

London model of dual color superconductor

Jiří Hošek^{1,2,*}

¹Department of Theoretical Physics, Nuclear Physics Institute, Czech Academy of Sciences, 25068 Řež (Prague), Czech Republic

²Institute of Experimental and Applied Physics, Czech Technical University in Prague, Husova 240/5, 110 00 Prague 1, Czech Republic

Abstract. The Meissner effect for the chromoelectric field \vec{E}_a is a property of the non-perturbative QCD vacuum medium *assumed* to explain the observed confinement of color. The color dielectric function ϵ of such a medium should vanish. By Lorentz invariance $\epsilon\mu = 1$ i.e., its color magnetic permeability μ should diverge. The assumption based on analogy is well motivated: Ordinary superconductor is the physical medium with $\mu = 0$, confining the opposite magnetic charges (would they exist). For the first successful phenomenological description of both the Meissner effect *and* superconductivity within Maxwell equations Fritz London guessed two equations for the superconductivity current. We adapt his arguments to QCD and come with two analogous manifestly non-Abelian London-like equations. One equation describes the dual Meissner effect. The analogy is, however, not perfect: We can only speculate that there is some sort of chromo-magnetic superfluidity in QCD phenomena to which the second equation might be ascribed. Moreover, from more advanced Ginzburg-Landau (GL) theory of superconductivity it follows that the Meissner effect is a consequence of spontaneous breaking of the underlying global $U(1)$ symmetry. This is certainly not the case of the dual Meissner effect. Fortunately, the derived London-like equations suggest a strongly color-paramagnetic behavior. We interpret it as a guide to the phenomenological Ginzburg-Landau-like manifestly gauge-invariant low-momentum QCD with the confining vacuum which behaves as a perfect color paramagnet ($\mu = +\infty$).

1 Introduction

Our trust in QCD relies mainly on its distinguished *perturbative* property of asymptotic freedom [1]. It perfectly describes the experimental facts in hadron physics caused by the short-distance interactions of the colored quarks and the colored gluons [2]. Another experimental fact, also expected to be described by QCD, the permanent confinement of the colored quarks and the colored gluons inside the colorless hadronic jails, stays unexplained [3]. It is, however, well described by the extensive numerical lattice QCD computations. In general terms the color confinement is attributed to the peculiar properties of the *non-perturbative* QCD vacuum medium. The intuitive bag-model picture [4–6] is that the non-perturbative QCD vacuum is a medium which does not allow for the penetration of the chromoelectric field,

*e-mail: hosek@ujf.cas.cz

of which the colored quarks and the colored gluons are the sources: Its chromo-dielectric function ϵ must vanish.

As in the Lorentz-invariant theories the vacuum must look the same in all Lorentz frames [7] i.e.,

$$\epsilon, \mu = 1, \quad (1)$$

the confining QCD vacuum must behave simultaneously as a perfect color paramagnet: its chromo-magnetic permeability diverges: $\mu \rightarrow \infty$.

An admiration deserves the old highly imaginative Copenhagen spaghetti vacuum [8] of randomly oriented chromo-magnetic fluxes motivated by [9]. It is interesting that some lattice calculations [10, 11] support such a picture.

Because of the enormous credit of QCD and the knowledge of the microscopic theory of superconductivity (BCS) the majority of most ambitious attempts pretends to understand the color confinement directly from the first principles: That the QCD vacuum should exhibit the dual Meissner effect was suggested by G. t'Hooft, S. Mandelstam and Y. Nambu [12–14]. The analogy with superconductivity led these authors to viewing the confining QCD vacuum as a condensate of the chromo-magnetic monopoles, the dual image of the condensate of the electrically charged Cooper pairs. It is interesting that some lattice calculations [15] support such a picture. The recent critical view of such a vacuum is presented in [16].

The right general idea can have several distinct realizations. Ours is humble. For finding a description of the dual Meissner effect we develop here the dual version of the first successful macroscopic theory of superconductivity of F. London [17]. To the best of our knowledge such a description does not exist. It might be useful as a guide to the manifestly gauge-invariant Ginzburg-Landau-like phenomenological model of the confining medium of the sort advocated by R. Friedberg and T. D. Lee [18].

