A Skyrme force for $\Xi^-$ hypernuclei

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Abstract. Experimental data for the cascade hypernuclei $^{12}_2\text{C}$, $^{12}_2\text{Be}$, and $^{13}_2\text{B}$ are analyzed within a Skyrme-Hartree-Fock theoretical approach and optimal Skyrme parameters are determined. The important role of deformation for $^{13}_2\text{B}$ is pointed out. Predictions for $^7_2\text{H}$ and $^{10}_2\text{Li}$ are made.

1 Introduction

While the properties of $\Lambda$ hypernuclei and the $\Lambda N$ interaction are fairly well known [1–3], only a few data regarding the heavier $\Xi$ hyperons are available that are furthermore often hampered by ambiguities in their identification and interpretation [4–6]. Recently, new data on $\Xi^-$ hypernuclei have become available at the J-PARC and KEK experimental facilities [7]. In this work we try to determine an optimal set of $\Xi N$ interaction parameters within the Skyrme-Hartree-Fock (SHF) theoretical approach, capable to describe consistently most data [8].

We first briefly review the available data for the $\Xi$ hypernuclei $^{15}_2\text{C}$ ($^{14}_2\text{N}+\Xi^-$), $^{13}_2\text{B}$ ($^{12}_2\text{C}+\Xi^-$), and $^{12}_2\text{Be}$ ($^{11}_2\text{B}+\Xi^-$), which are mostly nuclear emulsion events occurring after a $K^- p \to {K'}^+ \Xi^-$ reaction. Regarding $^{15}_2\text{C}$, analysis of the KEK-E373-T2 (KISO) event [6, 9] provided clear evidence that this is a (Coulomb-assisted) strongly-bound hypernucleus, produced in the reaction $\Xi^- + ^{14}_2\text{N} \to ^{15}_2\text{C} \to ^{10}_8\text{Be} + ^{5}_3\text{He}$. It is interpreted [6, 10–12] as production of a $\Xi^- 1p$ state with a removal energy $B_\Xi = 1.03 \pm 0.18$ MeV, leaving $^{10}_8\text{Be}$ in an excited state after its decay. Compatible results were obtained by the J-PARC-E07-T006 (IBUKI) event, $B_\Xi = 1.27 \pm 0.21$ MeV [13], and also KEK-E176 #14-03-35, $B_\Xi = 1.18 \pm 0.22$ MeV [14]. The combined analysis of KISO and IBUKI yields $B_\Xi = 1.13 \pm 0.14$ MeV [13].

Recently larger $B_\Xi$ values interpreted as possible $\Xi^- 1s$ bound states [11] have been extracted for the J-PARC-E07-T010 (IRRAWADDY), $B_\Xi = 6.27 \pm 0.27$ MeV, and the KEK-E373-T3 (KINKA), $B_\Xi = 8.00 \pm 0.77$ MeV or $B_\Xi = 4.96 \pm 0.77$ MeV, events. As the error bars are non overlapping, at most one of these values may correspond to the ground state energy. We demonstrate in the following that the KINKA (8.00 MeV) result yields the best consistency with the other data.

Regarding the $^{12}_2\text{Be}$ hypernucleus, in the BNL-E885 counter experiment [15] the cross section for the $^{12}_2\text{C}(K^-, K^+)^{12}_2\text{Be}$ reaction was interpreted by assuming a $\Xi^-$-nucleus Woods-Saxon (WS) potential with a depth of about 14 MeV. The result is consistent with the estimate $V_{WS} < 20$ MeV of the preceding KEK-E224 [16], and $V_{WS} = 17 \pm 6$ MeV in $^8_2\text{He}$ ($^8_2\text{Li}+\Xi^-$) of BNL-E906 [17], while the sequel J-PARC-E05 experiment on $^{12}_2\text{Be}$ remained so far inconclusive, reporting a preliminary possible one-peak interpretation with $B_\Xi \approx 6.3$ MeV [18].

