

## Higher Twists

Vladimir M. Braun<sup>1,\*</sup>

<sup>1</sup>Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany

**Abstract.** The higher twist corrections refer to a certain class of contributions to hard processes in strong interactions that are suppressed by a power of the hard scale. This is a very broad field of research which is becoming more and more important as the accuracy of the available experimental data increases. I give an overview of some relevant basic theory concepts and technical developments, and briefly discuss a few phenomenological applications.

### 1 Introduction

The notion of twist was introduced in 1971 in the paper by Gross and Treiman [1] who noticed that “it is no longer the dimension alone that determines the importance of an operator near the light-cone, but rather the difference between the dimension and spin”. They called this quantity the “twist” on an operator,  $\tau = d - s$ . For a paradigm example of the traceless and symmetric in all indices quark-antiquark operator with many derivatives

$$O_{\mu\mu_1\dots\mu_n} = \bar{q}\gamma_\mu D_{\mu_1}\dots D_{\mu_n}q, \quad \tau = (n + 3) - (n + 1) = 2. \quad (1)$$

Contributions of the operators with the lowest twist give the dominant contribution to light-cone dominated processes and those of higher twist (HT) are power-suppressed. A similar observation was done around the same time by Brandt and Preparata [2].

Today, the name “twist” is used more broadly as a basis for several distinct classification schemes. The original definition  $\tau = \text{dimension} - \text{spin}$  is sometimes referred to as “geometric twist”. Operators with different geometric twist do not mix under renormalization and their matrix elements define independent nonperturbative quantities. In contrast, “collinear twist” is defined as  $\tau = \text{dimension} - \text{spin projection on plus direction}$ . This concept goes back to the light-cone formalism by Kogut and Soper [3] who decompose the quark and gluon fields in “good” (dynamically independent) and “bad” components. Each replacement of a “good” by a “bad” component adds one unit of collinear twist. The utility of this concept is that collinear twist counting is directly related, see e.g. [4], to power suppression of the corresponding contribution in light-cone dominated processes, i.e. those that can be treated using collinear factorization. The price to pay is that parton distributions of a given collinear twist contain contributions of lower geometric twists which are dynamically independent. The classical example of such “contamination” is provided by the Wandzura-Wilczek contribution [5] to the structure function  $g_2(x, Q^2)$  which will be discussed in what follows.

The subject of HT effects is getting prominence fuelled by very high accuracy of the experimental data from LHC, JLAB, KEKII, and on the 10 years scale from the Electron-Ion

---

\*e-mail: vladimir.braun@ur.de

Collider (EIC) [6]. Interpretation of these data requires the corresponding theory precision so that higher-twist corrections cannot be ignored. Maybe more importantly, as emphasized already by Politzer [7], the higher-twist contributions provide one with insight on quark-gluon correlations and quantum interference effects in hadrons. Thus their study allows one to learn more about hadron structure.

HT contributions do not have any simple partonic interpretation. Generally speaking, they are generated by a non-vanishing parton off-shellness/transverse momentum in two-particle parton distributions and the contributions of multiparton correlation functions that take into account coherent hard scattering from a parton pair. These two effects are physically distinct, but are related by exact QCD equations of motion (EOM) so that they must be taken into account simultaneously. A good example is provided by the twist-three light-cone distribution amplitudes (LCDAs) of the  $\pi$ -meson, schematically

$$\begin{aligned}
 \langle 0 | \bar{q}(z) \gamma_5 q(0) | \pi(q) \rangle & \mapsto \phi_p(x) \\
 \langle 0 | \bar{q}(z) \sigma_{+-} \gamma_5 q(0) | \pi(q) \rangle & \mapsto \phi_\sigma(x) \\
 \langle 0 | \bar{q}(z) \sigma_{+\perp} g F_{+\perp}(\alpha z) \gamma_5 q(0) | \pi(q) \rangle & \mapsto \Phi_3(x_1, x_2, x_3)
 \end{aligned} \tag{2}$$

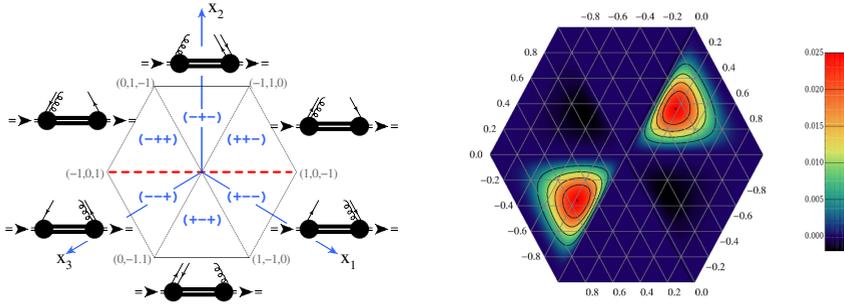
that contribute at subleading  $1/Q$  accuracy in hard processes involving pion emission with large momentum. One can show [8] that the two-particle LCDAs  $\phi_p(x)$ ,  $\phi_\sigma(x)$  are given by exact relations in terms of the three-particle LCDA  $\Phi_3(x_1, x_2, x_3)$ .

Thanks to EOM there are several possibilities to identify independent degrees of freedom by choosing a convenient operator basis. The most natural choice is probably a multipartonic (longitudinal) basis in which case contributions of “bad” field components are systematically eliminated in terms of the contributions with extra gluon fields (or quark-antiquark pairs). This is the closest one can do to maintain partonic interpretation and Lorentz covariance. This approach was followed in most of the early works on higher twists, see [7, 9–14]. A “transverse” basis [15], where HT operators are built of a quark-antiquark pair and transverse covariant derivatives presents another viable option that simplifies diagrammatic calculation of the coefficient functions. An example of the application of this technique and its equivalence to the covariant approach can be found in [16]. Worth mentioning is also the  $SL(2)$ -covariant operator basis of Refs. [17, 18] that makes explicit symmetry properties and greatly simplifies the evolution equations. The one-loop evolution equations for all twist-three and twist-four operators in QCD are known [9, 10, 13, 14, 18]. Some of these equations are completely integrable [19] and can be solved explicitly, see [20] for a review.

