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Excited States of "Elementary" Particles in Gauge-Higgs Theories

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Abstract. We show that the gauge and scalar fields which surround static ions in the Ginzburg-Landau model of superconductivity have a spectrum of excitations which are potentially observable. This ties in with earlier results along these lines found in a variety of gauge Higgs theories. Excitations of this type would appear as a mass spectrum of the "elementary" particles in the theory.

1 Introduction

Composite systems (molecules, atoms, nuclei, hadrons...) generally have a spectrum of excitations. What about non-composite systems: charged "elementary" particles like quarks and leptons? If the particle is charged, then by Gauss's Law it is accompanied by a surrounding gauge (and possibly other) fields. If these surrounding fields interact with themselves, could they not also exhibit a spectrum of excitations? This is certainly the case for color electric flux tubes in a confining gauge theory, which are known to have a discrete spectrum of excitations [1, 2]. It is a reasonable guess that the gauge and scalar fields surrounding a charged source in a gauge Higgs theory (Fig. 1) would also have a discrete set of excitations. This would look like a mass spectrum of the isolated elementary particle.

Two gauge Higgs theories are known to describe reality. One is the Ginzburg-Landau effective action for superconductivity, which is a non-relativistic version of an abelian gauge Higgs theory with a double-charged Higgs field. The other is the electroweak sector of the standard model, which may also be an effective theory. So we would be looking for excitations of static charges in a superconductor, or mass excitations of quarks and leptons. Perturbatively, no such thing is found. But the lattice supplies non-perturbative information. The electroweak sector is a chiral gauge theory, and the lattice formulation is so far problematic. We therefore concentrate, in these proceedings, on superconductors.

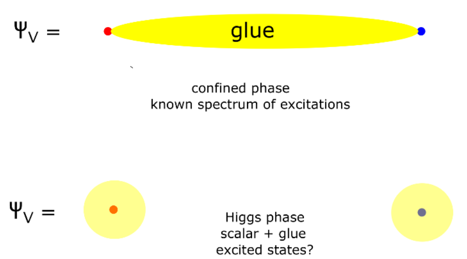

Figure 1. In a confining gauge theory, the color flux tube emanating from static charged sources has a discrete spectrum of fluctuations. We suggest that the same may be true of the gauge and scalar fields surrounding charged sources in the Higgs phase of a gauge Higgs theory.

2 Pseudomatter operators

Let us begin with the simplest case: the free Maxwell field with a static charged source in an infinite volume. The ground state is

$$|\Psi_{\mathbf{x}}\rangle = \overline{\psi}(\mathbf{x})\rho(\mathbf{x};A)|\Psi_0\rangle \ , \tag{1}$$

where $\overline{\psi}(\mathbf{x})$, operating on the vacuum, creates the static charge, and

$$\rho(\mathbf{x};A) \;=\; \exp\left[-i\frac{e}{4\pi}\int d^3z\, A_i(\mathbf{z})\frac{\partial}{\partial z_i}\frac{1}{|\mathbf{x}-\mathbf{z}|}\right] \ . \tag{2}$$

$\rho(\mathbf{x}, A)$ is a *pseudomatter* operator. This is an operator which (i) is a functional of the gauge field only; and (ii) transforms like a matter field at point $\mathbf{x}$ *except* under global gauge transformations in the center of the gauge group. Such transformations do not affect the gauge field. In this case

$$g(x) = e^{i\theta} \quad , \quad \rho \to \rho \quad , \quad \Psi_{\mathbf{x}} \to e^{-i\theta}\Psi_{\mathbf{x}} \ . \tag{3}$$

We have shown in ref. [3] that it is spontaneous breaking of the global center subgroup of the gauge group that distinguishes the Higgs phase from the massless and confining phases of a gauge Higgs theory. This symmetry breaking transition coincides, in the confinement to Higgs transition, with a transition from separation-of-charge (S_c)