Finally, there are several older KEK-E176 emulsion data [14, 19] that can be understood as possible formation of $^{13}_2\text{B}$, but all of them are ambiguous. In particular, the events KEK-E176 #10-9-6, $B_\Xi = 0.82 \pm 0.17$ MeV, and KEK-E176 #13-11-14, $B_\Xi = 0.82 \pm 0.14$ MeV, might both be interpreted as $^{13}_2\text{B}$ ($1p$) states [19–21].
2 Formalism

In the present work, we study $\Xi$ hypernuclei with $A = 12, 13, \text{ and } 15$ and focus on the $\Xi^{-} 1s$ and $1p$ states and their hyperon separation energy.

$$B_\Xi \equiv E[(n, p, -)] - E([n, p, \Xi^{-}]) = E^{[4-1](Z+1)} - E^2Z.$$  \hspace{1cm} (1)

We employ a model based on the self-consistent SHF method [22, 23], first extended to the theoretical description of $\Lambda$ hypernuclei in Ref. [24], and now used for $\Xi^{-}$ hyperons here. The fundamental SHF local energy-density functional of hypernuclear matter is written as

$$\varepsilon_{\text{SHF}} = \varepsilon_N + \varepsilon_Y,$$  \hspace{1cm} (2)

and depends on the one-body densities $p_0$, kinetic densities $\tau_q$, and spin-orbit currents $J_q$.

$$[\rho_q, \tau_q, J_q] = \sum_{i=1}^{N_q} p_i^q \left[ |\phi_q^i|^2, |\nabla \phi_q^i|^2, \phi_q^i \cdot (\nabla \phi_q^i \times \sigma) / i \right],$$  \hspace{1cm} (3)

where $\phi_q^i$ ($i = 1, N_q$) are the self-consistently calculated single-particle (s.p.) wave functions of the $N_q$ occupied states for the species $q = n, p, Y$ in a hypernucleus.

The functional $\varepsilon_N$ is the usual nucleonic part [22, 23] and a possible standard parametrization for the hyperonic part is [10, 20, 24–26]

$$\varepsilon_Y = \frac{\tau_y}{2m_Y} + a_0 \rho_Y \rho_N + a_3 \rho_Y^2 \rho_N^2 + a_1 (\rho_Y \tau_N + \rho_N \tau_y)$$

$$- a_2 (\rho_Y \Delta \rho_N + \rho_N \Delta \rho_Y) / 2 - a_4 (\rho_Y \nabla \cdot J_N + \rho_N \nabla \cdot J_Y),$$  \hspace{1cm} (4)

from which one obtains the corresponding hyperonic SHF mean fields

$$V_Y = a_0 \rho_N + a_1 \tau_N - a_2 \Delta \rho_N - a_4 \nabla \cdot J_N + a_3 \rho_N^2,$$  \hspace{1cm} (5)

$$V_N^{(Y)} = a_0 \rho_Y + a_1 \tau_Y - a_2 \Delta \rho_Y - a_4 \nabla \cdot J_Y + a_3 \alpha \rho_Y \rho_N^{\alpha-1},$$  \hspace{1cm} (6)

and a hyperon effective mass

$$\frac{1}{2m_Y} = \frac{1}{2m_Y} + a_1 \rho_N,$$  \hspace{1cm} (7)

which appear in the SHF Schrödinger equation

$$\left[ \nabla \cdot \frac{1}{2m_Y^2} \nabla - V_q(r) - \varepsilon_q V_C(r) + iW_q(r) \cdot (\nabla \times \sigma) \right] \phi_q^i(r) = \varepsilon_q^i \phi_q^i(r),$$  \hspace{1cm} (8)

where $V_C$ is the Coulomb field and $W_q$ the nucleonic spin-orbit mean-field [23]. The ‘three-body’ parameter $\alpha$ is kept here to its standard value of 2, but also an alternative value of 7/6, used in several NN Skyrme forces, is evaluated.

Contrary to $\Lambda$ hypernuclei, the Coulomb interaction is very important for $\Xi^{-}$ hypernuclei discussed here and provides a substantial part of the $\Xi^{-}$ binding. An approximate c.m. correction is applied as usual [22, 23, 27] by replacing the bare masses: $1/m_q \rightarrow 1/m_q - 1/M$, where $M = (N_n + N_p)m_N + N_Y m_Y$ is the total mass of the (hyper)nucleus. Solving the Schrödinger equation provides the wave functions $\phi_q^i(r)$ and the s.p. energies $-\varepsilon_q^i$ for the different s.p. levels $i$ and species $q$. We use in this work the standard nucleonic Skyrme force SLy4 [28], but the results for hyperonic observables hardly depend on that choice [29].