## 2 Twist three

Twist-three contributions in inclusive and semiinclusive reactions in collinear factorization can be described in terms of a quark-antiquark-gluon correlation function (CF) (and in addition a three-gluon CF, for flavor singlet). It is a function of two variables and is conveniently presented in barycentric coordinates  $x_1 + x_2 + x_3 = 0$  where  $x_1$ ,  $x_2$  and  $-x_3$  correspond to the momentum fractions carried by the quark, gluon and antiquark, respectively, see Fig. 1. This CF describes quantum-mechanical interference between scattering from a parton pair and a single parton, with six different regions as shown on the left panel. A simple light-front wave function overlap model [21] for the u-quark CF is shown on the right panel as an illustration. The scale-dependence of the twist-three CFs is well understood [13, 14, 22–25]. A summary of the one-loop evolution kernels can be found in [25].

The information on twist-three CFs can be obtained in particular from measurements of the structure function  $g_2(x, Q^2)$  in polarized deep-inelastic scattering (DIS) and single transverse spin asymmetries in semiinclusive reactions. In the first case a certain integral over



**Figure 1.** Domain interpretation (left) and the light-front wave function overlap model [21] (right) for the twist-three  $u$ -quark-antiquark-gluon correlation function in the proton.

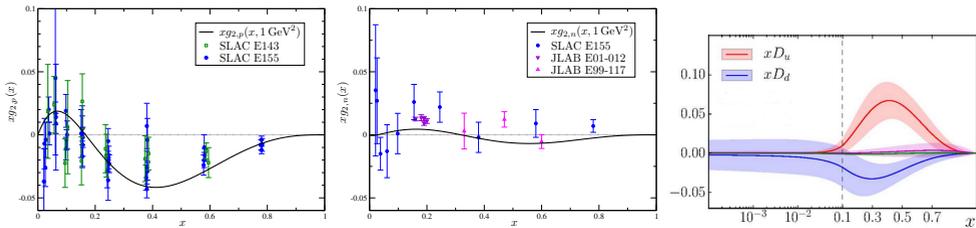
the whole hexagon in Fig. 1 enters, and in the second case the CF on the horizontal line  $x_2 = 0$  gives the main contribution, the so-called Qiu-Sterman function [26]. Proposals and exploratory studies exist of various twist-three matrix elements in lattice QCD [27–31].

The structure function  $g_2(x, Q^2)$  provides a paradigm case of a *collinear twist-three* observable (suppressed as  $1/Q$ ) that receives contributions of *geometric twist-two* and *-three*.

$$\begin{aligned}
 g_2^{(\tau_2)}(x, Q^2) &= g_1(x, Q^2) - \int_x^1 \frac{dy}{y} g_1(y, Q^2) && \Leftarrow \text{Wandzura-Wilczek contribution} \\
 g_2^{(\tau_3)}(x, Q^2) &\stackrel{!}{=} D(x, Q^2) - \int_x^1 \frac{dy}{y} D(y, Q^2) && \Leftarrow \text{convenient parametrization} \quad (3)
 \end{aligned}$$

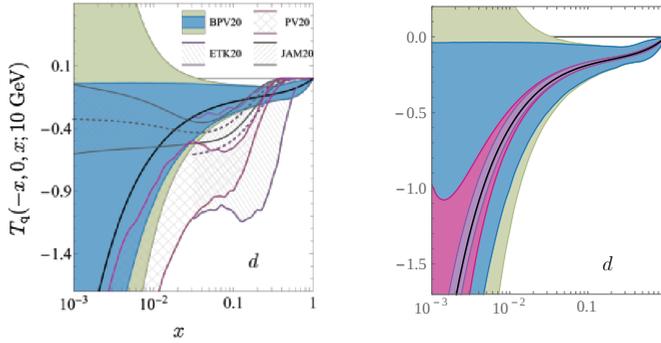
The first equation is exact in QCD, the second one is just a convenient parametrization. The D-function is given by a certain integral of the quark-antiquark-gluon CF sketched in Fig. 1, see e.g. [21].

Experimental data on the structure function  $g_2(x, Q^2)$  so far are not very precise, see the first two panels in Fig. 2. The Wandzura-Wilczek contribution is known to be dominant, which makes the extraction of the twist-three part difficult. A recent attempt [32] is shown in Fig. 2 on the right panel.



**Figure 2.** Left and middle panels: The structure function  $g_2(x, Q^2)$  for the proton and the neutron, respectively. Right panel: The twist-three contributions  $D(x, Q^2)$  for the  $u$ -quark and the  $d$ -quark in the proton. Figure adapted from [32].

Better data exist for the first nontrivial moment  $d_2 = 6 \int dx x^2 g_2^{(\tau_3)}(x)$ . A recent lattice calculation [30] obtains  $d_2^{(p)} = 0.0105(68)$  and  $d_2^{(n)} = -0.0009(70)$  for the proton and the neutron,



**Figure 3.** The  $d$ -quark contribution to the Qiu-Sterman function extracted from the collinear limit of the Sivers function. The existing determinations are summarized on the left panel (adapted from [36]). The projected accuracy for EIC is shown in magenta on the right panel (EIC impact studies, courtesy of A. Vladimirov).

respectively. In this work also a rather complete compilation of the experimental data on  $d_2$  can be found. The smallness of  $d_2$  was expected and supported by older estimates based on QCD sum rules [33, 34] and the instanton liquid model [35].