2 The QCD London equations

The equations of motion corresponding to the perturbative QCD Lagrangian $L_{QCD} = -\frac{1}{4}F_a^{\mu\nu}F_{a\mu\nu}$ (QCD Maxwell equations) have the form

$$\text{div}\vec{E}_a = -gf_{abc}\vec{A}_b \cdot \vec{E}_c \equiv j_a^0 \quad (2)$$

$$\text{rot}\vec{B}_a - \frac{\partial}{\partial t}\vec{E}_a = -gf_{abc}\vec{A}_b \times \vec{B}_c \equiv \vec{J}_a \quad (3)$$

Here the chromo-electric field \vec{E}_a and the chromo-magnetic field \vec{B}_a are defined as particular components of the covariant color gluon field tensor

$$F_{a\mu\nu} = \partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} + gf_{abc}A_{b\mu}A_{c\nu} \quad (4)$$

In the gauge $A_a^0 = 0$ used here, in which the quantization is natural [2] they are defined as

$$\vec{E}_a = -\frac{\partial}{\partial t}\vec{A}_a \quad (5)$$

$$\vec{B}_a = \text{rot}\vec{A}_a - \frac{1}{2}gf_{abc}\vec{A}_b \times \vec{A}_c \quad (6)$$

We assume that the chromo-electric and chromo-magnetic fields remain the appropriate degrees of freedom also in the strong-coupling regime. Consequently, the QCD Bianchi identities which follow from the form of the covariant gauge field tensor (4) must remain intact. In the chosen gauge they have the form

$$\text{div}\vec{B}_a = +gf_{abc}\vec{A}_b \text{rot}\vec{A}_c \equiv k_a^0 \quad (7)$$

$$\text{rot}\vec{E}_a + \frac{\partial}{\partial t}\vec{B}_a = +gf_{abc}\vec{E}_b \times \vec{A}_c \equiv \vec{k}_a \quad (8)$$

The pseudo-scalar chromo-magnetic density k_a^0 is a sign for the existence of the chromo-magnetic monopoles [19, 20]. Their condensate, as an analog of the condensate of the Cooper pairs in superconductors, is at the heart of papers [12–14].

It is our understanding that the pseudo-vector chromo-magnetic current \vec{k}_a is another indication of the relevance of non-trivial chromo-magnetic configurations in the confining QCD vacuum medium. In any case its rot will be present in the time derivative of the QCD London current.

In accord with London's prediction we assume that there is an appropriate, yet unknown, manifestly gauge-invariant effective QCD Lagrangian at strong coupling which yields, **upon appropriate, yet unknown, condensation**, the QCD Maxwell equations in the form

$$\text{div}\vec{E}_a = J_a^0 \quad (9)$$

$$\text{rot}\vec{B}_a - \frac{\partial}{\partial t}\vec{E}_a = \vec{J}_a \quad (10)$$

Our intention is to fix the form of J_a^0 and \vec{J}_a in such a way that they result in the Meissner effect for the chromo-electric field.

The analogy is not perfect: In superconductivity the right hand sides of the Maxwell equations are the quantities external to the Maxwell equations, being determined, often approximately, by the matter fields. In QCD the right hand sides of the QCD Maxwell equations are heavily restricted by the gauge and Lorentz invariance. For their construction we can use merely \vec{E}_a , \vec{B}_a and \vec{A}_a .

As a consequence, we cannot set J_a^0 equal to zero. The Gauss law for $\text{div}\vec{E}_a$ is a constraint on the physical states of the 'Great Big' Schroedinger equation of QCD at strong coupling [2]. It could provide an information about the confinement in the London approximation.

First we apply the operation rot to the Eq.(8), use the QCD equations (9) and (10) modified for the strong coupling, and get

$$\vec{\nabla} J_a^0 + \left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\vec{E}_a + \frac{\partial}{\partial t}\vec{J}_a \equiv \text{rot}\vec{k}_a \quad (11)$$

This equation suggests to postulate the first QCD London-like equation in the form

$$\boxed{\frac{\partial}{\partial t}\vec{J}_a = -\vec{\nabla} J_a^0 - \mu^2\vec{E}_a + \text{rot}\vec{k}_a} \quad (12)$$

provided we define the Meissner effect of the chromo-electric field as an effectively massive vector field

$$\left[\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) - \mu^2\right]\vec{E}_a = 0 \quad (13)$$

Here μ is the scale in the current \vec{J}_a , and $\lambda = 1/\mu$ is the London penetration length of the chromo-electric field. It should be of the order of the typical hadron size. We should keep in mind that, although rather natural, it is just an assumption based on analogy. Its reliability can be justified only a posteriori.