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confinement in the confining phase, to color (C) confinement in the Higgs phase.[1] Thus the Higgs and confinement phases are physically distinct. In the massless phase to Higgs transition, the symmetry breaking transition coincides with massless vector bosons becoming massive. For details we refer the reader to the cited reference. Two other examples of pseudomatter operators include, first, any transformation to a physical gauge (e.g. axial, Coulomb) defined by $F[A] = 0$, and $\rho^*(\mathbf{x}, A)$ is in fact the gauge transformation to Coulomb gauge in an abelian theory at infinite volume. Secondly, and most relevant for this work, eigenstates $\zeta_n(\mathbf{x}; U)$ of the lattice Laplacian operator D^2

$$\sum_{\mathbf{y}} (-D^2)_{\mathbf{xy}} \zeta_n(\mathbf{y}) = \lambda_n \zeta(\mathbf{x}) \tag{4}$$

are pseudomatter operators, where

$$(-D^2)_{\mathbf{xy}} = \sum_{k=1}^{3} \left[2\delta_{\mathbf{xy}} - U_k(\mathbf{x})\delta_{\mathbf{y},\mathbf{x}+\hat{k}} - U_k^\dagger(\mathbf{x}-\hat{k})\delta_{\mathbf{y},\mathbf{x}-\hat{k}} \right] . \tag{5}$$

There are, of course, excitations of the free electromagnetic field in the presence of a static charge. But these are simply some number of photons in the background of a Coulombic electric field. Theories in which there are interacting gauge and Higgs fields surrounding a static charge can, in principle, have a richer variety of localized excitations, which cannot be interpreted as simply the ground state plus some set of propagating particle excitations.

3 The Ginzburg-Landau model

This is an effective model of superconductivity, with action (cf. [4])

$$\begin{aligned} S \;=\; \int d^4x \Big\{ \frac{1}{2}\rho_s \bigg(\frac{1}{\upsilon^2} (\partial_0 \xi + 2eA_0)^2 \\ +(\partial_k \xi - 2eA_k)^2 \bigg) + \frac{1}{2}(E^2 - B^2) \Big\} . \end{aligned} \tag{6}$$

It can be derived from the underlying microscopic BCS theory. The factor of $2e$ indicates that the scalar field is double-charged (Cooper pairs). On the lattice

$$\begin{aligned} S_{eff} &= -\beta \sum_{plaq} \mathrm{Re}[UUU^*U^*] \\ &\quad -\gamma \sum_x \mathrm{Re} \sum_{k=1}^{3} \phi^*(x) U_k^2(x) \phi(x+\hat{k})] \\ &\quad -\frac{\gamma}{\upsilon^2} \sum_x \mathrm{Re}[\phi^*(x) U_0^2(x) \phi(x+\hat{t}) , \end{aligned} \tag{7}$$

where $\phi(x) = e^{i\xi(x)}$, $\beta = 1/e^2 \approx 10.9$, $\upsilon \sim 10^{-2}$. We consider physical states with $+/-$ static charges at points $\mathbf{x}, \mathbf{y}$ of the form

$$|\Phi_n(\mathbf{x}, \mathbf{y})\rangle = Q_n(\mathbf{x}, \mathbf{y}) |\Psi_0\rangle , \tag{8}$$

with

$$\begin{aligned} Q_{2n-1}(\mathbf{x}, \mathbf{y}) &= \overline{\psi}(\mathbf{x}) \zeta_n(\mathbf{x}) \zeta_n^*(\mathbf{y}) \psi(\mathbf{y}) \\ Q_{2n}(\mathbf{x}, \mathbf{y}) &= \overline{\psi}(\mathbf{x}) \phi(\mathbf{x}) \zeta_n^*(\mathbf{x}) \zeta_n(\mathbf{y}) \phi^*(\mathbf{y}) \psi(\mathbf{y}) . \end{aligned} \tag{9}$$