Another source of information on the twist-three quark-antiquark-gluon correlation function is provided by the Qiu-Sterman function [26]. It can in principle be extracted, e.g., from the collinear limit of the Sivers function, but the results are so far very uncertain, see the left panel in Fig. 3. The situation is going to improve significantly with the data from EIC, see the right panel for an impact study. The projected accuracy is indicated by the magenta region.

Due to the time constraints I will not discuss here subleading twist-three LCDAs of pseudoscalar and vector mesons. This is a large separate topic where the formalism is well-developed and realistic models based on the conformal expansion [37] are available, see e.g. [8, 38, 39].

### 3 Twist four and renormalons

The structure of twist-four contributions to DIS is well-known since many years [9–15, 22]. A generalization to one-particle inclusive production in  $e^+e^-$ -annihilation also exists [40]. The one-loop evolution equations for all twist-four correlation functions are known [9, 10, 14, 18]. The main problem is that the nonperturbative input required for the description of twist-four effects is very complicated; for the DIS case one finds seven independent three-parton and four-parton distributions (for one quark flavor) [12] and there is no obvious hierarchy. The phenomenology is inconclusive; it is generally accepted that twist-four corrections in DIS are “small”, but this statement is difficult to quantify.<sup>1</sup> A crucial difference to twist-three effects is that the twist-four contributions are not associated with particular power-suppressed *observables*, but rather give rise to  $1/Q^2$  corrections to the amplitudes/cross sections that receive the leading-twist contributions as well.

A separation of the leading power/leading twist and  $1/Q^2$  twist-four contributions is in fact a nontrivial problem. The reason is that twist-four operators suffer from quadratic UV

<sup>1</sup>In the case of twist-four LCDAs a systematic expansion is possible in conformal spin, which is analogous to the partial wave expansion in quantum mechanics. This construction is widely used to build models, see e.g. [8, 41, 42], with many applications to weak exclusive B-decays.

divergences so that their determination from, e.g., lattice QCD would need a prescription how these divergences are regularized. This prescription, in turn, affects the procedure to treat higher-order perturbative contributions at leading power due to the so-called renormalons. I will briefly explain this phenomenon and argue that the “renormalon problem” naturally leads to “renormalon models” for the power corrections with very few parameters. A detailed discussion can be found in [43, 44].

The key observation is that a leading-twist calculation “knows” about the necessity to add a power correction. Consider the DIS structure function  $F_2(x, Q^2)$  as an example. It can be written as a convolution

$$F_2(x, Q^2) = 2x \int_x^1 \frac{dy}{y} C(y, Q^2/\mu^2) q\left(\frac{x}{y}, \mu^2\right) \left(1 + \frac{D_2(x)}{Q^2}\right). \quad (4)$$

Here  $C(y) = \delta(1-y) + \sum_{n=0}^{\infty} c_n \alpha_s^{n+1}$ ,  $\alpha_s = \alpha_s(\mu)$ ,  $q(x, \mu^2)$  is the quark distribution function (PDF) and  $D_2(x)$  is a twist-four contribution of interest.

Imagine the separation between the coefficient function (CF) and the quark PDF is done using explicit cutoff at  $|k| = \mu$ . The CF will then be modified compared to usual calculation by terms  $\sim \mu^2/Q^2$ :

$$C_2(y)|^{\text{cut}} = \delta(1-y) + \sum_{n=0}^{\infty} c_n \alpha_s^{n+1} - \frac{\mu^2}{Q^2} d_2(x) + \mathcal{O}\left(\frac{\mu^4}{Q^4}\right). \quad (5)$$

The dependence on  $\mu$  must cancel: Logarithmic terms  $\ln Q^2/\mu^2$  in CFs against the  $\mu$ -dependence in PDFs, and power-suppressed terms  $\mu^2/Q^2$  against the higher-twist contributions. This implies that  $D_2(x)$  in the cutoff scheme must have the form

$$D_2(x) = \mu^2 2x \int_x^1 \frac{dy}{y} d_2(x) q\left(\frac{x}{y}\right) + \delta D_2(x), \quad (6)$$

where the first term is related to quadratic UV divergences in matrix elements of twist-4 operators (in this scheme).

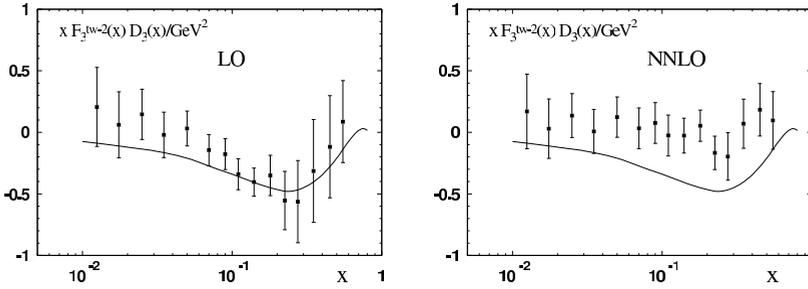
Using dimensional regularization power-like terms in the CFs do not appear. Instead, the coefficients  $c_k$  diverge factorially with the order  $k$ . The sum of the pert. series in the CF is only defined to a power accuracy and this ambiguity (renormalon ambiguity) must be compensated by adding a non-perturbative higher-twist correction. A detailed analysis [44] shows that large-order behavior of the coefficients (the renormalons) is in one-to-one correspondence with the sensitivity to extreme (small or large) loop momenta in Feynman diagrams. IR renormalons in twist-two CFs are compensated by UV renormalons in matrix elements of twist-four operators. At the end the same picture re-appears: only the details depend on the factorization method.