It should be noted that the $\Xi^{-}$ hypernuclei decay into double-$\Lambda$ hypernuclei by the $\Xi N$–$\Lambda \Lambda$ coupling [30]. Therefore, the $\Xi N$ interaction should have an imaginary part to represent
Table 1. The removal energies $B_{\Xi}$ (in MeV) for the $\Xi^{-}\,1s$ and $1p$ states of $^{13}_{\Xi}B$, $^{12}_{\Xi}Be$, $^{10}_{\Xi}Li$, and $^{7}_{\Xi}H$, obtained with $\Xi N$ Skyrme forces of different parameters $\alpha$, $a_0[\text{MeV fm}^3]$, $a_2[\text{MeV fm}^5]$, $a_3[\text{MeV fm}^3\alpha]$ and $a_1 = a_4 = 0$. With these parameters, the $\Xi^{-}\,1s$ and $1p$ removal energies of $^{15}_{\Xi}C$ are fixed to 8.00 and 1.13 MeV, respectively. $^{13}_{\Xi}B$ values in brackets are for spherical calculations.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$a_2$</th>
<th>$a_0$</th>
<th>$a_3$</th>
<th>$^{13}_{\Xi}B$</th>
<th>$^{13}_{\Xi}B$</th>
<th>$^{12}_{\Xi}Be$</th>
<th>$^{10}_{\Xi}Li$</th>
<th>$^{7}_{\Xi}H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-200.3</td>
<td>704.8</td>
<td></td>
<td>6.62 (6.71)</td>
<td>0.86 (-0.12)</td>
<td>5.83</td>
<td>4.42</td>
<td>1.59</td>
</tr>
<tr>
<td>10</td>
<td>-198.7</td>
<td>641.3</td>
<td></td>
<td>6.49 (6.65)</td>
<td>0.75 (-0.11)</td>
<td>5.72</td>
<td>4.23</td>
<td>1.31</td>
</tr>
<tr>
<td>2</td>
<td>-196.9</td>
<td>576.8</td>
<td></td>
<td>6.37 (6.59)</td>
<td>0.65 (-0.09)</td>
<td>5.61</td>
<td>4.04</td>
<td>1.05</td>
</tr>
<tr>
<td>30</td>
<td>-194.7</td>
<td>509.3</td>
<td></td>
<td>6.25 (6.53)</td>
<td>0.56 (-0.08)</td>
<td>5.51</td>
<td>3.86</td>
<td>0.80</td>
</tr>
<tr>
<td>40</td>
<td>-192.5</td>
<td>443.0</td>
<td></td>
<td>6.14 (6.46)</td>
<td>0.47 (-0.07)</td>
<td>5.40</td>
<td>3.67</td>
<td>0.56</td>
</tr>
</tbody>
</table>

| 0        | -498.6| 551.3 |       | 6.44 (6.84)   | 0.84 (-0.07)   | 5.87           | 4.37           | 1.52     |
| 10       | -470.8| 503.0 |       | 6.32 (6.76)   | 0.73 (-0.06)   | 5.76           | 4.17           | 1.25     |
| 7/6      | -439.4| 449.3 |       | 6.21 (6.69)   | 0.63 (-0.06)   | 5.64           | 3.98           | 1.00     |
| 30       | -409.0| 397.3 |       | 6.10 (6.61)   | 0.54 (-0.05)   | 5.53           | 3.79           | 0.76     |
| 40       | -378.9| 345.5 |       | 6.02 (6.54)   | 0.47 (-0.04)   | 5.42           | 3.62           | 0.54     |

the decay width. However, since we have so far no useful experimental information on this coupling by Refs. [9, 11, 13–16], here the imaginary part is omitted.