The idea is to diagonalize the (rescaled) transfer matrix $\mathcal{T}$ in a subspace spanned by N states $\{|\Phi_n(\mathbf{x}, \mathbf{y})\rangle\}$, and check whether the eigenstates in the subspace are close to eigenstates in the full Hilbert space.[2] If so, we can obtain the low-lying spectrum of the +/- charges in the superconductor. Defining $Q_\alpha(\mathbf{x}, \mathbf{y}, t)$ as the operator $Q_\alpha(\mathbf{x}, \mathbf{y})$ at Euclidean time t, we compute matrix elements

$$\begin{aligned} [\mathcal{T}]_{\alpha\beta}(R) &= \langle \Phi_\alpha | \mathcal{T} | \Phi_\beta \rangle \\ &= \langle Q_\alpha^\dagger(\mathbf{x}, \mathbf{y}, 1) Q_\beta(\mathbf{x}, \mathbf{y}, 0) \rangle \end{aligned} \tag{10}$$

and overlaps

$$\begin{aligned} [O]_{\alpha\beta}(R) &= \langle \Phi_\alpha | \Phi_\beta \rangle \\ &= \langle Q_\alpha^\dagger(\mathbf{x}, \mathbf{y}, 0) Q_\beta(\mathbf{x}, \mathbf{y}, 0) \rangle . \end{aligned} \tag{11}$$

The states Φ_n do not form an orthogonal basis. The standard method for finding the orthogonal eigenstates Ψ_n of a matrix with components in a non-orthogonal basis is via solving the generalized eigenvalue problem, which in our case is

$$[\mathcal{T}]\vec{\upsilon}_n = \lambda_n [O] \vec{\upsilon}^{(n)} \ , \ |\Psi_n(\mathbf{x}, \mathbf{y})\rangle = \sum_{\alpha=1}^{N} \upsilon_\alpha^{(n)} |\Phi_\alpha(\mathbf{x}, \mathbf{y})\rangle . \tag{12}$$

The $|\Psi_n\rangle$ states are eigenstates of $\mathcal{T}$ in the subspace spanned by the $|\Phi_\alpha\rangle$, in the sense that

$$\langle \Psi_m | \mathcal{T} | \Psi_n \rangle = \lambda_n \delta_{mn} \ , \ \langle \Psi_m | \Psi_n \rangle = \delta_{mn} , \tag{13}$$

which can be readily seen from (12) and the identity

$$\vec{\upsilon}^{(m)\dagger} \cdot [O] \vec{\upsilon}^{(n)} = \delta_{mn} . \tag{14}$$

The $|\Psi_n\rangle$ are not, in general, eigenstates of the operator $\mathcal{T}$ in the full Hilbert space. We then consider evolving these states for Euclidean time T (with $R \equiv |\mathbf{x} - \mathbf{y}|$),

$$\begin{aligned} \mathcal{T}_{nn}(R, T) &= \langle \Psi_n | \mathcal{T}^T | \Psi_n \rangle \\ &= \sum_{i,j} \upsilon_i^{(n)*} \langle \Phi_i | \mathcal{T}^T | \Phi_j \rangle \upsilon_j^{(n)} . \end{aligned} \tag{15}$$

Integrating out the massive (i.e. static) fermion fields generates a pair of Wilson lines. The numerical computation of $\langle \Phi_i | \mathcal{T}^T | \Phi_j \rangle$ involves expectation values of products of Wilson lines, terminated by pseudomatter fields (either ζ_n or $\phi\zeta_n^*$), as sketched in Fig. 2. From the $\langle \Phi_i | \mathcal{T}^T | \Phi_j \rangle$ we can determine the $\mathcal{T}_{nn}(R, T)$, and on general grounds

$$\begin{aligned} \mathcal{T}_{nn}(R, T) &\equiv \langle \Psi_n | \mathcal{T}^T | \Psi_n \rangle \\ &= \sum_k |c_k(R)|^2 e^{-E_k(R)T} . \end{aligned} \tag{16}$$

where E_k is an energy eigenvalue minus the vacuum energy. If $\Psi_n(\mathbf{x}, \mathbf{y})$ has a large overlap with one excited energy eigenstate Ψ_i^{exact}, and very small overlap with other

[1] S_c confinement is a stronger condition than color confinement; very roughly speaking it corresponds to the tendency to form metastable color electric flux tubes, which decay via string breaking, cf. [3].

[2] By "rescaled" we mean that $\mathcal{T}$ is the transfer matrix divided its lowest eigenvalue. This removes the vacuum energy from the excitation spectrum, and occurs automatically in the Euclidean time correlators $\langle QQ \rangle$ in (10).