The quadratic term in  $\mu$  in Eq. (5) is spurious since its sole purpose is to cancel the similar contribution to the CF  $\Rightarrow$  does not contribute to any physical observable. Assuming, however, that the “true” twist-four term is of the same order, one gets a *renormalon model*

$$D_2(x) = \varkappa \Lambda_{\text{QCD}}^2 2x \int_x^1 \frac{dy}{y} d_2(x) q\left(\frac{x}{y}\right), \quad \varkappa = \mathcal{O}(1), \quad (7)$$

where  $\varkappa$  is the only free parameter. To one-loop accuracy [45]

$$d_2 = -\frac{4}{[1-x]_+} + 4 + 2x + 12x^2 - 9\delta(1-x) - \delta'(1-x). \quad (8)$$



**Figure 4.** Twist-4 correction to  $xF_3(x, Q^2)$  as extracted from the (revised) CCFR data. The data points [46] are overlaid with the shape obtained from the “renormalon model” for the  $1/Q^2$  power correction. Figure adapted from [44].

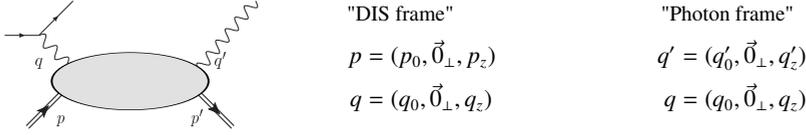
Noticeable features of this approximation is that the shape of the twist-four correction does not depend on the target, and that the correction is enhanced at  $x \rightarrow 1$ :  $D_2(x)/Q^2 \sim \Lambda^2/[Q^2(1-x)]$ . This analysis was done in the past for  $F_2(x, Q^2)$ ,  $F_L(x, Q^2)$ ,  $F_3(x, Q^2)$  and  $g_1(x, Q^2)$  etc. An example is shown in Fig. 4 where the data points [46] on the twist-four correction to  $xF_3(x, Q^2)$  are overlaid with the shape obtained from the renormalon model. This analysis also confirms the expectation from renormalons that the size of the twist-four correction is decreasing with the increasing order of perturbation theory in leading twist.

The power of the renormalon-based analysis is that this technique can be used for Minkowski-space observables where the operator product expansion is not applicable — the Drell-Yan process, event shapes in  $e^+e^-$  annihilation etc., see [43, 44] for a review. Notable other applications are to power corrections in quasi- and pseudo-parton distributions [47, 48] and in transverse momentum dependent (TMD) distributions [49]. For a recent review on renormalons in the heavy quark pole mass definition see [50].

## 4 Kinematic power corrections in off-forward reactions

For definiteness, I will speak of the deeply-virtual Compton scattering (DVCS)  $\gamma^*(q) + N(p) \rightarrow \gamma(q') + N(p')$ ,  $q^2 = -Q^2 \gg \Lambda_{QCD}^2$ ,  $q'^2 = 0$ , but the problem is generic for all hard processes involving a momentum transfer between the initial and final state hadrons. This reaction involves three helicity amplitudes — helicity conserving and helicity flip for transversely polarized photons, and with a longitudinally polarized photon in the initial state — of which the first one contributes at leading twist and the other two are power suppressed. As well known, the definition of helicity amplitudes depends on the frame of reference. Going over to a different frame, the leading-twist amplitude gets modified by power corrections  $(\sqrt{-t}/Q)^n$  and  $(m/Q)^n$  where  $t$  is the Mandelstam momentum transfer variable and  $m$  is the nucleon mass, and also the subleading power amplitudes are generated even if they were absent (neglected) in the original frame.

Which frame is the best? In DIS there is a natural choice: the photon-nucleon scattering plane (in four dimensions) is defined as longitudinal and orthogonal directions as transverse. In DVCS the four external momenta are not coplanar and there are many possibilities, see Fig. 5. The leading-twist approximation in DVCS is therefore convention-dependent: taking into account only the leading-power contribution in the “DIS frame” and the “photon frame” will produce different results for the observables [51–53]. The difference can be large in certain kinematic regions, see Fig. 6. The Bethe-Heitler contribution (red dashes)



**Figure 5.** Two examples of possible choices of longitudinal and transverse directions in DVCS

is essentially an electromagnetic background. The blue curve shows the leading-twist result in the “DIS frame” using the GK12 GPD model [54]. The green curve includes in addition the kinematic power corrections  $t/Q^2$  and  $m^2/Q^2$ . This curve is very close numerically to the leading-twist result in the “photon frame”, see [51]. One sees that the difference is quite substantial.

Power corrections that repair the frame dependence (and restore electromagnetic gauge invariance) in DVCS come from two sources. First, there are corrections  $(\sqrt{-t}/Q)^n$  and  $(m/Q)^n$  to the matrix elements of twist-two operators that are analogous to the target mass corrections in DIS [55]. Second, there are power corrections due to contributions of higher-twist operators that are obtained from the twist-two ones by adding total derivatives, which are called the descendants. Schematically

$$\begin{aligned}
 T\{j(x)j(0)\} = \sum_N \left\{ & A_N^{\mu_1 \dots \mu_N} \underbrace{\mathcal{O}_{\mu_1 \dots \mu_N}^N}_{\text{twist-2 operators}} + B_N^{\mu_1 \dots \mu_N} \underbrace{\partial^\mu \mathcal{O}_{\mu, \mu_1 \dots \mu_N}^N}_{\text{descendants of twist 2}} \right. \\
 & + C_N^{\mu_1 \dots \mu_N} \underbrace{\partial^2 \mathcal{O}_{\mu_1 \dots \mu_N}^N}_{\text{descendants}} + D_N^{\mu_1 \dots \mu_N} \underbrace{\partial^\mu \partial^\nu \mathcal{O}_{\mu, \nu, \mu_1 \dots \mu_N}^N}_{\text{descendants}} + \dots \left. \right\} \\
 & + \text{quark-gluon operators} \tag{9}
 \end{aligned}$$

