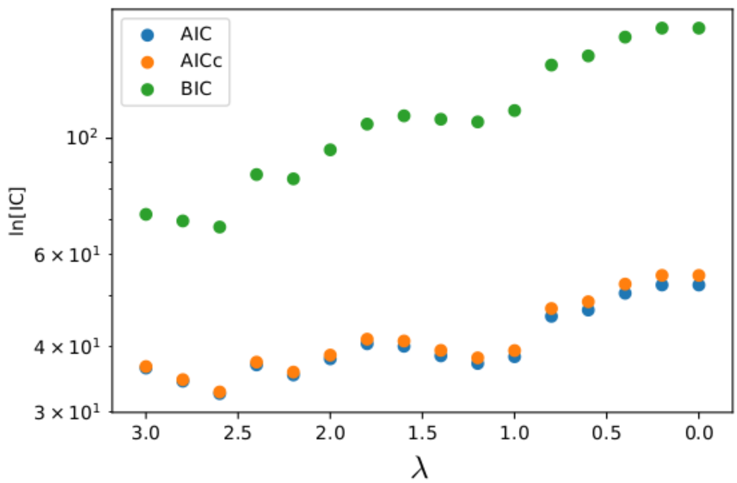

Figure 1. The Akaike (blue), a modified version known as 'corrected Akaike' (orange), and Bayesian (green) information criteria for various values of the regularization parameter λ. Please note the logarithmic scale of the vertical axis. (Taken from [5]).

Table 1. Summary of the features of the two fits.

	M: Minuit	L: LASSO + IC
no. of resonances	14	9
no. of parameters	25	17
χ^2 / n.d.f.	2.4	3.2

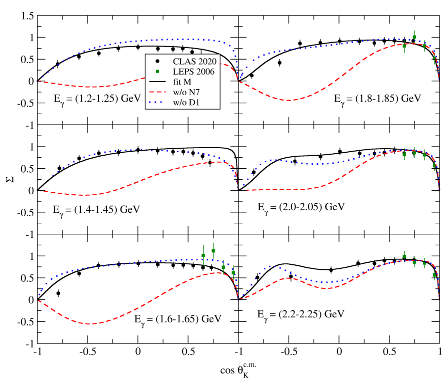

Figure 2. Photon beam asymmetry data compared to the full "fit M" results (solid line). Same fit, with the $N(1720)$ $3/2^+$ (dashed line) and $\Delta(1900)$ $1/2^-$ (dotted line) resonances omitted. The data are from CLAS [6] and LEPS [7] experiments.

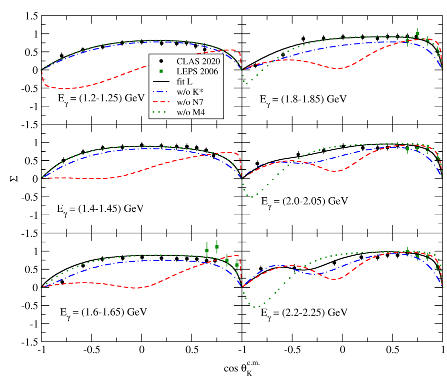

Figure 3. Photon beam asymmetry data as in Fig. 2, compared to "fit L" results (solid line). Same fit, with the $K^*(892)$ (dash-dotted line), $N(1720)$ $3/2^+$ (dashed line) and $N(2060)$ $5/2^-$ (dotted line) resonances omitted.

figures is the importance of the $N(1720)\ 3/2^{+}$ resonance, given the disagreement with the data when it is not included in the fits.

In conclusion, the application of LASSO regularization in combination with information criteria provides a tool for automatic selection of parameters based on their information content. This can be especially useful in our case, where the number of possible combinations of resonances is prohibitively large in the region of interest.

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References

[1] B. Guegan, J. Hardin, J. Stevens, and M. Williams, JINST **10**, P09002 (2015)

[2] J. Landay, M. Döring, C. Fernández-Ramírez, B. Hu and R. Molina, Phys. Rev. C **95**, 015203 (2017)

[3] J. Landay, M. Mai, M. Döring, H. Haberzettl and K. Nakayama, Phys. Rev. D **99**, 016001 (2019)

[4] T. Hastie, R. Tibshirani, and J. Friedman *The Elements of Statistical Learning: Data Mining, Inference, and Prediction* (Springer Series in Statistics, New York, 2009)

[5] P. Bydžovský, A. Cieplý, D. Petrellis, D. Skoupil and N. Zachariou, Phys. Rev. C **104**, 065202 (2021)

[6] N. Zachariou et al., Phys. Lett. B **827**, 136985 (2022)

[7] H. Kohri *et al.* (LEPS Collaboration), Phys. Rev. Lett. **97**, 082003 (2006)

[8] D. Skoupil and P. Bydžovský, Phys. Rev. C **97**, 025202 (2018)

[9] D. Skoupil and P. Bydžovský, Phys. Rev. C **93**, 025204 (2016)

[10] H. Akaike, IEEE Transactions on Automatic Control, **19** (6), 716 (1974)

[11] G. Schwarz, Ann. Stat. **6**(2), 461 (1978)

[12] L. De Cruz *Bayesian model selection for electromagnetic kaon production in the Regge-plus-resonance framework* (PhD Thesis, Ghent University, 2012)

[13] P. Zyla *et al.*, Prog. Theor. Exp. Phys. 2020, 083C01 (2020)