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Meson-baryon scattering and $\Lambda(1405)$ in chiral effective field theory

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Abstract. We investigated the meson-baryon scattering using time-order perturbation theory (TOPT) based on covariant chiral effective field theory. The effective potential is defined as the sum of two-particle irreducible contributions of time-ordered diagrams, and the renormalized scattering amplitude is obtained by solving the integral equation, which is derived self-consistently in TOPT. Our developed formalism has been successfully applied to the πN scattering at leading order, and it has been extended to the meson-baryon scattering in $S = -1$ sector and found the two-pole structure of the $\Lambda(1405)$ resonance.

1 Introduction

Meson-baryon scattering contains very interesting phenomena, such as the πN scattering data can be used to extract the sigma term [1], $\sigma_{\pi N}$, which is a key input in the evaluation of the neutralino-nucleon cross section [2]; the $\bar{K}N$ interaction, due to its strong attractive nature, is important in strangeness nuclear physics, e.g. the $\Lambda(1405)$ resonance, $\bar{K}NN$ and multi-$\bar{K}$ nuclei, and the kaon-condensate in neutron stars [3]. Furthermore, the study of meson-baryon scattering can deepen our understanding of SU(3) dynamics in nonperturbative QCD.

Chiral unitary approach [4], which extends the applicable range of low-energy chiral effective field theory to the resonance region by iterating the interaction kernel in the Lippmann-Schwinger or Bethe-Salpeter equations ($T = V + VGT$), has been extensively applied to the studies of meson-baryon scattering processes. However, in practice, the on-shell factorization is often used in solving the scattering equation by neglecting the off-shell effects. Furthermore, the finite momentum cutoff or subtraction constant is introduced to renormalize the one-loop Green function [5], which results in the cutoff or model dependence of the scattering amplitude.

To avoid these approximations or obstacles, in this work, we tentatively propose a renormalized framework for meson-baryon scattering using TOPT with the covariant chiral Lagrangians [6, 7], which accompanies the research line of baryon-baryon scattering in Refs. [8–10]. Such a framework allows us to obtain the effective potential and the scattering equation on an equal footing. By including the off-shell effects of potential and utilizing the subtractive renormalization, we obtain a renormalized scattering amplitude at leading order (LO). We have successfully applied the developed formalism to the pion-nucleon and $S = -1$ meson-baryon scatterings.

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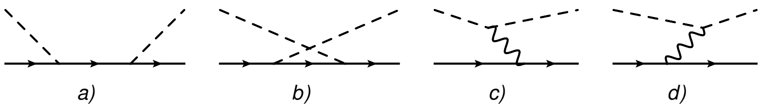

Figure 1. Time-ordered diagrams contributing to the LO meson-baryon potential.

2 Theoretical framework

Here we briefly present the major ingredients of our framework, the details can be found in Refs.[6, 7]. First is to obtain the coupled-channel scattering equations in TOPT,

$$\begin{aligned} T^{M_f B_f, M_i B_i}(E; \boldsymbol{p}', \boldsymbol{p}) =& V^{M_f B_f, M_i B_i}(E; \boldsymbol{p}', \boldsymbol{p}) \\ &+ \sum_{M,B} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} V^{M_f B_f, MB}(E; \boldsymbol{p}', \boldsymbol{k})\, G^{MB}(E)\, T^{MB, M_i B_i}(E; \boldsymbol{k}, \boldsymbol{p}), \end{aligned} \tag{1}$$

where M_iB_i, M_fB_f and MB denote initial, final and intermediate particle channels. The two-body Green function, which is obtained according to the diagrammatic rules of TOPT [8], has the form

$$G^{MB}(E) = \frac{1}{2\,\omega_M \omega_B} \frac{-m_B}{E - \omega_M - \omega_B + i\epsilon}\,, \tag{2}$$

where $\omega_i \equiv \sqrt{\boldsymbol{p}_i^2 + m_i^2}$ is the energy of the $i_{\rm th}$ hadron with the mass m_i and momentum $\boldsymbol{p}_i$.

