Path integral study of the Casimir effect in a chiral medium

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Abstract. The Casimir effect is a remarkable macroscopic feature of QED, while recent lattice studies have also shown its potential nontrivial consequences in QCD. In light of having a better understanding of the Casimir effect, it is advantageous to have a self-contained path integral formulation of the phenomenon. I will show how the Casimir effect between two uncharged plates in the presence of a chiral medium, modeled with an axion term $\theta\widetilde{F}_{\mu\nu}F_{\mu\nu}$, can be formulated in terms of the path integral, and how such a formulation leads to a 3D effective action of the restricted electromagnetic field.

1 Introduction

The Casimir effect, and its associated force, describes how even neutral objects experience forces through their disturbance of the vacuum structure. This phenomenon is well known in QED, and has been verified in a multitude of experiments, see e.g. [1–7]. While the Casimir effect is not necessarily proof of the physicality of vacuum energy [8], the formulation in terms of vacuum energies remains convenient for practical calculations.

The Casimir effect is not only of theoretical interests, it also plays an important role in the production and operation of micro(electro)mechanical systems, as on the nanometer scale the Casimir force is non-negligible and needs to be taken into account [9–12]. In such situations it is beneficial to be able to tune the Casimir force to a desired strength to create novel

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applications, e.g with a repulsive Casimir force. A repulsive Casimir force in configurations with dielectric materials is however only possible in the case that reflection symmetry is broken, see the no-go theorem of [13]. Such a broken reflection symmetry can be obtained in a straightforward way by using a geometrically non-symmetric setup. Keeping the symmetric geometry, another way out is to modify the vacuum (or better said, medium) to be chiral, an effect not present with dielectric materials. There is also a third possibility of applying boundary conditions (e.g. at plates), consistent with making plates out of non-dielectric materials.

There is also growing proof from lattice simulations that the Casimir effect has nontrivial consequences in $(1+1)$d $\mathbb{C}P^{N-1}$ and $(2+1)$d Yang-Mills models [14, 15]. Specifically in $(2+1)$d Yang-Mills theory, there seems to be an interesting interplay between the Casimir effect, the deconfinement transition, and a dynamically generated mass scale which differs from the lowest glueball mass. Moreover, the QCD quark-gluon plasma has also been motivated to form a chiral medium with asymmetric behavior for left-and right quarks, potentially leading to novel phenomena [16]. The Casimir effect in QCD is also related to the phenomenological MIT bag model [1, 4, 17, 18], where hadrons are modelled as spherical bags containing quarks.

In this talk we apply the path integral techniques developed in [19, 20] to a situation involving a chiral medium and perfect electromagnetic conductors, in light of eventually moving towards $(1 + 1)$d $\mathbb{C}P^{N-1}$ and $(2 + 1)$d Yang-Mills theories. We will pay particular attention to imposing these boundary conditions in a gauge invariant manner. The presented methodology is quite flexible, and where possible, we check against existing results.

2 Setup

We start from the Euclidean QED action augmented with a θ-term

$$S = \int \mathrm{d}^4 x \left[\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{i}{4} \theta F_{\mu\nu} \widetilde{F}_{\mu\nu}\right], \tag{1}$$

where we take θ to be a classical background field which we will use to model our media of interest. We will only consider θ to be a continuous piecewise linear function of the z-coordinate, as this resembles the effective action of Weyl Semimetals (WSMs) [21, 22]. Only the derivative of $\theta(z)$ will enter the equations of motion, and the following calculations, therefore we define $\beta(z) = \partial_z \theta(z)$, which is now a piecewise constant function.

Despite its simplicity, the Casimir force between parallel plates is already an interesting case study. We will apply so-called hard boundary conditions on the plates, which we place at $z = \pm\frac{L}{2}$, as this will allow us to implement them conveniently in the path integral formalism. Instead of perfect electric conductor (PEC) or perfect magnetic conductor (PMC) boundary conditions, we choose to use their generalization: perfect electromagnetic conductor (PEMC) [23] boundary conditions

$$n_\nu F_{\mu\nu} + i\alpha_\pm n_\nu \widetilde{F}_{\mu\nu} \Big|_{z=\pm\frac{L}{2}} = 0 , \tag{2}$$

with $\alpha_\pm$ real parameters and $n_\mu = \delta_{z\mu}$. This type of boundary conditions arises naturally in the description of chiral or bi-isotropic materials [24], and can be shown to be the most general class of gauge invariant boundary conditions [25]. The PEC (PMC) case can be recovered by setting $\alpha \to \infty$ ($\alpha \to 0$).