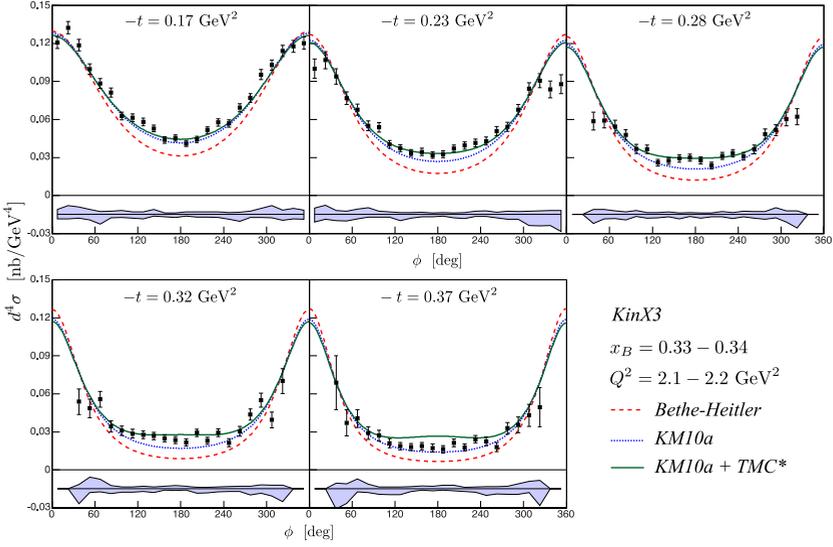
Contributions of the descendants have to be taken into account, whereas the additional quark-antiquark-gluon (and more complicated) operators can be omitted for the present purpose as they involve new nonperturbative matrix elements and cannot be responsible for the restoration of Lorentz invariance. The problem is that matrix elements of some of the descendant operators on free quarks vanish, so that the corresponding coefficient functions cannot be calculated in the standard manner. Moreover, descendant operators are related to the  $\bar{q}Fq$  operators by EOM, for the simplest case

$$\partial^\mu O_{\mu\nu} = 2i\bar{q}gF_{\nu\mu}\gamma^\mu q, \quad O_{\mu\nu} = (1/2)[\bar{q}\gamma_\mu \overleftrightarrow{D}_\nu q + (\mu \leftrightarrow \nu)]. \tag{10}$$

Thus the separation of “kinematic” higher-twist contributions of the descendant operators from the “genuine” higher-twist corrections due to quark-gluon correlations poses a nontrivial problem [57, 58]. The suggested solution (see also [59]) is that the coefficient functions of descendants are related to those of twist-two operators by conformal symmetry, schematically

$$A_N^{\mu_1 \dots \mu_N} \xrightarrow{O(4,2)} \{B_N, C_N, D_N, \dots\}^{\mu_1 \dots \mu_N}, \tag{11}$$

and do not need to be calculated explicitly. This approach was used to calculate the twist-four kinematic corrections [60] used in [51]. The extension of these results up to twist-six accuracy for scalar targets is reported in [61]. Possible applications beyond DVCS include, e.g., the studies of  $t$ -channel processes like  $\gamma^* \gamma \rightarrow \pi\pi$ , see [62].



**Figure 6.** The (unpolarized) DVCS cross section as a function of the azimuthal angle  $\phi$  [56] compared to the GK12 GPD model [54].

## 5 Beyond the twist expansion

Not all power corrections  $1/Q^k$  can be obtained from the expansion near the light cone. An instructive example [63] is provided by a calculation of the pion transition form factor  $\gamma^* \rightarrow \gamma\pi$  is the Brodsky-Lepage formalism [64]. Using a simple model for the quark-antiquark component of the pion light-front wave function

$$\Psi^{\bar{q}q}(x, k_{\perp}^2) \sim \phi_{\pi}(x) \exp\left[-\frac{k_{\perp}^2}{2\sigma x\bar{x}}\right], \quad \bar{x} = 1 - x, \quad (12)$$

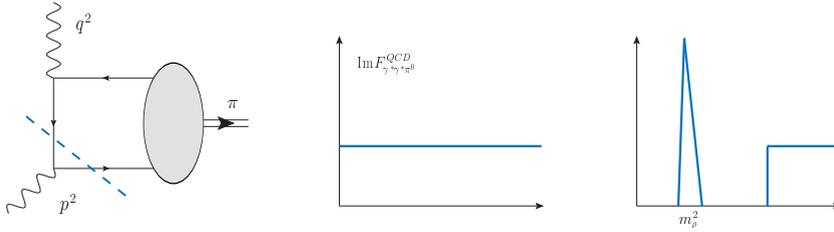
where  $\sigma = O(\Lambda_{QCD}^2)$  is a nonperturbative width parameter, one obtains [63]

$$F_{\gamma^*\gamma\pi}^{\bar{q}q}(Q^2) = \frac{\sqrt{2}f_{\pi}}{3Q^2} \int_0^1 \frac{dx}{x} \phi_{\pi}(x) \left[1 - \exp\left(-\frac{xQ^2}{2\bar{x}\sigma}\right)\right], \quad (13)$$

so that for  $Q^2 \rightarrow \infty$

$$F_{\gamma^*\gamma\pi}(Q^2) = \frac{\sqrt{2}f_{\pi}}{3Q^2} \int_0^1 \frac{dx}{x} \phi_{\pi}(x) + \underbrace{\frac{1}{Q^4}[\text{higher Fock states}]}_{\text{higher twists}} - \underbrace{\frac{4\sqrt{2}f_{\pi}\sigma}{Q^4}}_{\text{end-point (Feynman)}}. \quad (14)$$