Second, we apply rot to $\frac{\partial}{\partial t}\vec{J}_a$, use the identity $\vec{k}_a = -\frac{1}{2}gf_{abc}\frac{\partial}{\partial t}(\vec{A}_b \times \vec{A}_c)$ and get the second London equation

$$\boxed{\text{rot}\vec{J}_a = +\mu^2\vec{B}_a + \frac{1}{2}gf_{abc}[\mu^2 - \text{rot rot}](\vec{A}_b \times \vec{A}_c)} \quad (14)$$

Third, checking the consistency of (14) with the modified QCD Maxwell equation (10) we obtain

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\vec{B}_a = \text{rot}\vec{J}_a + \frac{\partial}{\partial t}\vec{k}_a - \vec{\nabla}k_a^0 \quad (15)$$

We interpret this equation as the London-type indication that the medium which exhibits the Meissner effect for the chromo-electric field behaves at the same time as a phenomenological perfect color paramagnet.

Unfortunately, as far as we can see the London-like approximation does not offer any useful constraint on J_a^0 .

3 The chromo-magnetic super-fluid?

In ordinary superconductivity one London equation describes superconductivity, the other describes the Meissner effect. We believe it is legitimate to ask whether the QCD London equation (14) describes some chromo-magnetic super-flow of the strongly interacting color gluon matter. We are aware of only one candidate. It is suggestive to speculate that it might be responsible for the observed almost perfect fluidity of the strongly interacting colored gluon plasma [21, 22]. As the analysis of the corresponding experimental data is behind our ability we merely add three supportive remarks.

First, the perturbative QCD Maxwell equations (2) and (3) are certainly appropriate for the description of the *short-distance color-gluon phenomena*. The current \vec{j}_a then can hardly be compared with a macroscopic Ohm-like flow. On the other hand the QCD Maxwell equations (9) and (10) are the effective ones, intended for the description of the *large-distance color-gluon phenomena*. Hence, the long-distance flow property associated with the London-like equation (14) could have the reasonable macroscopic physical sense.

Second, association of (14) with the macroscopic flow of a strongly coupled color-gluon super-fluid calls for the interpretation of the genuinely non-Abelian pseudo-vector magnetic current \vec{k}_a contained in it. At the London phenomenological level such a description should incorporate **unspecified** chromo-magnetic degrees of freedom in the strongly interacting gluon plasma. We find encouraging that in more explicit realizations of such a 'magnetic scenario' the plasma contains, besides the colored gluons, also the chromo-magnetic monopoles, dyons or the magnetic strings [23]. In their condensed form these topological objects are suggested for the explanation of the color confinement. The reliability of such a picture is supported by the lattice QCD simulations [10, 11].

Third, our ultimate task which refers to (1) is to describe a Lorentz-invariant vacuum medium of QCD which behaves as a perfect para-magnet. According to our understanding it means that, when disturbed, the vacuum medium should willingly transmit the chromo-magnetic field over macroscopic distances.

4 Towards the GL-like perfect color para-magnet

Presentation of this subject here [24] differs slightly from the oral one. It tries to reflect and incorporate into the text the critical remarks of Jeff Greensite during the talk at the Conference. I am grateful to Jeff Greensite and Dima Antonov for useful discussions.

Our intention is to identify the important differences in our analogy [25], and extract the possible useful lessons for the description of color confinement by an effective GL-like theory provided the first step, the London-like description of the dual Meissner effect, makes sense. The analogy remains useful, but we have to identify where it falters.

1. The effective mass μ , common to the whole octet of the chromo-electric fields \vec{E}_a , which appears in the QCD London equations (12) and (14) *cannot be a consequence of any spontaneous breakdown of global symmetry*: There are no interacting matter fields which could condense when the gauge interaction is switched off. When the (strong) gauge interaction would be switched off there would remain only the free gluon fields A_a . How at the strong coupling the chromo-electric field acquires the extra degree of freedom remains at the London level unclear. To quote J. D. Bjorken [2]: "It seems necessary to solve QCD in order to formulate it."