While the core nuclei $^{14}_{\Xi}N$ and $^{11}_{\Xi}B$ are (nearly) spherical, $^{12}_{\Xi}C$ is an axially-deformed oblate nucleus [20, 31–36]. In our approach, the deformed SHF Schrödinger equation is solved in cylindrical coordinates $(r, z)$ within the axially-deformed harmonic-oscillator basis [22, 23]. The geometric quadrupole deformation parameter of the nuclear core is expressed as

$$\beta_2 \equiv \sqrt{\frac{\pi}{5}} \frac{(2r^2 - r^2)}{(r^2 + z^2)}.$$  

As there are currently not enough data to determine uniquely the five $\Xi N$ interaction parameters $a_i$, we proceed as follows. The effective-mass parameter $a_1$ is kept zero, since recent Brueckner-Hartree-Fock calculations [37] indicate a rather flat $\Xi^{-}\text{s.p.}$ spectrum and thus $m_\Xi^*/m_\Xi$ close to unity. Also the spin-orbit parameter is disregarded, $a_4 = 0$, as the inclusion of $\Xi^{-}$ spin-orbit splitting is clearly premature. For the same reason we do not introduce further parameters for the isospin dependence of the interaction at this stage.

The surface-energy parameter $a_2$ is essential to determine the shape of the $\Xi^{-}$ mean field $V_\Xi$ in the hypernucleus, Eq. (5), which is important for comparison with the WS mean field that was used in the experimental analysis of BNL-E885 for $^{12}_{\Xi}Be$, Refs. [9, 15],

$$V_{\text{WS}}(r) = \frac{-V_0}{1 + \exp[(r - 2.52\,\text{fm})/0.65\,\text{fm}]}$$  

with $V_0 = 12, 14, 16, 18, 20$ MeV and $V_0 \approx 14$ MeV identified as the preferred value. Motivated by the equivalent parameter value ($a_2^{\text{SLN}} \approx 20\,\text{MeV fm}^5$) of the recently derived SLL4 $\Lambda N$ Skyrme force [26], we use several trial values of this parameter, $a_2 = 0, 10, \ldots, 40\,\text{MeV fm}^5$.

For fixed $a_1 = a_4 = 0$ and chosen $a_2$, the remaining volume parameters $a_0$ and $a_3$ are then determined by fitting the removal energies $B_{\Xi} = 8.00$ and 1.13 MeV for the $\Xi^{-}\,1s$ and $1p$ states in $^{15}_{\Xi}C$, respectively, as claimed for the KINKA and KISO+IBUKI events. The resulting three-parameter force is termed SLX3.
3 Results and discussion

Table 1 lists the parameter values \(a_{0,2,3}\) obtained, together with the \(\Xi^-\) 1s and 1p removal energies of \(^{13}\Xi\)B, \(^{12}\Xi\)Be, \(^{10}\Xi\)Li, and \(^{7}\Xi\)H that are predicted. Strongly-bound 1p states with positive removal energies are only found for deformed \(^{13}\Xi\)B nuclei, but not in spherical approximation (numbers in brackets). \(^{10}\Xi\)Li and \(^{7}\Xi\)H are bound for all versions of the \(\XiN\) force, although these nuclei are probably too light systems to be calculated reliably within the SHF approach. One obtains reasonable values for the parameters \(a_0\) and \(a_3\). For comparison, in the SLL4 \(\Lambda\)N Skyrme force [25, 26], the equivalent optimal parameters are \(a_{0,3}^{\Lambda\Xi} \approx 320\) MeV fm\(^3\) and \(a_{3,3}^{\Lambda\Xi} \approx 700\) MeV fm\(^6\). For increasing surface parameter \(a_2\), more \(\Xi^-\) binding is provided by the associated terms in Eqs. (4,5), and therefore both \(a_0\) and \(a_3\) decrease in magnitude.

