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XVth Quark Confinement and the Hadron Spectrum

Modified TMD Factorization and Sub-leading Power Corrections

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Abstract. Collinear factorization and transverse-momentum-dependent (TMD) factorization are two complementary approaches to perform QCD calculations of Drell-Yan differential cross sections. The former is designed to correctly describe the behavior of the observable at large values of the gauge boson transverse momentum q_T, while the latter accounts for non-perturbative effects relevant at small q_T. We present basic features and first numerical results of a novel method which is related to both previous frameworks and allows for an improved description of the intermediate-q_T region.

1 Introduction

The Drell-Yan process plays a central role in the study of vector gauge bosons and allows us also to extract information about the internal structure of the proton, like the distribution of quark spin and angular momentum. QCD calculations of Drell-Yan multi-differential cross sections have been performed using two different frameworks, namely collinear factorization [1–4] and TMD factorization [5–12], the former designed to describe physical cross sections at high values of the magnitude of the gauge-boson transverse momentum $q_T \equiv \sqrt{\mathbf{q}_T^2}$, and the latter accounting for features relevant at sufficiently small values of q_T. In the collinear factorization framework, the gauge boson transverse momentum has a purely perturbative origin rooted in the hard scattering process that involves purely collinear partons. In TMD factorization instead, $\mathbf{q}_T$ is due to the fact that the colliding partons have non-vanishing non-perturbative intrinsic transverse momentum inside the initial-state hadrons, as described by transverse-momentum-dependent parton distributions functions (TMDPDFs) [10, 13–15].

1.1 Collinear factorization

For two protons $h_{A,B}$ collinear to the z−axis and colliding in the center-of-mass (CM) frame, the following factorization formula for Drell-Yan holds at leading power [9, 10, 16] :

$$\frac{d\sigma_{h_A h_b \to X l\bar{l}}}{dQ^2\, dy\, d\mathbf{q}_T^2} = \sum_{a,b} \int_{x_A}^{1} \frac{dz_a}{z_a} \int_{x_B}^{1} \frac{dz_b}{z_b} H_{ab}\left(\alpha_s\left(\mu^2\right), Q^2, \mathbf{q}_T^2, \frac{x_A}{z_a}, \frac{x_A}{z_a}\right) f_{a\leftarrow h_A}\left(\alpha_s\left(\mu^2\right), z_a\right) f_{b\leftarrow h_B}\left(\alpha_s\left(\mu^2\right), z_b\right), \tag{1}$$

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where y is the gauge boson rapidity and Q^2 is the invariant mass of the final lepton pair. The variables $x_{A(B)}$ are defined as $x_{A(B)} = \sqrt{Q^2/s}\, e^{\pm y}$ where s is the CM energy squared. The value of the renormalization scale μ^2 is typically set equal to Q^2. Non-perturbative information encoded in the parton distribution functions (PDFs) $f_{a(b)\leftarrow h_{A(B)}}$ is extracted from experimental data. Using perturbative QCD, the coefficients H_{ab} are given by

$$\begin{aligned} H_{ab}\left(\alpha_s\left(\mu^2\right), Q^2, \mathbf{q}_T^2, \frac{x_A}{z_a}, \frac{x_B}{z_b}\right) &= \delta^{(2)}\left(\mathbf{q}_T\right)\delta\left(1-\frac{x_A}{z_a}\right)\delta\left(1-\frac{x_B}{z_b}\right)\sigma_{ab}^{\text{Born}}\delta_{ah_A}\delta_{bh_B} \\ &+ \sum_n \alpha_S\left(\mu^2\right)^n H_{ab}^{[n]}\left(Q^2, \mathbf{q}_T^2, \frac{x_A}{z_a}, \frac{x_B}{z_b}\right), \end{aligned} \tag{2}$$