The first term in this expression is the usual pQCD result at leading power [64], the second (higher-twist) term comes from the contributions of higher Fock states, and the last term arises from the exponential correction in Eq. (13) under the assumption that  $\phi_{\pi}(x)_{x \rightarrow 1} = O(1-x)$ . This last contribution originates from the end-point region  $1-x = O(\sigma/Q^2)$  and large transverse distances between the quark and the antiquark in the pion,  $|x_{\perp}^2| \sim 1/\sigma \sim 1/\Lambda_{QCD}^2$ , so that it is not seen to any order in the light-cone (twist) expansion. Such terms arise naturally in models and are usually referred to as end-point contributions. They can be present already



**Figure 7.** A schematic representation of the LCSR technique: Consider a dispersion relation for the form factor with two nonzero photon virtualities (left) and modify the perturbative spectral density (middle) by the resonance + continuum model with the parameters fixed by duality.

at leading power and invalidate collinear factorization theorems, e.g., for the  $B \rightarrow \pi \ell \bar{\nu}_\ell$  form factor.

End-point contributions present a new type of nonperturbative corrections that should be taken into account. They have to be properly defined and separated from the “usual” higher-twist corrections to prevent double counting. A common theory approach to estimate such contributions are the light-cone sum rules (LCSRs) [65–68] based on dispersion relations and quark-hadron duality. The simplest example is provided, again, by the pion transition form factor  $\gamma^* \rightarrow \gamma \pi$  [69]. The idea is to consider a more general process with two virtual photons,  $p^2 < 0$ ,  $q^2 < 0$ , write a dispersion relation in  $p^2$  and replace the calculated in QCD perturbative spectral density by the resonance + continuum model with the parameters fixed by duality; put  $p^2 \rightarrow 0$  at the end. For the state-of-the-art calculations of the  $\gamma^* \rightarrow \gamma \pi$  form factor in this technique see [70–73].

The LCSR approach proves to be very flexible. It has several modifications and has been applied to many exclusive reactions, most notably to weak B-decays, see e.g. [74–78]. This is a large field of research that cannot be covered in this talk.

## 6 Brief summary and further developments

I have given a brief overview of the research on higher twist corrections in QCD that arise from subleading contributions in light-cone dominated hard processes. The subject is very broad and the choice of topics in this review is of course influenced by my own work and experience. I apologize for being unable to mention many important contributions; the bibliography would explode if I tried. The take-home message from this report is that basic theory of higher-twist corrections is in a good shape, but phenomenology is lagging behind. An obvious reason is that higher twist effects are small in most situations and are difficult to disentangle unambiguously from leading twist contributions. Twist-three contributions are special in this respect since they can be directly related to observables, at least in principle. With extremely high statistical precision of experimental data expected from the new generation of particle accelerators, most notably the EIC, determination of the twist-three correlation functions in the nucleon can be achievable, albeit a challenging task.

In conclusion, I want to mention briefly two more research directions that are currently receiving increasing attention. The first one concerns power corrections in the formalism of TMD factorization. The motivation for this research comes from the desire to be able to describe  $p_T$ -dependent observables in semi-inclusive reactions (where  $p_T$  could be, e.g., the hadron transverse momentum in SIDIS or the transverse momentum of a Drell-Yan pair) in

the whole kinematic region from small  $p_t$  where TMD factorization is applicable, to large  $p_T$  that can be described in collinear factorization. At EIC, the regions of applicability of these approaches do not overlap, so one needs to extend them including power suppressed effects. There are several recent attempts to do this from the TMD side, see [79–81]. This generalization is not straightforward, e.g., the structure of rapidity divergences in twist-three TMDs proves to be nontrivial [82].

The second topic concerns going beyond leading power in jet observables and related quantities. This environment is specific in that all ingredients in factorization formulas are perturbatively calculable (if one stays at the parton level), so that a proliferation of distributions at subleading powers is not of concern. Soft-collinear effective theory (SCET) offers a natural framework for these studies, and in the last years this field was very active: computing power corrections at fixed order, subleading power regularization and resummation, resummation of large logarithms and the role of Glauber regions was addressed. There exists a large literature on this subject, see e.g. [83–89]. Threshold resummation beyond leading power is another relevant theme in LHC and EIC context, see [90] and references therein.

## Acknowledgments

This work was supported in part by the Research Unit FOR2926 funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under grant 409651613.