But nothing prevents us to consider the non-zero vacuum condensates of the gauge-invariant combinations of the gauge fields exhibiting in the GL-like effective Lagrangian the long-range order [18] characteristic of quantum super-fluids. In fact, namely the properties of the confining vacuum described by R. Friedberg and T. D. Lee we would like to convert into appropriate formulas. Later it might be useful to refer to their rather tolerant statement "there is no reason why it (the confining vacuum) should remain invariant under a local color gauge transformation".

2. The Monte Carlo simulations of the quenched $SU(2)$ QCD [26] apparently support the heuristic value of our London-like equations: (i) They understand the dual Meissner effect for the chromo-electric field in terms of its effective mass. (ii) They suggest to describe it by the dimension 2 colorless condensate of two gauge potentials of Gubarev and Zakharov [27, 28]

$$\langle A_a^\mu A_{\mu a} \rangle_0 \quad (16)$$

in accord with the remark of Ref.[18].

3. Our idea is to relate the scale μ in our London-like equations with the gauge-invariant condensates of the uniform generically non-Abelian uniform gauge fields [29]: Their definition is that all their gauge-invariant combinations are constant in space-time, and are *gauge-equivalent to constant, non-commuting gauge potentials*.

At the Conference I have presented an attempt to find an effective gauge-invariant GL-like theory for $SU(3)$ arguing intuitively as follows.

4.1 Towards GL-like $SU(3)$

Looking at the general definitions

$$\vec{E}_a = -\vec{\nabla} A_a^0 - \frac{\partial}{\partial t} \vec{A}_a - g f_{abc} \vec{A}_b A_c^0 \quad (17)$$

$$\vec{B}_a = \text{rot} \vec{A}_a - \frac{1}{2} g f_{abc} \vec{A}_b \times \vec{A}_c \quad (18)$$

we conclude that in the canonical gauge $A_a^0 = 0$ the chromo-electric and the chromo-magnetic fields around the constant non-commuting vector potentials behave differently: The chromo-electric field is a small field and its vacuum value is zero; the chromo-magnetic field is the large field, and its vacuum value equals the constant commutator term.

The basic argument in favor of the effective GL-like description of color gluon dynamics at strong coupling is the following: For an intuitive, physical understanding of the non-perturbative QCD quantum ground state the QCD Lagrangian $L_{QCD} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}$, clearly appropriate for the perturbative expansion around the zero value of the fields, is impractical (though possible, as the lattice QCD documents). An effective low-momentum QCD Lagrangian seems much more efficient. When the gauge fields $F_a^{\mu\nu}$ become large at small momenta, their higher powers should be equally important as the lowest, second power in L_{QCD} .

Consider the effective Lagrangian in the simplest possible form

$$\mathcal{L}_{eff} = -\frac{1}{4}F_a^{\mu\nu}F_{a\mu\nu} + \kappa_1 t + \kappa_2 t^2 \quad (19)$$

where

$$-\frac{1}{4}F_a^{\mu\nu}F_{a\mu\nu} = \frac{1}{2}(\vec{E}_a^2 - \vec{B}_a^2) \quad (20)$$

$$t \equiv \frac{1}{6}f_{abc}F_{a\mu}{}^\nu F_{b\nu}{}^\rho F_{c\rho}{}^\mu = \frac{1}{2}f_{abc}\vec{B}_a \cdot (\frac{1}{3}\vec{B}_b \times \vec{B}_c - \vec{E}_b \times \vec{E}_c) \quad (21)$$

The first term in (19) is necessary, including the sign. The second term is the only dimension six term provided we do not consider the term with covariant derivatives. The third term is sufficient to guarantee that the corresponding Hamiltonian is bounded from below. In general the Hamiltonian is

$$\mathcal{H}_{eff} = \vec{\nabla} \cdot (A_a^0 \vec{\mathcal{E}}_a) + \vec{E}_a \cdot \frac{\partial \mathcal{L}_{eff}}{\partial \vec{E}_a} - \mathcal{L}_{eff} \quad (22)$$

where $\vec{\mathcal{E}}_a$ is a given function of \vec{E}_a and \vec{B}_a .

