

Four-body Faddeev-type calculation of the $\bar{K}NNN$ system

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Abstract. A quasi-bound state in the $\bar{K}NNN$ system caused by strong interactions was studied using dynamically exact four-body Faddeev-type equations. Binding energy and width of the K^-ppn state were calculated using three antikaon-nucleon and three nucleon-nucleon interaction models.

The attractive nature of $\bar{K}N$ interaction has stimulated theoretical and experimental searches for K^- bound states in different systems. In particular, many efforts were devoted to the lightest possible system $\bar{K}NN$, see e.g. [1]. All theoretical results agree that a quasi-bound state in the K^-pp system exists, but the predicted binding energies and widths are quite diverse. Several experimental evidences of the K^-pp quasi-bound state existence were already reported. The estimated characteristics of the state are different from each other and also from the theoretical predictions.

Some theoretical works were devoted to the question of the quasi-bound state caused by strong interactions in the heavier $\bar{K}NNN$ system with different quantum numbers [2–5]. However, more accurate calculations within Faddeev-type equations are needed. Only these dynamically exact equations written in momentum representation can treat energy dependent $\bar{K}N$ potentials, necessary for study of this system, exactly.

We solved four-body Faddeev equations in the Grassberger-Sandhas (GS) form [6]. If separable potentials leading to the corresponding separable T -matrices

$$T_\alpha(z) = |g_\alpha\rangle\tau_\alpha(z)\langle g_\alpha| \quad (1)$$

are used as an input, and the three-body T -matrices $\bar{T}_{\alpha\beta}^\tau(z)$ are presented in a separable form

$$\bar{T}_{\alpha\beta}^\tau(z) = |\bar{g}_\alpha^\tau\rangle\bar{\tau}_{\alpha\beta}^\tau(z)\langle\bar{g}_\beta^\tau|, \quad (2)$$

the four-body equations can be written as

$$\bar{X}_{\alpha\beta}^{\sigma\rho}(z) = \bar{Z}_{\alpha\beta}^{\sigma\rho}(z) + \sum_{\tau,\gamma,\delta} \bar{Z}_{\alpha\gamma}^{\sigma\tau}(z)\bar{\tau}_{\gamma\delta}^\tau(z)\bar{X}_{\delta\beta}^{\tau\rho}(z) \quad (3)$$

with four-body transition $\bar{X}_{\alpha\beta}^{\sigma\rho}$ and kernel $\bar{Z}_{\alpha\beta}^{\sigma\rho}$ operators

$$\bar{X}_{\alpha\beta}^{\sigma\rho}(z) = \langle\bar{g}_\alpha^\sigma|(\bar{G}_0)_{\alpha\alpha}(z)\bar{U}_{\alpha\beta}^{\sigma\rho}(z)(\bar{G}_0)_{\beta\beta}(z)|\bar{g}_\beta^\rho\rangle, \quad (4)$$

$$\bar{Z}_{\alpha\beta}^{\sigma\rho}(z) = (1 - \delta_{\sigma\rho})\langle\bar{g}_\alpha^\sigma|(\bar{G}_0)_{\alpha\beta}(z)|\bar{g}_\beta^\rho\rangle. \quad (5)$$

The four-body operators $\bar{U}_{\alpha\beta}^{\sigma\rho}(z)$ and $(\bar{G}_0)_{\alpha\beta}(z)$ in Eqs.(4,5) enter the Faddeev-type GS equations with separable two-body potentials and three-body amplitudes in an arbitrary form.

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They are defined as

$$\bar{U}_{\alpha\beta}^{\sigma\rho}(z) = \langle g_\alpha | G_0(z) U_{\alpha\beta}^{\sigma\rho}(z) G_0(z) | g_\beta \rangle, \quad (6)$$

$$(\bar{G}_0)_{\alpha\beta}(z) = \delta_{\alpha\beta} \tau_\alpha(z), \quad (7)$$

where $G_0(z)$ is a free Green function in four-body space, and $U_{\alpha\beta}^{\sigma\rho}(z)$ is a four-body transition operator. The high indices σ, ρ, τ define a partition, which could be $3 + 1$ or $2 + 2$ type, while the low indices α, β define two-body subsystems of the particular three-body subsystem or the $2 + 2$ partition, denoted by the high index.

The input for the system Eq.(3) are $\bar{K}N$ and NN potentials. We used two-body interaction models constructed for our three-body calculations [1]. Namely, three separable models of the $\bar{K}N$ interaction were used: two phenomenological potentials with one- $V_{\bar{K}N}^{1,\text{SIDD}}$ or two-pole $V_{\bar{K}N}^{2,\text{SIDD}}$ structure of the $\Lambda(1405)$ resonance and a chirally motivated potential $V_{\bar{K}N}^{\text{Chiral}}$ with two-pole structure. All three potentials describe low-energy K^-p scattering and the $1s$ level shift of kaonic hydrogen with equally high accuracy.