with $H_{ab}^{[n]}$ determined from a partonic calculation [1, 4]. The last expression contains logarithmically enhanced terms of the form $\ln^k \mathbf{q}_T^2/Q^2$ which needs to be resummed to all orders to obtain reliable theory predictions [2, 3, 17, 18]. The resummed differential cross section in the limit $\mathbf{q}_T^2/Q^2 \ll 1$ is given by [1–4, 17–19]:

$$\begin{aligned} &\left[\frac{d\sigma_{h_A h_B\to X l\bar{l}}}{dQ^2\, dy\, d\mathbf{q}_T^2}\right]_{\text{Res}} = \\ &\sum_{c,a,b}\int_{x_A}^1 \frac{dz_a}{z_a}\int_{x_B}^1 \frac{dz_b}{z_b}\int \frac{d^2\mathbf{b}_T}{(2\pi)^2} e^{i\mathbf{q}_T\cdot\mathbf{b}_T}\sigma_{c\bar{c}}^{\text{Born}} C_{c\leftarrow a}\left(\alpha_s\left(b_0^2/\mathbf{b}_T^2\right), \frac{x_A}{z_a}\right) C_{\bar{c}\leftarrow b}\left(\alpha_s\left(b_0^2/\mathbf{b}_T^2\right), \frac{x_B}{z_b}\right) \\ &\times S(Q^2, b_0^2/\mathbf{b}_T^2) f_{a\leftarrow h_A}\left(\alpha_s\left(b_0^2/\mathbf{b}_T^2\right), z_a\right) f_{b\leftarrow h_B}\left(\alpha_s\left(b_0^2/\mathbf{b}_T^2\right), z_b\right), \end{aligned} \tag{3}$$

where $\mathbf{b}_T$ is the impact parameter, $b_0^2 = 4e^{-2\gamma_E}$, $C_{c(\bar{c})\leftarrow a(b)}$ denote perturbative coefficients and $S(Q^2, b_0^2/\mathbf{b}_T^2)$ is the Sudakov form factor [1–4, 8, 19]. The last expression allows us to define, order-by-order in QCD perturbation theory, the fixed-order (FO) contribution to the cross section as

$$\left[\frac{d\sigma_{h_A h_B\to X l\bar{l}}}{dQ^2\, dy\, d\mathbf{q}_T^2}\right]_{\text{FO}} = \frac{d\sigma_{h_A h_B\to X l\bar{l}}}{dQ^2\, dy\, d\mathbf{q}_T^2} - \left[\frac{d\sigma_{h_A h_B\to X l\bar{l}}}{dQ^2\, dy\, d\mathbf{q}_T^2}\right]_{\text{Res}}, \tag{4}$$

where the most singular terms in the limit $\mathbf{q}_T^2/Q^2$ are removed via a subtraction of partonic collinear divergences using PDFs. Combining fixed-order and resummed contributions, the cross section derived in the collinear factorization framework describes well experimental data at sufficiently high q_T, but fails to do so in the small-q_T regime. The non-perturbative contributions that are responsible for this discrepancy are captured by TMD factorization.

1.2 TMD factorization

In TMD factorization [5–7, 9–13, 20, 21], the differential cross section at leading power is given by

$$\begin{aligned} &\frac{d\sigma_{h_A h_B\to X l\bar{l}}}{dQ^2\, dy\, d\mathbf{q}_T^2} = \sum_c \sigma_{c\bar{c}}^{\text{Born}} H\left(\alpha_s\left(\mu^2\right), Q^2/\mu^2\right) \\ &\times \int \frac{d^2\mathbf{b}_T}{(2\pi)^2} e^{i\mathbf{b}_T\cdot\mathbf{q}_T} F_{c\leftarrow h_A}\left(\alpha_s\left(\mu^2\right), \mathbf{b}_T^2, x_A, \zeta_A\right) F_{\bar{c}\leftarrow h_B}\left(\alpha_s\left(\mu^2\right), \mathbf{b}_T^2, x_B, \zeta_B\right), \end{aligned} \tag{5}$$