Reformulating the parameters $\alpha_\pm$ as $\alpha'_\pm = \arctan(\alpha_\pm)$, the boundary conditions become

$$\cos\alpha'_\pm n_\nu F_{\mu\nu} + i \sin\alpha'_\pm n_\nu \widetilde{F}_{\mu\nu} \Big|_{z=\pm\frac{L}{2}} = 0 , \tag{3}$$

so that they can also be seen as a finite 'duality' rotation, mixing $F_{\mu\nu}$ and $\widetilde{F}_{\mu\nu}$, applied to $n_\mu F_{\mu\nu} = 0$ with angle $\alpha'_\pm$.

These boundary conditions can be implemented using Lagrange multiplier fields $b^a, a = 1, 2$ which are defined on the surfaces with $z = \frac{L}{2}$ and $z = -\frac{L}{2}$, such that only configurations obeying the boundary conditions survive after integration over b^a

$$S_{\rm bc} = -\frac{1}{2}\int \mathrm{d}^4x \left[b_i^2\delta\Big(z - \frac{L}{2}\Big)n_\mu\Big(F_{\mu i} + i\alpha_+\widetilde{F}_{\mu i}\Big) + b_i^1\delta\Big(z + \frac{L}{2}\Big)n_\mu\Big(F_{\mu i} + i\alpha_-\widetilde{F}_{\mu i}\Big)\right] . \tag{4}$$

The b^a fields have only three components, as a z-component would be irrelevant due to the asymmetry of $F_{\mu\nu}$ and $\widetilde{F}_{\mu\nu}$. From now on we will label the t, x, y directions with latin indices $i, j, \cdots$. Choosing to work in Feynman gauge by adding a gauge fixing term $\int \frac{1}{2}(\partial A)^2$ to the action, we obtain

$$S = -\frac{1}{2}\int \mathrm{d}^4x \left[A_\mu\Big(\delta_{\mu\nu}\partial^2 + i\varepsilon_{\mu\nu\rho 3}\beta(z)\partial_\rho\Big)A_\nu\right] + S_{\rm bc} , \tag{5}$$

as starting point.

Due to the z-dependence of $\beta(z)$ we only Fourier transform the t, x, y coordinates. Furthermore the presence of the $\varepsilon_{\mu\nu\rho 3}$, which reduces to ε_{ijk}, can be eliminated by moving to a different orthonormal basis E^r_μ, with $E^r_\mu E^{s\dagger}_\mu = \delta^{rs}$. Arbitrary vectors v_μ can then be decomposed in terms of this new basis as $v_\mu = E^r_\mu v_r$. The $r = 0, 3$ vectors are analogues of timelike and longitudinal polarizations

$$E^0_i = \frac{k_i}{|\mathbf{k}|}, \quad E^0_3 = 0, \quad E^3_i = 0, \quad E^3_3 = 1 , \tag{6}$$

where $|\mathbf{k}| = \sqrt{k_i k_i}$, while $r = 1, 2$ vectors are transversal

$$E^1_\mu = \frac{1}{\sqrt{2}}\Big(\widetilde{E}^1_\mu + i\widetilde{E}^2_\mu\Big), \quad E^2_\mu = \frac{1}{\sqrt{2}}\Big(\widetilde{E}^1_\mu - i\widetilde{E}^2_\mu\Big) \tag{7}$$

and are built out of real vectors that obey

$$\widetilde{E}^2_i = \varepsilon_{ijk}\frac{k_k}{|\mathbf{k}|}\widetilde{E}^1_j, \quad \widetilde{E}^1_i k_i = \widetilde{E}^2_i k_i = 0 \tag{8}$$

which do not have to be defined explicitly for our purposes. After these transformations our action takes the form

$$S = \frac{1}{2}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^2}\,\mathrm{d}z \left[A_r(\mathbf{k}, z)K_{rs}\Big(\mathbf{k}, \partial_z^2\Big)A_s(-\mathbf{k}, z)\right] + S_{\rm bc} , \tag{9}$$

where the kinetic operator is now diagonal in the r, s indices

$$K_{rs}\Big(\mathbf{k}, \partial_z^2\Big) = \mathrm{diag}\left(\partial_z^2 - |\mathbf{k}|^2, \quad \partial_z^2 - \Big(k_c^\star\Big)^2(z), \quad \partial_z^2 - k_c^2(z), \quad \partial_z^2 - |\mathbf{k}|^2\right), \tag{10}$$