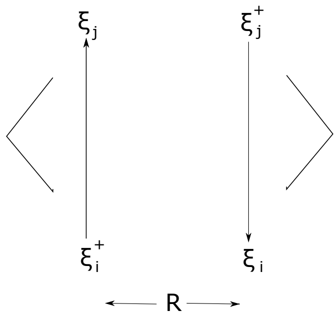


Figure 2. Schematic representation of $[\mathcal{T}]_{ji}(R)$, which is a correlator of Wilson lines, separated by distance R, terminated at either end by pseudomatter fields, here denoted ξ_i.

energy eigenstates, then we may expect that for some range of $T_{min} \leq T \leq T_{max}$

$$\mathcal{T}_{nn}(R,T) \approx |c_i(R)|^2 e^{-E_i(R)T} , \tag{17}$$

and in that case we may extract the excitation energy $E_i(R)$ from a logarithmic plot of $\mathcal{T}_{nn}(R,T)$.

4 Excitations

Figure 3 is a log plot of $\mathcal{T}_{nn}(R,T)$ vs. T at $R = 5.83, \gamma = 0.6$, for $n = 1,2,3$, on a $12^3 \times 36$ lattice. From the slopes, we deduce that $E_1 = 0, E_2 = 0.46, E_3 = 0.55$ in lattice units.

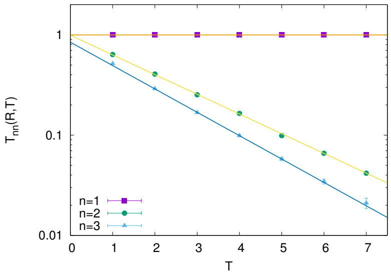


Figure 3. $\mathcal{T}_{nn}(R,T)$ vs. T at $R = 5.83$, for $n = 1,2,3$ together with their best fits. The corresponding energies $E_n(R)$ are extracted from the slope of exponential fits on a log plot. This data was obtained on a $12^3 \times 36$ lattice volume at $\beta = 10.9$ and $\gamma = 0.60$.

We compute the photon mass from the time correlator

$$\begin{aligned} G(t) &= \frac{1}{3}\sum_{i=1}^{3}\langle \mathcal{A}_i(t)\mathcal{A}_i(0)\rangle \\ \mathcal{A}_i(t) &= \frac{1}{L^3}\sum_{\mathbf{x}} \mathrm{Im}[\phi^\dagger(\mathbf{x},t)U_i^2(\mathbf{x},t)\phi(\mathbf{x}+\hat{\imath})] \, . \end{aligned} \tag{18}$$

The results, at $\beta = 1/e^2, \gamma = 0.6$ on a $16^3 \times 36$ lattice volume, are shown in Fig. 4. A fit to a single exponential

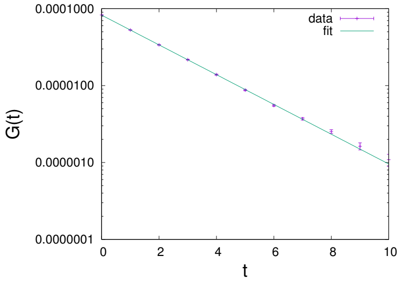


Figure 4. Determination of the photon mass at $\beta = 10.9, \gamma = 0.6$ from the time correlator $G(t)$ of space averaged gauge invariant link variables $\mathcal{A}_i$ on a $16^3 \times 36$ lattice volume. The straight line on the log plot is a best fit of $ae^{-m_{ph}t}$ to the data, where m_{ph} is the photon mass, and a is a constant.

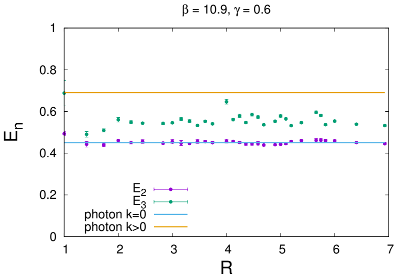


Figure 5. Energies $E_n(R)$ vs. R for $n = 2,3$ again at $\gamma = 0.60$ on a $12^3 \times 36$ lattice volume. The lower solid line represents the mass of a massive photon, and the upper line is the energy of a massive photon at the lowest possible momentum on this finite periodic lattice.