where the hard factor H is computed in perturbative QCD [11, 22, 23] and $F_{i\leftarrow h_{A,B}}$ describes the non-perturbative intrinsic transverse momentum distribution of the parton i inside the

colliding hadrons. Here the renormalization scale μ^2 is typically set equal to Q^2 and the same choice is made for the rapidity renormalization scale $\zeta_{A(B)}$ [24–26]. The TMDPDFs can be matched onto collinear PDFs as [13, 15, 27]

$$F_{a\leftarrow h_A}\left(\alpha_s\left(\mu^2\right), \mathbf{b}_T^2, x_A, \zeta_A\right) = \sum_{n,r} \left(|\mathbf{b}_T|M\right)^n C^{(n)}_{a\leftarrow r}\left(\alpha_s\left(\mu^2\right), \mathbf{b}_T^2, \frac{x_A}{z_a}, \zeta_A\right) \otimes f^{(n)}_{r\leftarrow h_A}\left(\alpha_s\left(\mu^2\right), z_a\right), \tag{6}$$

where M is typically identified with the colliding hadron mass, n stands for the twist of the matching and $\otimes$ denotes the Mellin convolution in z_a.

The distribution in eq. (5), including the matching of TMDPDFs onto PDFs at NNLO and NNLL resummation, allows for a good fit to experimental data at small q_T as shown in fig. (1), but is not suitable at intermediate/large q_T, see also [28]. For this purpose, matching corrections to the cross section should be taken into account. Our approach provides a convenient method to account for these effects and identify power corrections to structure functions appearing in the decomposition of the hadronic tensor.

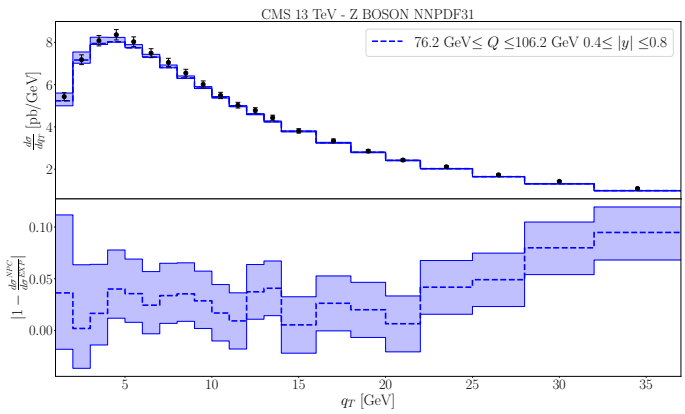

Figure 1. Drell-Yan Z-boson production at the LHC: comparison between the theoretical prediction in TMD factorization and CMS experimental data [28]. TMDPDFs are matched onto collinear PDFs at NNLO and are extracted from a fit to several data sets in [23]. The resummation in the Drell-Yan cross section is performed up to NNLL accuracy.

2 Modified TMD factorization and power corrections

In the spirit of the work by Catani et al. [1, 3, 17, 18, 33], we worked out a phenomenological modification of the factorization formula in eq. (1) where the hard factor depends *both* on the collinear momentum components $z_{a(b)}$ *and* on the intrinsic transverse momenta $\mathbf{k}_{a(b)T}$ of the partons in the underlying scattering process. In our formalism, collinear divergences are subtracted using TMDPDFs instead of collinear PDFs, which enables the resummation of all logarithmically enhanced contributions in $\mathbf{q}_T^2/Q^2$ as well as the inclusion of a set of sub-leading power corrections. By matching to the fixed-order cross section in the unpolarized case, the expansion of the full hadronic tensor in the collinear limit leads to a series of power corrections to the standard TMDPDFs, which we write as a perturbative coefficient convoluted with the TMDPDFs at leading power. In detail, the Drell-Yan cross section in our