and we have defined $k_c^2(z) = |\mathbf{k}|^2 + ig\beta(z)$. Consequently it is significantly easier in the E^r_μ basis to construct the Green's function K^{-1}, as it can be obtained by sewing the solutions for constant β

$$\Big(\partial_z^2 - k_c^2\Big)\phi_\beta(\mathbf{k}, z, z') = \delta(z - z'), \quad \phi_\beta(\mathbf{k}, z, z') = -\frac{1}{2k_c}e^{-k_c|z-z'|} \tag{11}$$

together. We want to stress that the Green's function K^{-1} can be constructed without any knowledge of the boundary conditions. The boundary conditions that are contained within $S_{\rm bc}$ are enforced afterwards by integration over b_i^a.

Going back to the boundary conditions, the relevant part of the action $S_{\rm bc}$ can be rewritten as a source term[1]

$$S_{\rm bc} = \frac{1}{2}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\,\mathrm{d}z\, b_i^a(-\mathbf{k}) V_{i\mu}^a(\mathbf{k}, z) A_\mu(\mathbf{k}, z) \tag{12}$$

and by the standard redefinition of the fields $A \to A + K^{-1}J$ we can rewrite the action as

$$S = S_A + S_b \tag{13}$$

with

$$\begin{aligned} S_A &= \frac{1}{2}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^2}\,\mathrm{d}z \left[A_r(\mathbf{k}, z) K_{rs}\Big(\mathbf{k}, \partial_z^2\Big) A_s(-\mathbf{k}, z)\right] \\ S_b &= \frac{1}{2}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} b_i^a(\mathbf{k}) \mathcal{K}_{ij}^{ab}(\mathbf{k}) b_j^b(-\mathbf{k}) \end{aligned} \tag{14}$$

where

$$\mathcal{K}_{ij}^{ab} = \int \mathrm{d}z\,\mathrm{d}z'\; V_{i\mu}^a(-\mathbf{k}, z) K_{\mu\nu}^{-1}(\mathbf{k}, z - z') V_{j\nu}^b(\mathbf{k}, z')\;. \tag{15}$$

Writing the b_i^a fields in the E_μ^r basis as $b_r = (b_\parallel, b_L, b_R)$ the boundary action can be brought into the form

$$S_b = \frac{1}{2}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\left[b_R^a(\mathbf{k}) \mathbb{K}^{ab}(\mathbf{k}) b_R^b(-\mathbf{k}) + b_L^a(\mathbf{k}) \Big(\mathbb{K}^{ab}(\mathbf{k})\Big)^\star b_L^b(-\mathbf{k})\right] \tag{16}$$

where we have integrated out $b_\parallel$, which results in an irrelevant L-independent constant.

The vacuum energy density originating from the boundary conditions S_b consequently follows as[2]

$$\mathcal{E}_b = \frac{1}{T\mathcal{V}_2}\,\mathrm{Re}\log\det(\mathbb{K}) = \mathrm{Re}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\log(|\mathbb{K}|) \tag{17}$$

while for the Casimir effect arising from the dependence of the medium on L it is convenient to use the Jacobi formula for the derivative of a determinant to directly obtain the Casimir force

$$F_A = -\frac{\mathrm{d}\mathcal{E}_A}{\mathrm{d}L} = -\frac{1}{2}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\int \mathrm{d}z\,\mathrm{tr}\left[\frac{\mathrm{d}K}{\mathrm{d}L}K^{-1}(z - z')\Big|_{z=z'}\right] \tag{18}$$

3 Results

3.1 Thin parallel plates in a chiral medium

The first case we consider is the Casimir effect between parallel plates in a chiral medium. Specifically this corresponds to setting $\beta(z) = \beta$ constant, such that in the E_μ^r basis the Green's function takes the relatively simple form

$$K_{rs}^{-1}(z - z') = \mathrm{diag}\left(\phi_0(z - z'), \phi_\beta^\star(z - z'), \phi_\beta(z - z'), \phi_0(z - z')\right) \tag{19}$$

where the ϕ_β are the Green's functions for constant β (11). The QED case can then be recovered by setting $\beta \to 0$.

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[1] The ∂_z derivatives in the $V_{i\mu}^a$ are written in terms of $\delta'\Big(z \pm \frac{L}{2}\Big)$.