The vacuum energy in the canonical gauge takes the form

$$\langle \mathcal{H}_{eff} \rangle_0 = -\langle \mathcal{L}_{eff} \rangle_0 = \langle \frac{1}{2}\vec{B}_a^2 - \kappa_1 [\frac{1}{6}f_{abc}\vec{B}_a \cdot (\vec{B}_b \times \vec{B}_c)] - \kappa_2 [\frac{1}{6}f_{abc}\vec{B}_a \cdot (\vec{B}_b \times \vec{B}_c)]^2 \rangle_0 \quad (23)$$

First, because of the Lorentz invariance of the Lagrangian the vacuum is Lorentz-invariant. Consequently, it fulfills the important property (1). Second, because the second term can be both positive and negative, the energy is minimal at non-zero value of the colorless scalar $f_{abc}\vec{B}_a \cdot (\vec{B}_b \times \vec{B}_c)$ characterized by a constant chromo-magnetic field. It is tempting to conclude that the vacuum behaves as a perfect color paramagnet i.e., it behaves simultaneously as a confining medium ($\epsilon = 0$). The problem is that the Lagrangian (19) does not allow one to compute the necessary components of \vec{A}_a in terms of the dimension-full parameters in the Lagrangian to get the fixed constant \vec{B}_a . To the best of my knowledge for $SU(3)$ the classification of the uniform gauge fields does not exist. Below we outline the road to the GL $SU(2)$ following closely the classification done by L. S. Brown and W. I. Weisberger [29] (we use the metric (+ - - -)).

4.2 Towards GL-like $SU(2)$

General analysis of the $SU(2)$ uniform gauge fields is done in terms of the three eigenvalues of the symmetric matrix

$$Y_{ab} = A_a^\mu A_{b\mu}$$

To characterize the fields we shall work in the gauge which diagonalizes Y and gives

$$A_a^\mu A_{\mu b} = \lambda_a \delta_{ab} \quad (24)$$

Here we restrict ourselves only to the purely "magnetic" configuration characterized by all three $\lambda_a < 0$. In the fixed gauge and Lorentz frames

$$A_1^\mu = (0, \vec{i}\sqrt{-\lambda_1})$$

$$A_2^\mu = (0, \vec{j}\sqrt{-\lambda_2})$$

$$A_3^\mu = (0, \vec{k}\sqrt{-\lambda_3})$$

In accord with the previous subsection in the gauge $A_a^0 = 0$ the constant chromo-magnetic fields

$$\begin{aligned} B_3^z &= \sqrt{\lambda_1 \lambda_2} \\ B_2^y &= \sqrt{\lambda_1 \lambda_3} \\ B_1^x &= \sqrt{\lambda_2 \lambda_3} \end{aligned}$$

are non-zero and the chromo-electric fields vanish.

According to the definition all gauge invariant (and Lorentz-invariant) quantities formed from the uniform fields are expressed in terms of Y and its eigenvalues. For example,

$$\begin{aligned} F_a^{\mu\nu} F_{a\mu\nu} &= (\text{tr}Y)^2 - \text{tr}Y^2 = \sum_{a \neq b} \lambda_a \lambda_b \\ \frac{1}{2} D^\sigma F_a^{\mu\nu} D_\sigma F_{a\mu\nu} &= \text{tr}Y \text{tr}Y^2 - \text{tr}Y^3 = \sum_a \lambda_a \sum_{b \neq a} \lambda_b^2 \\ \frac{1}{3} \epsilon_{abc} F_{av}^\mu F_{b\lambda}^\nu F_{c\mu}^\lambda &= \det Y = \lambda_1 \lambda_2 \lambda_3 \end{aligned}$$

The program for $SU(2)$ seems well defined: Find the minimal effective GL-like Lagrangian with the Lorentz-invariant ground state characterized by the fixed gauge-invariant condensates of constant chromo-magnetic field:

$$\mathcal{L}_{eff} = -\frac{1}{4} F_a^{\mu\nu} F_{a\mu\nu} + A \frac{1}{2} D^\sigma F_a^{\mu\nu} D_\sigma F_{a\mu\nu} + B \frac{1}{3} \epsilon_{abc} F_{av}^\mu F_{b\lambda}^\nu F_{c\mu}^\lambda + \dots \quad (25)$$

The ingredients are: (i) Eight algebraically independent gauge-invariant polynomials of the $SU(2)$ field tensor F found by Roskies [30]. (ii) Their covariant derivatives. (iii) The dimension-full parameters A, B, \dots . Unfortunately, the price for the departure from $SU(3)$ is not low. The colorless baryons are the bosons, the colorless diquarks.