## References

- [1] D.J. Gross, S.B. Treiman, *Phys. Rev. D* **4**, 1059 (1971)
- [2] R.A. Brandt, G. Preparata, *Nucl. Phys. B* **27**, 541 (1971)
- [3] J.B. Kogut, D.E. Soper, *Phys. Rev. D* **1**, 2901 (1970)
- [4] R.L. Jaffe, X.D. Ji, *Nucl. Phys. B* **375**, 527 (1992)
- [5] S. Wandzura, F. Wilczek, *Phys. Lett. B* **72**, 195 (1977)
- [6] R. Abdul Khalek et al., *Nucl. Phys. A* **1026**, 122447 (2022), 2103.05419
- [7] H.D. Politzer, *Nucl. Phys. B* **172**, 349 (1980)
- [8] V.M. Braun, I.E. Filyanov, *Z. Phys. C* **48**, 239 (1990)
- [9] E.V. Shuryak, A.I. Vainshtein, *Nucl. Phys. B* **199**, 451 (1982)
- [10] E.V. Shuryak, A.I. Vainshtein, *Nucl. Phys. B* **201**, 141 (1982)
- [11] R.L. Jaffe, M. Soldate, *Phys. Rev. D* **26**, 49 (1982)
- [12] R.L. Jaffe, *Nucl. Phys. B* **229**, 205 (1983)
- [13] A.P. Bukhvostov, E.A. Kuraev, L.N. Lipatov, *Sov. Phys. JETP* **60**, 22 (1984)
- [14] A.P. Bukhvostov, G.V. Frolov, L.N. Lipatov, E.A. Kuraev, *Nucl. Phys. B* **258**, 601 (1985)
- [15] R.K. Ellis, W. Furmanski, R. Petronzio, *Nucl. Phys. B* **212**, 29 (1983)
- [16] I.V. Anikin, D.Y. Ivanov, B. Pire, L. Szymanowski, S. Wallon, *Nucl. Phys. B* **828**, 1 (2010), 0909.4090
- [17] V.M. Braun, A.N. Manashov, J. Rohrwild, *Nucl. Phys. B* **807**, 89 (2009), 0806.2531
- [18] V.M. Braun, A.N. Manashov, J. Rohrwild, *Nucl. Phys. B* **826**, 235 (2010), 0908.1684
- [19] V.M. Braun, S.E. Derkachov, A.N. Manashov, *Phys. Rev. Lett.* **81**, 2020 (1998), hep-ph/9805225
- [20] A.V. Belitsky, V.M. Braun, A.S. Gorsky, G.P. Korchemsky, *Int. J. Mod. Phys. A* **19**, 4715 (2004), hep-th/0407232

- [21] V.M. Braun, T. Lautenschlager, A.N. Manashov, B. Pirnay, Phys. Rev. D **83**, 094023 (2011), 1103.1269
- [22] I.I. Balitsky, V.M. Braun, Nucl. Phys. B **311**, 541 (1989)
- [23] X.D. Ji, C.h. Chou, Phys. Rev. D **42**, 3637 (1990)
- [24] Y. Koike, K. Tanaka, Phys. Rev. D **51**, 6125 (1995), hep-ph/9412310
- [25] V.M. Braun, A.N. Manashov, B. Pirnay, Phys. Rev. D **80**, 114002 (2009), [Erratum: Phys.Rev.D 86, 119902 (2012)], 0909.3410
- [26] J.w. Qiu, G.F. Sterman, Phys. Rev. Lett. **67**, 2264 (1991)
- [27] M. Gockeler, R. Horsley, D. Pleiter, P.E.L. Rakow, A. Schafer, G. Schierholz, H. Stuben, J.M. Zanotti, Phys. Rev. D **72**, 054507 (2005), hep-lat/0506017
- [28] S. Bhattacharya, K. Cichy, M. Constantinou, A. Metz, A. Scapellato, F. Steffens, Phys. Rev. D **102**, 111501 (2020), 2004.04130
- [29] S. Bhattacharya, K. Cichy, M. Constantinou, A. Metz, A. Scapellato, F. Steffens, Phys. Rev. D **104**, 114510 (2021), 2107.02574
- [30] S. Bürger, T. Wurm, M. Löffler, M. Göckeler, G. Bali, S. Collins, A. Schäfer, A. Sternbeck (RQCD), Phys. Rev. D **105**, 054504 (2022), 2111.08306
- [31] V.M. Braun, Y. Ji, A. Vladimirov, JHEP **05**, 086 (2021), 2103.12105
- [32] N. Sato, W. Melnitchouk, S.E. Kuhn, J.J. Ethier, A. Accardi (Jefferson Lab Angular Momentum), Phys. Rev. D **93**, 074005 (2016), 1601.07782
- [33] I.I. Balitsky, V.M. Braun, A.V. Kolesnichenko, Phys. Lett. B **242**, 245 (1990), [Erratum: Phys.Lett.B 318, 648 (1993)], hep-ph/9310316
- [34] E. Stein, P. Gornicki, L. Mankiewicz, A. Schafer, W. Greiner, Phys. Lett. B **343**, 369 (1995), hep-ph/9409212
- [35] J. Balla, M.V. Polyakov, C. Weiss, Nucl. Phys. B **510**, 327 (1998), hep-ph/9707515
- [36] M. Bury, A. Prokudin, A. Vladimirov, Phys. Rev. Lett. **126**, 112002 (2021), 2012.05135
- [37] V.M. Braun, G.P. Korchemsky, D. Müller, Prog. Part. Nucl. Phys. **51**, 311 (2003), hep-ph/0306057
- [38] P. Ball, V.M. Braun, Y. Koike, K. Tanaka, Nucl. Phys. B **529**, 323 (1998), hep-ph/9802299
- [39] P. Ball, V.M. Braun, A. Lenz, JHEP **05**, 004 (2006), hep-ph/0603063
- [40] I.I. Balitsky, V.M. Braun, Nucl. Phys. B **361**, 93 (1991)
- [41] P. Ball, V.M. Braun, Nucl. Phys. B **543**, 201 (1999), hep-ph/9810475
- [42] P. Ball, V.M. Braun, A. Lenz, JHEP **08**, 090 (2007), 0707.1201
- [43] M. Beneke, Phys. Rept. **317**, 1 (1999), hep-ph/9807443
- [44] M. Beneke, V.M. Braun, pp. 1719–1773 (2000), hep-ph/0010208
- [45] M. Dasgupta, B.R. Webber, Phys. Lett. B **382**, 273 (1996), hep-ph/9604388
- [46] A.L. Kataev, A.V. Kotikov, G. Parente, A.V. Sidorov, pp. 355–364 (1997), hep-ph/9706534
- [47] V.M. Braun, E. Gardi, S. Gottwald, Nucl. Phys. B **685**, 171 (2004), hep-ph/0401158
- [48] V.M. Braun, A. Vladimirov, J.H. Zhang, Phys. Rev. D **99**, 014013 (2019), 1810.00048
- [49] I. Scimemi, A. Vladimirov, JHEP **03**, 002 (2017), 1609.06047
- [50] M. Beneke, Eur. Phys. J. ST **230**, 2565 (2021), 2108.04861
- [51] V.M. Braun, A.N. Manashov, D. Müller, B.M. Pirnay, Phys. Rev. D **89**, 074022 (2014), 1401.7621
- [52] V.M. Braun, A.N. Manashov, D. Mueller, B. Pirnay, PoS **DIS2014**, 225 (2014), 1407.0815