[2] We denote with T and $\mathcal{V}_2$ the (infinite) length of the time integration interval and plate surface area respectively.

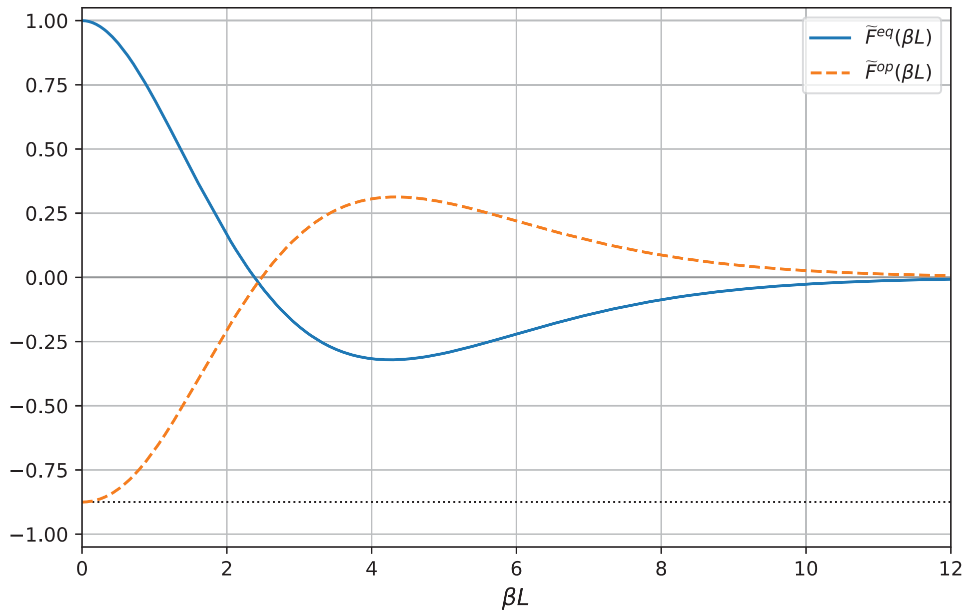

Figure 1. The Casimir force in a chiral medium relative to the QED Casimir force $\widetilde{F} = F_\beta(L)/F_{\text{qed}}(L)$, for the cases when $\alpha_+ = \alpha_-$ ($\widetilde{F}^{\text{eq}}$) and when $\alpha_- = 0$ ($\alpha_- = \pm\infty$) and $\alpha_+ = \pm\infty$ ($\alpha_+ = 0$) ($\widetilde{F}^{\text{op}}$). The dashed horizontal line corresponds to $-\frac{7}{8}$.

As there is no L-dependence in $K_{\mu\nu}, K^{-1}_{\mu\nu}$ it follows that, as expected, $F_A = 0$. Consequently we only need to consider the Casimir energy arising from the boundary conditions on the plates

$$\mathcal{E}_\beta = \text{Re} \int \frac{\text{d}^3\mathbf{k}}{(2\pi)^3} \log(|\mathbb{K}|) \ . \tag{20}$$

where the $|\mathbb{K}|$ determinant is given by

$$|\mathbb{K}| = 1 - \frac{k_c^2 + ik_c|\mathbf{k}|(\alpha_+ - \alpha_-) + |\mathbf{k}|^2\alpha_+\alpha_-}{k_c^2 - ik_c|\mathbf{k}|(\alpha_+ - \alpha_-) + |\mathbf{k}|^2\alpha_+\alpha_-} e^{-2k_cL} \ . \tag{21}$$

In the case that $\alpha_+ = \alpha_-$ we can bring (20) into closed form [20]

$$\mathcal{E}^{\text{eq}}_\beta(L) = \frac{\beta^4 L}{16\pi^2} \sum_{n=1}^{\infty} \left[\frac{K_1(\beta L n)}{\beta L n} - \frac{K_2(\beta L n)}{\beta^2 L^2 n^2} \right] \tag{22}$$

where K_1, K_2 are modified Bessel functions of the second kind. We note that this is the same form as in [26], even though our starting point and calculation method are different. Similarly when $\alpha_- = 0$ ($\alpha_- = \pm\infty$) and $\alpha_+ = \pm\infty$ ($\alpha_+ = 0$) the Casimir energy can be brought into the form

$$\mathcal{E}^{\text{op}}_\beta(L) = \frac{\beta^4 L}{16\pi^2} \sum_{n=1}^{\infty} (-1)^n \left[\frac{K_1(\beta L n)}{\beta L n} - \frac{K_2(\beta L n)}{\beta^2 L^2 n^2} \right] . \tag{23}$$