5 Conclusion and outlook

We consider our primary task fulfilled: Employing the popular analogy we came to one London-like equation describing the Meissner effect for the chromo-electric field. Such a view at the color confinement is in accordance with the successful bag model [4–6]. Moreover, it is apparently also supported by some lattice simulations [26].

The other London-like equation, in accordance with the analogy, suggests that there should be a sort of the chromo-magnetic superfluidity in the QCD phenomena at strong coupling i.e., at large distances.

I thank Tom Brauner for valuable suggestions, and Petr Beneš and Adam Smetana for pleasant collaboration.

References

- [1] D.J. Gross, F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973)
- [2] J.D. Bjorken, *Elements of Quantum Chromodynamics* (Birkhäuser Boston, Boston, MA, 1982), pp. 423–561, ISBN 978-1-4899-6691-9, https://doi.org/10.1007/978-1-4899-6691-9_5
- [3] J. Greensite, *An introduction to the confinement problem*, Vol. 821 (2011)
- [4] A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thorn, V.F. Weisskopf, Phys. Rev. D **9**, 3471 (1974)
- [5] A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thorn, Phys. Rev. D **10**, 2599 (1974)

- [6] P. Hasenfratz, J. Kuti, Phys. Rept. **40**, 75 (1978)
- [7] T.D. Lee, *Particle Physics and Introduction to Field Theory*, Vol. 1 (1981)
- [8] P. Olesen, Physica Scripta **23**, 1000 (1981)
- [9] N. Nielsen, P. Olesen, Nuclear Physics B **144**, 376 (1978)
- [10] M.N. Chernodub, V.I. Zakharov, Phys. Rev. Lett. **98**, 082002 (2007), hep-ph/0611228
- [11] A. Gorsky, V. Zakharov, Phys. Rev. D **77**, 045017 (2008)
- [12] G. 't Hooft, *Gauge Fields with Unified Weak, Electromagnetic, and Strong Interactions, in 1975 High-Energy Particle Physics Divisional Conference of EPS (includes 8th biennial conf on Elem. Particles)* (1975)
- [13] S. Mandelstam, Phys. Rept. **23**, 245 (1976)
- [14] Y. Nambu, Phys. Rev. D **10**, 4262 (1974)
- [15] A. Di Giacomo, JHEP **02**, 208 (2021), 2010.04232
- [16] M. Shifman (2022), 2210.06615
- [17] F. London, *Superfluids, Vol.I, Macroscopic Theory of Superconductivity*, Vol. 1 (John Wiley and Sons, Inc. (1950), 1981)
- [18] R. Friedberg, T.D. Lee, Phys. Rev. D **18**, 2623 (1978)
- [19] G. 't Hooft, Nucl. Phys. B **79**, 276 (1974)
- [20] A.M. Polyakov, JETP Lett. **20**, 194 (1974)
- [21] E. Shuryak, Rev. Mod. Phys. **89**, 035001 (2017)
- [22] U. Heinz, Journal of Physics A: Mathematical and Theoretical **42**, 214003 (2009)
- [23] J. Liao, E. Shuryak, Phys. Rev. C **75**, 054907 (2007)
- [24] J.H. P. Benes, A. Smetana, in preparation (...)
- [25] K. Matsuyama, J. Greensite, Phys. Rev. B **100**, 184513 (2019)
- [26] T. Suzuki, K. Ishiguro, Y. Mori, T. Sekido, Phys. Rev. Lett. **94**, 132001 (2005)
- [27] F.V. Gubarev, L. Stodolsky, V.I. Zakharov, Phys. Rev. Lett. **86**, 2220 (2001)
- [28] F.V. Gubarev, V.I. Zakharov, *On the emerging phenomenology of $\langle(A_{\mu}^a)^2_{min}\rangle$* (2000), pp. 52–63
- [29] L.S. Brown, W.L. Weisberger, Nuclear Physics B **157**, 285 (1979)
- [30] R. Roskies, Phys. Rev. D **15**, 1722 (1977)