In Fig. 1, the different SHF mean field potentials in the \(^{12}\Xi\)Be hypernucleus are plotted, including the local Coulomb field \(-V_C\), the strong mean field \(V_{\Xi}\), Eq. (5), and for comparison the WS mean field Eq. (10) with \(V_0 = 14, 16, 18\) MeV, as used in the analysis of BNL-E885 [9, 15]. One observes that the SHF results cover the range between the 14 and 18 MeV curves. We consider this as a very good agreement with the analysis of E885. It can clearly be seen that the zero-momentum value \(V_{\Xi}(0)\) alone is not a useful unique indicator of the interaction strength, as the shape of the mean field controlled by the parameter \(a_2\) plays an essential role. A value of about \(a_2 \approx 20 - 30\) MeV fm\(^5\) gives the closest correspondence to a WS shape, whereas \(a_2 = 0\) generates a flat or well shape in the core region.

The hypernuclei \(^{13}\Xi\)C and \(^{12}\Xi\)Be and their core nuclei discussed so far are spherical nuclei. The case of \(^{13}\Xi\)B is more delicate, as the core nucleus \(^{12}\Xi\)C is axially deformed in the SHF approach [20, 31–36]. Its deformation derived from the proton quadrupole moment \(Q_p\) is rather large [38],

\[
\beta \equiv \frac{\sqrt{5\pi}}{3} \frac{Q_p}{ZR_0^2} \approx -0.58 \pm 0.03
\]

with \(R_0 \approx 1.2 A^{1/3}\) fm. In the SHF approach the size of nuclear deformation can be controlled by adjusting the spin-orbit parameter of the \(NN\) Skyrme force [20, 31–36], and we follow this procedure to reproduce the proper \(\beta\) value. As discussed in detail in [20], the oblate deformation favors the binding of a \(\Xi^-\) 1p orbital because of the improved geometrical
overlap of wavefunction and embedding potential, such that this state becomes more bound than in spherical approximation. The results of Table 1 for $^{13}\Xi$B are obtained in this way and demonstrate that the proposed interpretation of the KEK-E176 events as $^{13}\Xi$B states might be solely due to the fact that $^{12}$C is a strongly deformed nucleus. Excellent agreement with the experimental value $B_{\Xi} \approx 0.82$ MeV is obtained with small values of $a_2$ and both choices of $\alpha$.

4 Summary

We have tried to fit all current data for the cascade hypernuclei $^{15}_2\Xi C$, $^{12}_2\Xi Be$, and $^{13}_2\Xi B$ within a global SHF approach for the $\Xi N$ interaction. Our main conclusions are:

1) Of all proposed $B_{\Xi}(^{15}_2\Xi s)$ values, the KINKA (8.00 MeV) interpretation seems to be the one most compatible with both the $^{12}_2\Xi Be$ and $^{13}_2\Xi B$ data, although slightly lower values down to about 7 MeV might also be possible and consistent with $^{12}_2\Xi Be$, but would not allow interpretation of the KEK-176 events as $^{13}_2\Xi B$ states.

2) Combining KINKA ($^{15}_2\Xi s$) and KISO+IBUKI ($^{15}_2\Xi s$) data, the predicted $\Xi^-$ mean field in $^{12}_2\Xi Be$ is very similar to the best-choice WS mean field for BNL-E885 with a depth of about 14–16 MeV [15], and also compatible with KEK-E224 [16] and BNL-E906 [17].

3) $^{13}_2\Xi B$ is a delicate case, as in the current model this hypernucleus and its core nucleus $^{12}_2\Xi C$ are strongly axially deformed, and the claimed $B_{\Xi} \approx 0.8$ MeV values of some KEK-E176 emulsion data might be explained as a consequence of this deformation, which energetically favors the extended $\Xi^-$ 1p orbit. Treating these nuclei as undeformed does not produce a sufficiently bound $\Xi^-$ 1p state (which is actually another possible interpretation of the data).

Furthermore, $^{2}_2H$ is predicted as a bound hypernucleus. The proposed Skyrme force SLX3 with moderate values of the parameter $a_2$ is thus able to fit satisfactorily all current data, although due to their limited accuracies and ambiguities a firm statement cannot yet be made. Hopefully a large number of emulsion events obtained in KEK-E373 and J-PARC-E07 will be analyzed soon for confrontation. Future improvements of the approach include constraining better the SHF $\Xi N$ interaction parameters, going beyond the mean-field treatment, and including the imaginary parts due to the $\Xi N-\Lambda\Lambda$ decay.

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