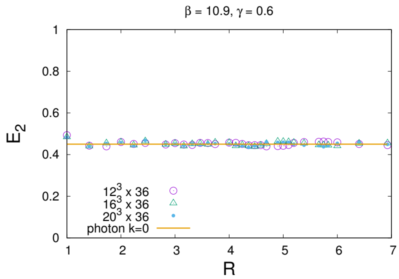
(i.e. a straight line on the log plot) gives a value for the photon mass $m_{ph} = 0.446(3)$ in lattice units.

Fig. 5 is a plot of E_1, E_2, E_3 vs. R. We see that $E_1 = 0$ and E_2 coincides with the mass of a photon. The upper solid line is next higher energy of a massive photon on the $12^3 \times 36$ lattice, with lowest non-zero momentum $|k| = 2\pi/12$, and this is $E_\gamma = 0.687$. The $E_3 \approx 0.56$ values lie well below this number. Therefore, E_3 cannot be interpreted as the ground state + a photon of non-zero momentum. It seems to be an excited state of the static charges.

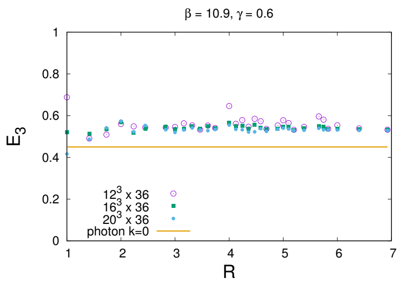
A larger volume doesn't change the energies much, but there is less scatter in the data (Fig. 6) for higher excitations E_3, E_4 at the larger volume.

5 Scaling

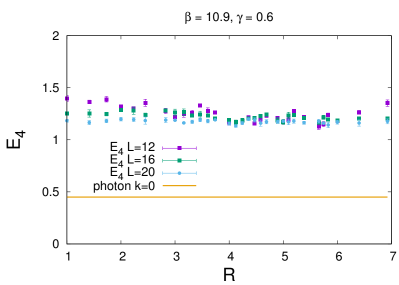
Figure 7 shows the results at a lower value of $\gamma = 0.25$, still at $\beta = 10.9$, on a $16^3 \times 36$ lattice volume. The scale is set by the London penetration depth λ_L, typically ~ 50 nm. The lattice spacing is given by $a = 2e\sqrt{\gamma}\lambda_L$, and from this we can convert to physical units. Figure 8 shows E_n in ev vs. R in nm, computed on a $16^3 \times 36$ lattice, for both



(a) E_2



(b) E_3



(c) E_4

Figure 6. Excitation energies $E_n(R)$ vs. R for lattice volumes $12^3, 16^3, 20^3 \times 36$. (a) $E_2(R)$; (b) $E_3(R)$, (c) $E_4(R)$. Note that fluctuations with R tend to decrease with higher volumes.

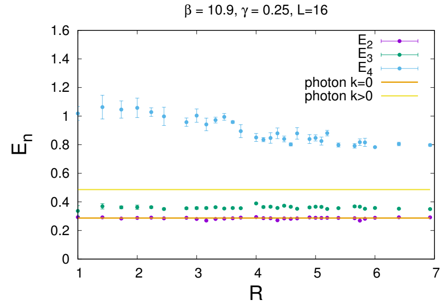


Figure 7. Energies $E_{2,3,4}(R)$, but this time at $\gamma = 0.25$, with $\beta = 10.9$ and lattice volume $16^3 \times 36$. The lower and upper solid lines represent the energies of a static massive photon, and a photon of minimal momentum for this periodic volume

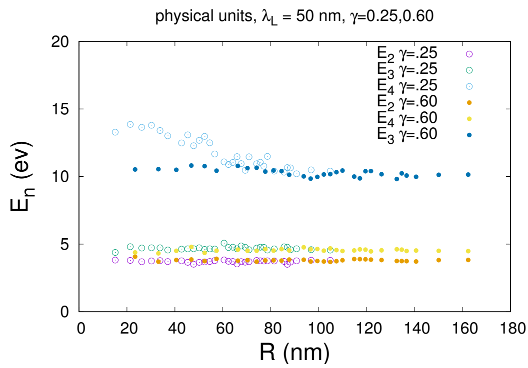


Figure 8. A test of scaling. Using a fixed London penetration depth of $\lambda_L = 50$ nm to set the scale, we convert both R and $E_{2,3,4}(R)$ to physical units at both $\gamma = 0.25$ and $\gamma = 0.60$, with $\beta = 10.9$ fixed.