formalism is given by

$$\frac{d\sigma_{h_A h_B \to X l\bar{l}}}{dQ^2\, dy\, d\mathbf{q}_T^2} =$$
$$\sum_{a,b} \int d^2\mathbf{k}_{aT} d^2\mathbf{k}_{bT} d^2\mathbf{q}'_T \int_{x_A}^1 \frac{dz_a}{z_a} \int_{x_B}^1 \frac{dz_b}{z_b} \delta^{(2)}\left(\mathbf{q}_T - \mathbf{q}'_T - \mathbf{k}_{aT} - \mathbf{k}_{bT}\right) \theta\left(\frac{(z_a - x_A)(z_b - x_B)}{x_A x_B} - \frac{\mathbf{q}_T'^2}{Q^2 + \mathbf{q}_T^2}\right)$$
$$\times \tilde{H}_{ab}\left(\alpha_s\left(\mu^2\right), Q^2, \mathbf{q}_T^2, \mathbf{q}_T'^2, \frac{x_A}{z_a}, \frac{x_B}{z_b}\right) F_{a\leftarrow h_A}\left(\alpha_s\left(\mu^2\right), \mathbf{k}_{aT}^2, z_a, \zeta_A\right) F_{b\leftarrow h_B}\left(\alpha_s\left(\mu^2\right), \mathbf{k}_{bT}^2, z_b, \zeta_B\right). \tag{7}$$

This expression shows that, according to the most general kinematic picture, $\mathbf{q}_T$ receives contributions from both perturbative parton emissions and non-perturbative intrinsic transverse-momentum parton distributions, related by four-momentum conservations. We explicitly checked up to NNLO in our framework that collinear divergences in the partonic calculations are taken care of by the TMDPDFs. The modified hard factor $\tilde{H}_{ab}$, which differs from both the hard coefficient in collinear factorization and the one in standard TMD factorization, is determined from eq. (7) by fixed-order matching and is free of singularities in the small-q_T limit. This definition of $\tilde{H}_{ab}$ guarantees that in the small-q_T limit, the Drell-Yan distribution in this formalism reproduces the results from TMD factorization. Notice that in eq. (7), the TMDPDFs are written in momentum space, while in TMD factorization, eq. (5), the impact-parameter space notation is used.

We stress that our approach differs from previous attempts to combine TMD and collinear factorization, as the one in [34] where the following formula was proposed,

$$\frac{d\sigma_{h_A h_B \to X l\bar{l}}}{dQ^2\, dy\, d\mathbf{q}_T^2} = g_1\left(\frac{\mathbf{q}_T^2}{Q^2}\right)\left[\frac{d\sigma_{h_A h_B \to X l\bar{l}}}{dQ^2\, dy\, d\mathbf{q}_T^2}\right]_{\text{TMD}} + g_2\left(\frac{\mathbf{q}_T^2}{Q^2}\right)\left[\frac{d\sigma_{h_A h_B \to X l\bar{l}}}{dQ^2\, dy\, d\mathbf{q}_T^2}\right]_{\text{high}\, q_T}, \tag{8}$$

which combines TMD factorization and collinear factorization results in a weighted sum where $g_{1,2}$ are non-perturbative model functions such that $g_1 \to 1$, $g_2 \to 0$ as $q_T \to 0$, while $g_1 \to 0$, $g_2 \to 1$ as $q_T \to \infty$. In our approach instead, TMD and collinear factorization results are combined through the perturbative matching coefficient $\tilde{H}$, which is an analytic function of $\mathbf{q}_T'^2$. Therefore a Taylor expansion can be performed for $\mathbf{q}_T'^2/Q^2 \ll 1$, which leads, through an integration by parts, to a smooth function involving derivatives of TMDPDFs and interpolating between the resummed and the fixed-order regimes, without the need to introduce the functions g_1 and g_2 above. Furthermore, the formula we provide for the cross sections enables improved fits of TMDPDFs parameters on larger data sets.