- [53] Y. Guo, X. Ji, K. Shiells, JHEP **12**, 103 (2021), 2109.10373
- [54] K. Kumerički, D. Mueller, Nucl. Phys. B **841**, 1 (2010), 0904.0458
- [55] O. Nachtmann, Nucl. Phys. B **63**, 237 (1973)
- [56] M. Defurne et al. (Jefferson Lab Hall A), Phys. Rev. C **92**, 055202 (2015), 1504.05453
- [57] V.M. Braun, A.N. Manashov, Phys. Rev. Lett. **107**, 202001 (2011), 1108.2394
- [58] V.M. Braun, A.N. Manashov, JHEP **01**, 085 (2012), 1111.6765
- [59] V.M. Braun, Y. Ji, A.N. Manashov, JHEP **03**, 051 (2021), 2011.04533
- [60] V.M. Braun, A.N. Manashov, B. Pirnay, Phys. Rev. Lett. **109**, 242001 (2012), 1209.2559
- [61] V.M. Braun, Y. Ji, A.N. Manashov (2022), 2211.04902
- [62] C. Lorcé, B. Pire, Q.T. Song (2022), 2209.11140
- [63] I.V. Musatov, A.V. Radyushkin, Phys. Rev. D **56**, 2713 (1997), hep-ph/9702443
- [64] G.P. Lepage, S.J. Brodsky, Phys. Rev. D **22**, 2157 (1980)
- [65] I.I. Balitsky, V.M. Braun, A.V. Kolesnichenko, Sov. J. Nucl. Phys. **44**, 1028 (1986)
- [66] I.I. Balitsky, V.M. Braun, A.V. Kolesnichenko, Nucl. Phys. B **312**, 509 (1989)
- [67] V.M. Braun, I.E. Filyanov, Z. Phys. C **44**, 157 (1989)
- [68] V.L. Chernyak, I.R. Zhitnitsky, Nucl. Phys. B **345**, 137 (1990)
- [69] A. Khodjamirian, Eur. Phys. J. C **6**, 477 (1999), hep-ph/9712451
- [70] S.S. Agaev, V.M. Braun, N. Offen, F.A. Porkert, Phys. Rev. D **83**, 054020 (2011), 1012.4671
- [71] S.V. Mikhailov, A.V. Pimikov, N.G. Stefanis, Phys. Rev. D **93**, 114018 (2016), 1604.06391
- [72] Y.M. Wang, Y.L. Shen, JHEP **12**, 037 (2017), 1706.05680
- [73] Y.L. Shen, J. Gao, C.D. Lü, Y. Miao, Phys. Rev. D **99**, 096013 (2019), 1901.10259
- [74] P. Ball, V.M. Braun, Phys. Rev. D **58**, 094016 (1998), hep-ph/9805422
- [75] P. Ball, R. Zwicky, Phys. Rev. D **71**, 014015 (2005), hep-ph/0406232
- [76] P. Ball, R. Zwicky, Phys. Rev. D **71**, 014029 (2005), hep-ph/0412079
- [77] A. Khodjamirian, T. Mannel, A.A. Pivovarov, Y.M. Wang, JHEP **09**, 089 (2010), 1006.4945
- [78] A. Khodjamirian, T. Mannel, Y.M. Wang, JHEP **02**, 010 (2013), 1211.0234
- [79] I. Balitsky, A. Tarasov, JHEP **05**, 150 (2018), 1712.09389
- [80] A. Vladimirov, V. Moos, I. Scimemi, JHEP **01**, 110 (2022), 2109.09771
- [81] M.A. Ebert, A. Gao, I.W. Stewart, JHEP **06**, 007 (2022), 2112.07680
- [82] S. Rodini, A. Vladimirov, JHEP **08**, 031 (2022), 2204.03856
- [83] I. Moulton, L. Rothen, I.W. Stewart, F.J. Tackmann, H.X. Zhu, Phys. Rev. D **95**, 074023 (2017), 1612.00450
- [84] I. Moulton, M.P. Solon, I.W. Stewart, G. Vita, JHEP **02**, 134 (2018), 1709.09174
- [85] C.H. Chang, I.W. Stewart, G. Vita, JHEP **04**, 041 (2018), 1712.04343
- [86] M. Beneke, M. Garny, R. Szafron, J. Wang, JHEP **03**, 001 (2018), 1712.04416
- [87] M.A. Ebert, I. Moulton, I.W. Stewart, F.J. Tackmann, G. Vita, H.X. Zhu, JHEP **12**, 084 (2018), 1807.10764
- [88] Z.L. Liu, B. Mecaj, M. Neubert, X. Wang, Phys. Rev. D **104**, 014004 (2021), 2009.04456
- [89] Z.L. Liu, B. Mecaj, M. Neubert, X. Wang, JHEP **01**, 077 (2021), 2009.06779
- [90] M. Beneke, M. Garny, S. Jaskiewicz, J. Strohm, R. Szafron, L. Vernazza, J. Wang, PoS **LL2022**, 068 (2022), 2207.14199