The effective potential $V(E; \boldsymbol{p}', \boldsymbol{p})$ is defined as the sum of two-particle irreducible time ordered diagrams of the meson-baryon scattering. At leading order, using the covariant Lagrangians,

$$\begin{aligned} \mathcal{L}_{\rm LO} \;=\; & \frac{F_0^2}{4} \left\langle u_\mu u^\mu + \chi_+ \right\rangle + \left\langle \bar{B} \left(i\gamma_\mu \partial^\mu - m \right) B \right\rangle + \frac{D/F}{2} \left\langle \bar{B}\gamma_\mu\gamma_5 [u^\mu, B]_\pm \right\rangle \\ & - \frac{1}{4} \left\langle V_{\mu\nu}V^{\mu\nu} - 2\mathring{M}_V^2 \left(V_\mu - \frac{i}{g}\Gamma_\mu \right)\left(V^\mu - \frac{i}{g}\Gamma^\mu \right) \right\rangle + g \left\langle \bar{B}\gamma_\mu [V^\mu, B] \right\rangle, \end{aligned} \tag{3}$$

where the definition of symbols can be found in Ref. [7], one can calculate the time-ordered diagrams shown in Fig. 1 to obtain the effective potential. It is worthy to note that the lowest-lying vector mesons are involved as the dynamical degrees of freedom in the effective Lagrangians. Because the vector-meson exchange contribution has a better ultraviolet behavior than the standard Weinberg-Tomozawa term, without changing the low-energy physics.

Next, we can divide the leading order potential into the one-baryon reducible part and irreducible part,

$$V_{\rm LO} = V_R + V_I, \quad \text{with} \quad V_R = V_a, \;\; V_I = V_{b+c+d}. \tag{4}$$

This allows us to rewrite the integral equation $T = V + VGT$ (the symbolic form of Eq. (1)) as follows,

$$\begin{aligned} T &= T_I + (1 + T_I\,G)\,T_R\,(1 + G\,T_I), \\ T_I &= V_I + V_I\,G\,T_I, \quad T_R = V_R + V_R\,G\,(1 + T_I\,G)\,T_R. \end{aligned}$$

It is found that the irreducible part T_I is finite in the removed regulator limit, while the reducible part T_R is divergent. The divergence terms in T_R can be systematically removed by using the subtractive renormalization [11]: replacing the meson-baryon propagator $G_{MB}(E)$ with the subtracted one $G^S_{MB}(E) = G_{MB}(E) - G_{MB}(m_B)$. Note that such subtraction, which is an analogy to the extended-on-mass-shell scheme [12], corresponds to the expansion around the threshold and can systematically remove the chiral power-counting breaking terms in the iterated amplitudes. Finally, we obtain a renormalized finite T-matrix.

Table 1. Pole positions of $\Lambda(1405)$ in the $S = -1$ sector (units are MeV).

		lower pole	higher pole
This work	$F_0 = F_\pi$	$1337.7 - i\,79.1$	$1430.9 - i\,8.0$
(LO)	$F_0 = 103.4$	$1348.2 - i\,120.2$	$1436.3 - i\,0.7$
NLO	Ref. [13]	$1381^{+18}_{-6} - i81^{+19}_{-8}$	$1424^{+7}_{-23} - i26^{+3}_{-14}$
	Ref. [14], Fit II	$1388^{+9}_{-9} - i114^{+24}_{-25}$	$1421^{+3}_{-2} - i19^{+8}_{-5}$
	Ref. [15], sol-2	$1330^{+4}_{-5} - i56^{+17}_{-11}$	$1434^{+2}_{-2} - i10^{+2}_{-1}$
	Ref. [15], sol-4	$1325^{+15}_{-15} - i90^{+12}_{-18}$	$1429^{+8}_{-7} - i12^{+2}_{-3}$

3 Results and discussion

As a simple application, we first studied the πN scattering up to LO using the proposed formalism in Ref. [6]. We found that phase shifts based on the non-perturbative renormalized amplitude are only slightly different from the ones obtained from the perturbative tree-level calculation. This indicates that our non-perturbative treatment is valid, since the chiral expansion has good convergence in the SU(2) sector.