Two-term separable nucleon-nucleon potentials $V_{NN}^{\text{TSA-A}}$ and $V_{NN}^{\text{TSA-B}}$ from our three-body calculations [1] were used here together with the new version of the NN potential V_{NN}^{TSN} , described in [7]. All three potentials reproduce the Argonne v18 NN phase shifts at low energies up to 500 MeV with the change of sign, so they are repulsive at short distances. All three models of nucleon-nucleon interaction give proper singlet and triplet NN scattering lengths and deuteron binding energy.

Separable versions of the three-body $\bar{K}NN$, NNN and the $2 + 2 \bar{K}N + NN$ amplitudes are not an input for the four-body equations, they should be calculated when the four-body system of equations Eq.(3) is being solved. We used Energy Dependent Pole Expansion/Approximation (EDPE/EDPA) method, suggested in [8] specially for the four-body Faddeev-type equations. It needs a solution of eigenequations for a fixed energy z_{fix} and an evaluation of the corresponding eigenvalues λ_n and eigenfunctions $g_{n\alpha}(p; z)$ first. Usually, z_{fix} is chosen to be equal to the binding energy $z_{\text{fix}} = E_B$ if a bound state in the system exists or $z_{\text{fix}} = 0$ if not. After that, the energy-dependent form-factors $g_{n\alpha}(p; z)$ and propagators $\Theta_{mn}(z)$ are calculated. Finally, the separable three-body amplitude is written as

$$X_{\alpha\beta}^{\text{EDPE}}(p, p'; z) = \sum_{m,n=1}^{\infty} g_{m\alpha}(p; z) \Theta_{mn}(z) g_{n\beta}(p'; z). \quad (8)$$

Therefore, the EDPE method needs only one solution of eigenvalue equations and calculations of integrals after that. The series Eq.(8) converges quite fast, it is accurate already with one term [8].

One of the three-body subsystems of the $\bar{K}NNN$ system, namely $\bar{K}NN$, was studied in our previous works [1]. The binding energy and width of the quasi-bound state in the K^-pp system, which is $\bar{K}NN$ with zero spin, were calculated. Our most recent values of the K^-pp quasi-bound state characteristics can be found in [7]. Another, spin one state of the $\bar{K}NN$ system, which is K^-np , can have two different quasi-bound states. One of them, kaonic deuterium, is an atomic state mainly caused by the Coulomb interaction between the antikaon and the proton. Another state, which is similar to the one in K^-pp , is caused by strong interactions. Since the pole corresponding to the "strong" K^-np state is situated very close to the K^-d threshold, theoretical prediction of its existence depends on the interaction models. In particular, calculations with the new NN potential V_{NN}^{TSN} together with the two-pole $\bar{K}N$ interaction models predict an existence of the strong quasi-bound state, while the use of V_{NN}^{TSA} leads to the negative result [7].

Three-body equations for the NNN subsystem, also necessary for the four-body calculations, were solved additionally. The remaining object to study is the amplitude of

the $\bar{K}N + NN$ partition. It is a special state consisting of a pair of non-interacting pairs of particles. It is described by a system of three-body equations, which were written down and solved separately. Partition $\bar{K}N + NN$ with zero isospin in each pair of particles has a quasi-bound state since $\bar{K}N$ with zero isospin has a quasi-bound state, the $\Lambda(1405)$ resonance, while the nucleon-nucleon pair can be bound into deuteron. The $2 + 2$ partition with isospin of the pairs equal to one does not have a quasi-bound state, so the fixed energy $z_{\text{fix}} = 0$ was set for this case. All three-body program codes were rewritten for evaluation of the separable versions of the three-body $\bar{K}NN$, NNN and "three-body" $\bar{K}N + NN$ amplitudes using the EDPE method.

The Faddeev-type system of equations Eq.(3) consists of 18 equations while all four particles are different. In the case of the $\bar{K}NNN$ system, the number of equations after antisymmetrization is reduced to five. However, our $\bar{K}N$ and NN potentials are spin- and isospin-dependent models, moreover, nucleon-nucleon interaction is described by the two-term potentials. Since only one term in the series Eq.(8) was used at the first step, the final number of the coupled integral equations to be solved for the $\bar{K}NNN$ system is equal to 18.

In contrast to our three-body calculations, here we did not take coupling between the $\bar{K}N$ and $\pi\Sigma$, $\pi\Lambda$ channels into account directly. We used exact optical $\bar{K}N$ potentials instead, which reproduce the elastic part of the coupled-channel amplitudes exactly. The inelasticity is taken into account through the energy-dependent imaginary parts of the potentials. We also did not take the Coulomb interaction into consideration assuming that it plays a minor role.