We also point out that our formalism is different from the one presented in [6] to work out next-to-leading power corrections to the Drell-Yan cross section in TMD factorization. In that study, the first term on the right-hand side of eq. (4) was computed in the $q\bar{q} \to g l\bar{l}$ channel by first performing an expansion in $\mathbf{k}_T^2/Q^2$ up to next-to-leading power, where $\mathbf{k}_T$ denotes the transverse momentum of the emitted gluon, and then carrying out the phase space integrals. In our formalism no expansion in powers of $\mathbf{k}_T^2/Q^2$ is performed.

For a first numerical analysis within our modified TMD factorization formalism, we determined the hard factor $\tilde{H}_{ab}$ at NLO from the right-hand side of eq. (4) in a partonic calculation involving the channels $q\bar{q} \to X l\bar{l}$ where $X = \{g, gg, q\bar{q}, \cdots\}$ and $q\,(\bar{q})$ has collinear(anti-collinear) momentum with respect to the z-axis. No assumption about the hierarchy between $\mathbf{q}_T^2$ and Q^2 has been made. Fig. (2) shows our results where the phenomenological parameters in the TMDPDFs were fixed according to the outcome of the fits presented in [23]. Our modified TMD factorization framework leads to a better agreement with experimental results in the intermediate-q_T region. Our analysis allows us to quantify the impact of a set of

power suppressed terms compared to standard TMD factorization: the contribution of these sub-leading effect turns out to be at least about 5% in the intermediate region, namely of comparable size or even larger than electroweak corrections that have recently been quantified to be around 1%-2% [35, 36].

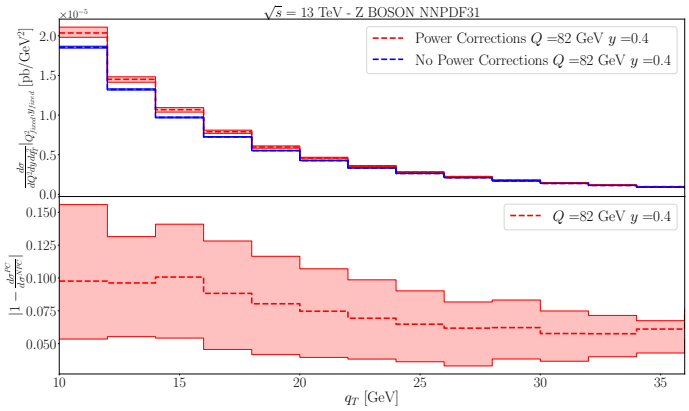

Figure 2. Comparison between theory predictions using our modified TMD factorization framework, with and without the power corrections that we can account for.

3 Conclusions

Modifying the collinear factorization formula for Drell-Yan by subtracting collinear divergences using TMDPDFs instead of standard PDFs allows us to include power suppressed terms in $\mathbf{q}_T^2/Q^2$ in the cross section. These terms provide additional positive (matching) contributions which improve the agreement between theory predictions and experimental data in the tail of the distribution. This enables future improved phenomenological extractions of TMDPDFs by fitting to enlarged sets experimental data. Furthermore, the fact that the hard factor in our modified TMD factorization formalism does not contain logarithmically enhanced terms makes this a smooth function in the limit $\mathbf{q}_T^2 \to 0$, opening up the possibility to reliably perform a numerical computation of $\tilde{H}$ using subtraction methods [17, 18, 33, 37], which can be automatized and implemented in Monte-Carlo event generators.

References

[1] S. Catani, D. de Florian, M. Grazzini, Nucl. Phys. B **596**, 299 (2001)
[2] S. Catani, D. de Florian, M. Grazzini, P. Nason, JHEP **07**, 028 (2003)
[3] D. de Florian, M. Grazzini, Phys. Rev. Lett. **85**, 4678 (2000)
[4] D. de Florian, M. Grazzini, Nucl. Phys. B **616**, 247 (2001)
[5] M.A. Ebert, F.J. Tackmann, JHEP **02**, 110 (2017)
[6] M.A. Ebert, I. Moult, I.W. Stewart, F.J. Tackmann, G. Vita, H.X. Zhu, JHEP **04**, 123 (2019)
[7] J.C. Collins, D.E. Soper, Nucl. Phys. B **194**, 445 (1982)
[8] J.C. Collins, D.E. Soper, G.F. Sterman, Nucl. Phys. B **250**, 199 (1985)
[9] J.C. Collins, D.E. Soper, G.F. Sterman, Adv. Ser. Direct. High Energy Phys. **5**, 1 (1989)