Next, in Ref. [7] we extended the TOPT framework to the $S = -1$ meson-baryon scattering, which has four coupled channels: $\bar{K}N$, $\pi\Sigma$, $\eta\Lambda$, and $K\Xi$. Using the LO effective potential, we exactly solved the coupled-channel scattering equation (Eq. (1)) in the isospin basis by taking into account the off-shell effects. In order to obtain the renormalized T-matrix, we utilized the subtractive renormalization which allows us to take the momentum cutoff to infinity. We would like to emphasize that no free parameters are introduced in the whole process. After performing the analytic continuation of the renormalized T-matrix into the complex s-plane, we find the two-pole structure of $\Lambda(1405)$ in the second Riemann sheet, and the corresponding pole positions are presented in Table 1. We can see that by changing the meson-decay constant, the width of lower pole is increasing, and the higher pole lies close and moves beyond the threshold of $\bar{K}N$ channel and its width decreases. In comparison with the previous NLO studies [13–15], we found that our LO results are consistent with Mai's results [15], in particular for the lower pole. The $\pi\Sigma$ invariant mass spectrum is presented in Ref. [7], which is in good agreement with experimental data. Furthermore, we calculated the coupling strengths of the initial/final transition channels, and found that the two poles of $\Lambda(1405)$ have different coupling natures: the lower pole couples predominantly to the $\pi\Sigma$ channel, and the higher pole couples strongly to the $\bar{K}N$ channel.

Finally, using the $I = 0,\ 1$ amplitudes of $\bar{K}N$ scattering in the coupled channels, we calculated the total cross section of $K^- p$ scattering to various channels, as shown in Fig. 2. Our LO prediction covers well the data of $K^- p \to \pi\Sigma$ cross section, and is slightly larger than the data of $K^- p \to K^- p, \pi^0\Lambda$. This will be improved by introducing the isospin-breaking effects and beyond LO studies.

4 Conclusion

In this work, we studied the meson-baryon scattering using the time-ordered perturbation theory based on covariant chiral effective field theory. The effective potential and the scattering equation are formulated within the same framework. By taking the vector mesons as the explicit degrees of freedom in the chiral effective Lagrangians, we achieved the renormalized scattering amplitude at leading order by solving the integral equations of meson-baryon scattering. The proposed formalism has been successfully applied to the πN scattering and the $S = -1$ meson-baryon scattering, and found the two-pole structure of the $\Lambda(1405)$ state.

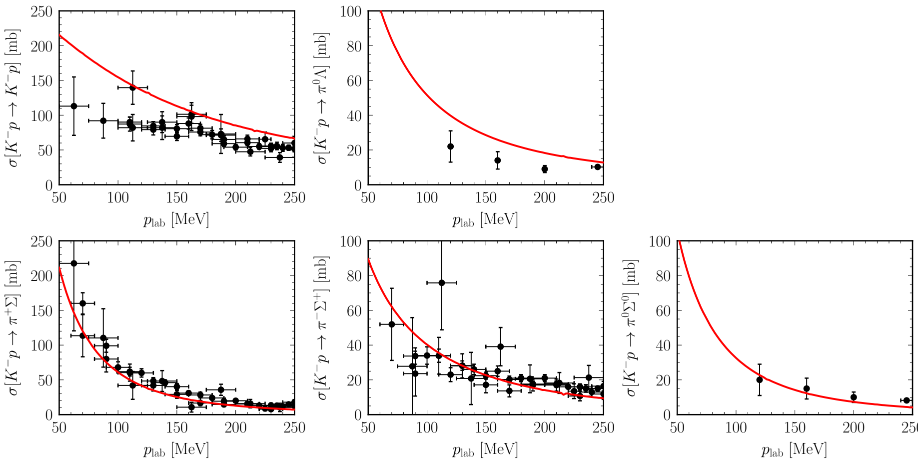

Figure 2. Total cross section of K^-p scattering to various channels as the function of incident K^- laboratory momentum $p_{\rm lab}$.

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