$\gamma = 0.25$ and $\gamma = 0.60$. In physical units and larger R values, the data points clearly overlap.

6 Is this observable?

We are a long way from accurate predictions. Still, if the effect is there, it might show up in photoelectron spectroscopy, comparing core level electron spectra in the normal and superconducting states. In fact one can argue that a similar effect – excitations of the field surrounding static charges – has *already* been seen in normal metals. The process, in a normal metal is this: A photon knocks out a core electron (bound to an atom), suddenly creating an isolated charge. In a normal metal, conduction electrons respond by screening the charge. In the screening response there is a near-continuum of excitations of the Fermi sea above the ground state of the screened charge. If the Fermi sea is left in an excitation above the ground state, this reduces the energy of the emitted electron, and this in turn affects the line shape in the photoelectron spectrum. From this picture, Doniach and Sunjic [5] derived the observed asymmetric lineshape for peaks in the X-ray photoemission spectrum. To quote from their article: *"Thus the maximum photoelectron energy corresponds to the ground state of the hole +metal, while photoelectrons emitted below the maximum correspond to events in which the hole + Fermi sea is left in an excited state. Excited states with energies very close (a fraction of an electron-volt) to the ground state are those in which the Fermi sea is excited by the creation of low energy conduction electron-hole pairs (i.e. charge density fluctuations)."* This is very similar to the process we are discussing in a superconductor. The difference is that in the superconductor phase, with screening from the Cooper pair condensate, a discrete spectrum of excitations of this condensate would lead to additional peaks in the spectrum, separated by a few ev from the main peak.

What is needed is a comparison of core emission spectra above and below the superconducting transition. Surprisingly, it seems that this comparison not been done yet.

6.1 Other theories

The results shown above are only the latest in a string of studies regarding excitations of static charges in gauge Higgs theories. Our previous work has looked at (i) SU(3) gauge Higgs theory with the Higgs scalar in the fundamental representation of the gauge group [6]; (ii) the abelian Higgs model with a double-charged ($q = 2$) Higgs scalar [7]; and (iii) Chiral U(1) gauge theory in the Smit-Swift lattice formulation with a Higgs scalar of charge $q = 1$ [8].

In each of these models we impose a unimodular constraint $|\phi| = 1$ for simplicity. And in each we find a spectrum of excitations of static charges. In SU(3) gauge Higgs theory, with the Higgs field in the fundamental representation, we have computed $E_n(R,T)$ on a $14^3 \times 32$ lattice volume, in the Higgs phase at $\beta = 5.5$, $\gamma = 3.5$. The lattice action is

$$\begin{aligned} S &= -\frac{\beta}{3}\sum_{plaq} \text{ReTr}[U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x)] \\ &\quad -\gamma \sum_{x,\mu} \text{Re}[\phi^\dagger(x)U_\mu(x)\phi(x+\widehat{\mu})] \end{aligned} \tag{19}$$

Because the Higgs field is in the fundamental representation, we can create color-neutral states using both pseudomatter fields

$$\Phi_n(R) = [\overline{q}^a(\mathbf{x})\zeta_n^a(\mathbf{x})] \times [\zeta_n^{\dagger b}(\mathbf{y})q^b(\mathbf{y})]\,\Psi_0 \quad (n = 1,2,3) \tag{20}$$

and the Higgs field

$$\Phi_4(R) = [\overline{q}^a(\mathbf{x})\phi^a(\mathbf{x})] \times [\phi^{\dagger b}(\mathbf{y})q^b(\mathbf{y})]\,\Psi_0 \tag{21}$$

We then diagonalize $\mathcal{T}$ in the four-dimensional subspace of Hilbert Space, and compute