Furthermore in the QED limit $\beta \to 0$ the Casimir energy can be written down for general $\alpha_\pm$ as

$$\mathcal{E}_{\text{qed}}(L, \alpha_+, \alpha_-) = -\frac{1}{8\pi^2 L^3} \text{Re}\,\text{Li}_4\!\left(e^{2i\alpha'}\right) \tag{24}$$

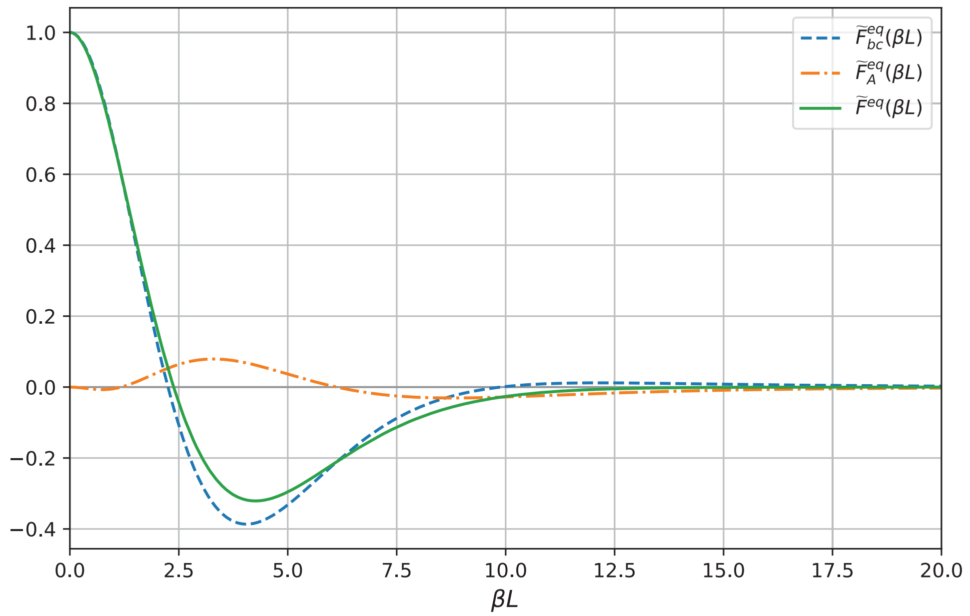

Figure 2. The Casimir force, in the case that the chiral medium is only present between the plates, relative to the QED Casimir force $\widetilde{F}^{\rm eq}(\beta L) = F^{\rm eq}(L,\beta)/F_{\rm qed}(L)$ and its decomposition into the part originating from the boundary conditions $\widetilde{F}^{\rm eq}_{\rm bc}(\beta L) = F^{\rm eq}_{\rm bc}(L,\beta)/F_{\rm qed}(L)$ and a part from the variable size of the chiral medium $\widetilde{F}^{\rm eq}_{A}(\beta L) = F^{\rm eq}_{A}(L,\beta)/F_{\rm qed}(L)$, $F_{\rm qed}(L) = -\frac{\pi^2}{240L^4}$. The total force $\widetilde{F}^{\rm eq}$ is the same as in Figure 1.

where $\alpha' = \alpha'_+ - \alpha'_- = \arctan\alpha_+ - \arctan\alpha_-$ and $\mathrm{Li}_n(x)$ is the polylogarithm. This corresponds to the Casimir force derived in [27]. The Casimir forces $F^{\rm eq}_{\beta} = -\frac{\mathrm{d}\mathcal{E}^{\rm eq}_{\beta}}{\mathrm{d}L}$ and $F^{\rm op}_{\beta} = -\frac{\mathrm{d}\mathcal{E}^{\rm op}_{\beta}}{\mathrm{d}L}$ are shown in Figure 1, relative to the QED Casimir force $F_{\rm qed}(L) = -\frac{\pi^2}{240L^4}$. It follows that for equal $\alpha_\pm$ the Casimir force is attractive at short distances, while becoming repulsive when $\beta L \gtrsim 2.4$. The Casimir force $\widetilde{F}^{\rm op}$ follows the same pattern, but with an opposite sign, and for $\beta L = 0$ becomes $-\frac{7}{8}F_{\rm qed}$ which is known as the Casimir force between a PEC and a PMC [28]. Note however that as $\left|\widetilde{F}\right| \leq 1$ this Casimir force is weaker than the conventional QED Casimir force, which is already proportional to L^{-4}.