[10] J. Collins, Int. J. Mod. Phys. Conf. Ser. **4**, 85 (2011)
[11] M.G. Echevarría, A. Idilbi, I. Scimemi, Phys. Lett. B **726**, 795 (2013)
[12] M.G. Echevarria, A. Idilbi, I. Scimemi, JHEP **07**, 002 (2012)
[13] M.G. Echevarria, I. Scimemi, A. Vladimirov, JHEP **09**, 004 (2016)
[14] X.d. Ji, J.p. Ma, F. Yuan, Phys. Rev. D **71**, 034005 (2005)
[15] A. Bacchetta, D. Boer, M. Diehl, P.J. Mulders, JHEP **08**, 023 (2008)
[16] G. Altarelli, R.K. Ellis, M. Greco, G. Martinelli, Nucl. Phys. B **246**, 12 (1984)
[17] G. Bozzi, S. Catani, D. de Florian, M. Grazzini, Phys. Lett. B **564**, 65 (2003)
[18] G. Bozzi, S. Catani, D. de Florian, M. Grazzini, Nucl. Phys. B **737**, 73 (2006)
[19] C. Balazs, C.P. Yuan, Phys. Rev. D **56**, 5558 (1997)
[20] M.G. Echevarria, A. Idilbi, I. Scimemi, Phys. Rev. D **90**, 014003 (2014)
[21] V. Bertone, I. Scimemi, A. Vladimirov, JHEP **06**, 028 (2019)
[22] I. Scimemi, A. Vladimirov, Eur. Phys. J. C **78**, 89 (2018)
[23] I. Scimemi, A. Vladimirov, JHEP **06**, 137 (2020)
[24] I. Scimemi, A. Vladimirov, JHEP **08**, 003 (2018)
[25] J.Y. Chiu, A. Jain, D. Neill, I.Z. Rothstein, JHEP **05**, 084 (2012)
[26] M.G. Echevarria, I. Scimemi, A. Vladimirov, Phys. Rev. D **93**, 054004 (2016)
[27] M.G. Echevarria, T. Kasemets, P.J. Mulders, C. Pisano, JHEP **07**, 158 (2015), [Erratum: JHEP 05, 073 (2017)]
[28] A.M. Sirunyan et al. (CMS), JHEP **12**, 061 (2019)
[29] R.D. Tangerman, P.J. Mulders, Phys. Rev. D **51**, 3357 (1995), hep-ph/9403227
[30] R.K. Ellis, W. Furmanski, R. Petronzio, Nucl. Phys. B **212**, 29 (1983)
[31] R.K. Ellis, W. Furmanski, R. Petronzio, Nucl. Phys. B **207**, 1 (1982)
[32] K. Goeke, A. Metz, M. Schlegel, Phys. Lett. B **618**, 90 (2005), hep-ph/0504130
[33] A. Broggio, C. Gnendiger, A. Signer, D. Stöckinger, A. Visconti, JHEP **01**, 078 (2016)
[34] J. Collins, L. Gamberg, A. Prokudin, T.C. Rogers, N. Sato, B. Wang, Phys. Rev. D **94**, 034014 (2016)
[35] R. Bonciani, L. Buonocore, M. Grazzini, S. Kallweit, N. Rana, F. Tramontano, A. Vicini, Phys. Rev. Lett. **128**, 012002 (2022)
[36] L. Cieri, G. Ferrera, G.F.R. Sborlini, JHEP **08**, 165 (2018)
[37] S. Catani, M.H. Seymour, Nucl. Phys. B **485**, 291 (1997), [Erratum: Nucl.Phys.B 510, 503–504 (1998)]