The $\bar{K}NNN$ system with zero isospin and spin one half ($K^-ppn - \bar{K}^0nnp$ in particle representation) was investigated, and binding energy and width of the quasi-bound state in the system were calculated. Three versions of the antikaon-nucleon interaction: $V_{\bar{K}N}^{1,\text{SIDD}}$, $V_{\bar{K}N}^{2,\text{SIDD}}$ and $V_{\bar{K}N}^{\text{Chiral}}$, - together with the three versions of the NN potentials: $V_{NN}^{\text{TSA-A}}$, $V_{NN}^{\text{TSA-B}}$ and V_{NN}^{TSN} , - used in the calculations allowed us to investigate the dependence of the results on the two-body input. The results are presented in Fig.1. It is seen that the four-body binding energies and widths, in the same way as those of the three-body $\bar{K}NN$ or $\bar{K}\bar{K}N$ systems, strongly depend on the model of the $\bar{K}N$ interaction. If the chirally motivated antikaon-nucleon potential $V_{\bar{K}N}^{\text{Chiral}}$ is used, the binding energy B_{K^-ppn} and the width Γ_{K^-ppn} of the $\bar{K}NNN$ quasi-bound state also significantly depend on the nucleon-nucleon potential. While one-pole $V_{\bar{K}N}^{1,\text{SIDD}}$ or two-pole $V_{\bar{K}N}^{2,\text{SIDD}}$ phenomenological antikaon-nucleon model is used, dependence on the NN potential is weak.

The previously calculated characteristics of the $\bar{K}NNN$ quasi-bound state with the same quantum numbers are also shown in the figure. The prediction [2], denoted as AY in the figure, with the largest binding energy and the smallest width was done using the G-matrix approach and an antikaon-nucleon interaction model, which does not reproduce experimental data. Variational calculations were performed by two groups, and the results are denoted as BGL [3] and OHHMH [4]. Both used a chirally motivated antikaon-nucleon potential, which is a problem for variational calculations in coordinate space since it is not possible to take the energy dependence of the potentials into account. The authors used different ways to deal with the energy dependence of the $\bar{K}N$ interaction model. The second group [4] fixed the energy at two different values, and the resulting widths differ drastically one from another. The authors of [5] (ME in Fig.1) used the same Faddeev-type four-body equations as we did. Moreover, two of the three antikaon-nucleon potentials they used are our phenomenological $V_{\bar{K}N}^{1,\text{SIDD}}$ and $V_{\bar{K}N}^{2,\text{SIDD}}$ models. The third one is a chiral potential. The results of [5] differ drastically from ours, namely, their binding energies are larger, while the widths are much smaller than ours for the same $V_{\bar{K}N}$. The nucleon-nucleon potentials used by us and by the authors of [5] are different, but it hardly can explain such differences. The reason can be an inappropriate treatment of the $2 + 2$ partition, which is not described in [5], or inaccuracy

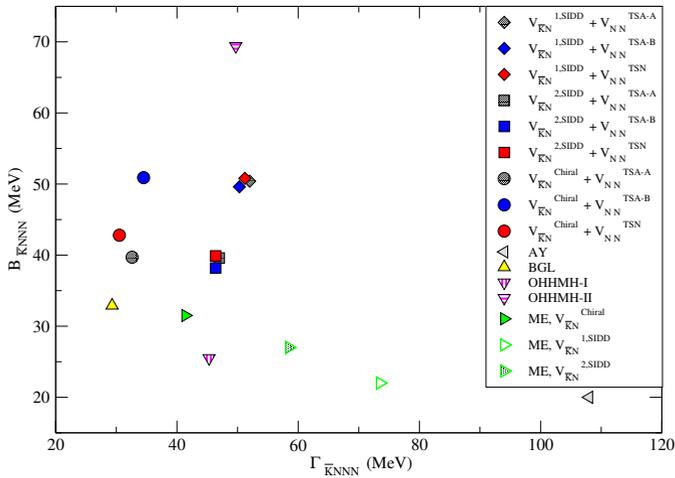


Figure 1. Binding energies $B_{\bar{K}NNN}$ (MeV) and widths $\Gamma_{\bar{K}NNN}$ (MeV) of the quasi-bound state of the $\bar{K}NNN$ state calculated using three different models of antikaon-nucleon interaction and three nucleon-nucleon models. The results of other authors are also presented.

during construction of the separable versions of the three-body subamplitudes and the $\bar{K}N + NN$ partition.

The binding energies B_{K^-ppn} in the K^-ppn system turned out to be quite close to those evaluated previously for the K^-pp system with the same two-body potentials. Namely $B_{K^-ppn} \sim 30.5 - 34.5$ MeV was obtained with the chirally motivated $\bar{K}N$ potential and $B_{K^-ppn} \sim 46.4 - 52.0$ MeV with phenomenological models of antikaon-nucleon interaction. The four-body widths $\Gamma_{K^-ppn} \sim 38.2 - 50.9$ MeV are smaller than the three-body ones.

A more detailed description of the calculation and discussion of the results can be found in [9].

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