3.2 Thin parallel plates with only a chiral medium in between

We now consider the case where the vacuum is only chiral in between the plates, in other words we have

$$\beta(z) = \begin{cases} \beta & \text{if} \quad z \in \left[-\frac{L}{2}, \frac{L}{2}\right] \\ 0 & \text{otherwise} \end{cases} \tag{25}$$

In this case the Green's function is given by

$$K^{-1}_{rs} = \mathrm{diag}\left(\phi_0(z-z'), D^{\star}(z-z'), D(z-z'), \phi_0(z-z')\right) \tag{26}$$

where $D(z-z')$ solves the 1D Green's function equation $\left(\partial_z^2 - k_c^2(z)\right)D(z-z') = \delta(z-z')$. The full expression of $D(z-z')$ is given in [20]. As the Green's function, and kinetic operator, now depend on L due to our definition of $\beta(z)$, the Casimir force will decompose as

$$F = F_{\rm bc} + F_A \tag{27}$$

where $F_{\rm bc}$ is the conventional Casimir force arising from the boundary conditions as in the previous section, while F_A arises from the fact that the chiral medium changes in size depending on L.

Starting with the Casimir energy density arising from the boundary conditions we have

$$\mathcal{E}_{\rm bc} = \mathrm{Re}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\log(|\mathbb{K}|)\,, \tag{28}$$

with the matrix determinant $|\mathbb{K}|$ given by

$$|\mathbb{K}| = \frac{(|\mathbf{k}|-k_c)^2 e^{k_c L}}{\left(|\mathbf{k}|^2+k_c^2\right)e^{2k_cL} - \left(|\mathbf{k}|^2-k_c^2\right)e^{-2k_cL}}\left[1 - \frac{k_c^2 + ik_c|\mathbf{k}|(\alpha_+-\alpha_-) + |\mathbf{k}|^2\alpha_+\alpha_-}{k_c^2 - ik_c|\mathbf{k}|(\alpha_+-\alpha_-) + |\mathbf{k}|^2\alpha_+\alpha_-}e^{-2k_cL}\right], \tag{29}$$

where $k_c^2 = |\mathbf{k}|^2 + i|\mathbf{k}|\beta$ i.e. $k_c^2(z)$ within the plates $z \in \left[-\frac{L}{2}, \frac{L}{2}\right]$. The force then follows as $F_{\rm bc} = -\frac{\mathrm{d}\mathcal{E}_{\rm bc}}{\mathrm{d}L}$. The F_A force has to be calculated by directly using (18), and follows as

$$\begin{aligned} F_A &= \frac{1}{2}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\left[\frac{i}{2}\beta|\mathbf{k}|\left(D\left(|\mathbf{k}|, \frac{L}{2}, \frac{L}{2}\right) + D\left(|\mathbf{k}|, -\frac{L}{2}, -\frac{L}{2}\right)\right) + \quad \text{c.c.} \quad \right] \\ &= -\beta\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}|\mathbf{k}|\,\mathrm{Im}\left\{D\left(|\mathbf{k}|, \frac{L}{2}, \frac{L}{2}\right)\right\}. \end{aligned} \tag{30}$$

Remarkably, the total force

$$F(L,\beta,\alpha_+,\alpha_-) = F_{\rm bc}(L,\beta,\alpha_+,\alpha_-) + F_A(L,\beta,\alpha_+,\alpha_-) \tag{31}$$

is the same as when $\beta(z)$ is constant. This could have been predicted by noting that from the perspective of the physical modes, the boundary conditions (2) are 'hard', and so there is no communication from the inside of the plates to the outside. Nonetheless the decomposition into F_{bc} and F_A is nontrivial, and is shown in Figure 2 for $\alpha_- = \alpha_+$, relative to the QED Casimir force.

4 Summary

We calculated the Casimir force between parallel perfect electromagnetically conducting plates in a chiral medium by using path integral techniques. The resulting Casimir force seems to be insensitive to whether the vacuum outside of the plates is chiral or not, even though in the former case it decomposes nontrivially into the force arising from the change in size of the chiral medium and from the boundary conditions.

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