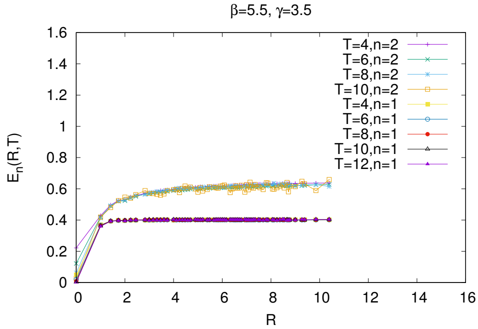
$$\begin{aligned} \mathcal{T}^T_{mn}(R) &= \langle\Psi_n|\mathcal{T}^T|\Psi_n\rangle \\ E_n(R,T) &= -\log\left[\frac{\mathcal{T}^T_{nn}(R)}{\mathcal{T}^{T-1}_{nn}(R)}\right] \end{aligned} \tag{22}$$

Of course the $\Psi_{1,2}$ states are orthogonal by construction. But in principle this orthogonality need not persist under Euclidean time evolution beyond $T = 1$. However, the rapid convergence of $\Psi_{1,2}$ to states with differing energies implies the near-orthogonality of the two states under Euclidean time evolution. In fact the overlap corresponding to off-diagonal matrix elements

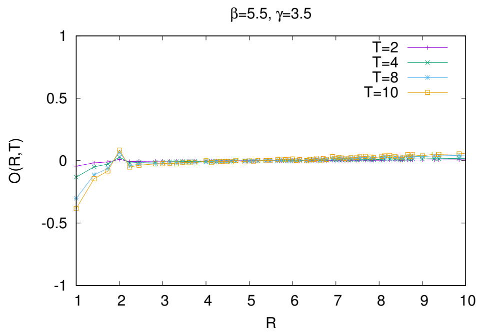
$$\begin{aligned} O(R,T) &= \frac{\langle\Psi_1|e^{-HT}|\Psi_2\rangle}{\sqrt{\langle\Psi_1|e^{-HT}|\Psi_1\rangle\langle\Psi_2|e^{-HT}|\Psi_2\rangle}} \\ &= \frac{\mathcal{T}_{12}(R,T)}{\sqrt{\mathcal{T}_{11}(R,T)\mathcal{T}_{22}(R,T)}} \end{aligned} \tag{23}$$

can be calculated for any R,T. This has the interpretation of an overlap between states obtained from $\Psi_{1,2}$ evolved for $T/2$ units of Euclidean time, and then normalized.

$E_n(R,T)$ for $T = 4-12$ is displayed in Fig. 9(a), and the overlap $O(R,T)$, is shown in Fig. 9(b). There seems to be clear evidence of an excited state in the spectrum, orthogonal to the ground state. The energy difference $E_2 - E_1$ is far smaller than the threshold for vector boson creation; the excited state cannot be regarded as the ground state plus a vector boson. It is clearly a localized excitation of the static charges.



(a) Energies



(b) Overlap

Figure 9. (a) Energies $E_{1,2}(R,T)$ of states Ψ_1, Ψ_2 after evolution for a period of $(T-1)/2$ units of Euclidean time, at lattice couplings $\beta = 5.5, \gamma = 3.5$. (b) The overlap $O(R,T)$ between states $\Psi_1(R)$ and $\Psi_2(R)$ which are evolved (and then normalized) for $T/2$ units of Euclidean time.

7 Conclusions

We have found, in the effective Ginzburg-Landau model of superconductivity, that the gauge+scalar fields surrounded a charged static fermion have a spectrum of localized excitations, which cannot be interpreted as simply the ground state plus some massive bosons. This conclusion seems robust. We see it in Ginzburg-Landau, abelian Higgs, SU(3) gauge Higgs, and chiral U(1) models, and this means that charged "elementary" particles can have a mass spectrum in gauge Higgs theories.

Excitations of screened ions have already been seen in normal metals. They *might* be observable in the superconducting phase, via X-ray photoemission spectroscopy, by comparing core-level spectra above and below the superconducting transition.

Given a lattice formulation of chiral gauge theories, with a positive transfer matrix and a sensible continuum limit, we could figure out the excitations of quarks and leptons in the electroweak theory by the means illustrated here. Going by previous results, the excitation energies should be above the ground state masses of quarks and leptons by an amount on the order of the Z mass. It would be nice to have that lattice formulation.

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