Production of lepton pairs from an arbitrarily magnetised QCD medium

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Abstract. We have estimated the rate of production of lepton pairs from a magnetised hot and dense QCD medium. We get rid of all kinds of previously considered approximations in terms of the strength of the magnetic field as well as the components of the momentum of the emitted lepton pairs. We find an enhancement in the rate in presence of an arbitrary strength of the magnetic field. With further consideration of an effective model scenario, we find the appearance of a gap in the rate. The implications of such a gap and other quasi-quark effects on the rate have been investigated in detail.

1 Introduction

Leptons, having smaller scattering cross section than the strongly interacting quarks, possess a large mean free path when produced in a strongly interacting medium. Thus they carry less contaminated information about the medium they are created in and can be used as a trustworthy signature [1].

This is the reason that the lepton-pairs or the so called dileptons from a strongly interacting quantum chromodynamics (QCD) medium are used as a penetrating probe and their productions have been explored extensively [2]. They are used as a thermometer for the QCD medium. They can also be used to study the production of various neutral vector mesons [1]. The leptons can be produced from different stages of the heavy ion collisions (HIC): initial hard collisions of partons, thermalised QCD medium or the quark gluon plasma (QGP), hadronic decays in the later stages etc. We will be particularly interested here in the thermalised leptons coming out of a QGP.

From our latest understanding, we also know that a very strong magnetic field (eB) is generated in the direction perpendicular to the reaction plane in the peripheral HICs. The non-engaging passing spectators engenders the magnetic field. The strength of such a magnetic field was estimated in one of the fast few calculations [3]. There the reaction plane is considered to be on the $x - z$ plane. A plot from that reference is shown in the left panel of Fig. 1. The solid lines represent Ultrarelativistic Quantum Molecular Dynamics model (UrQMD) calculations, whereas the dashed lines are obtained using a semi-analytical model.

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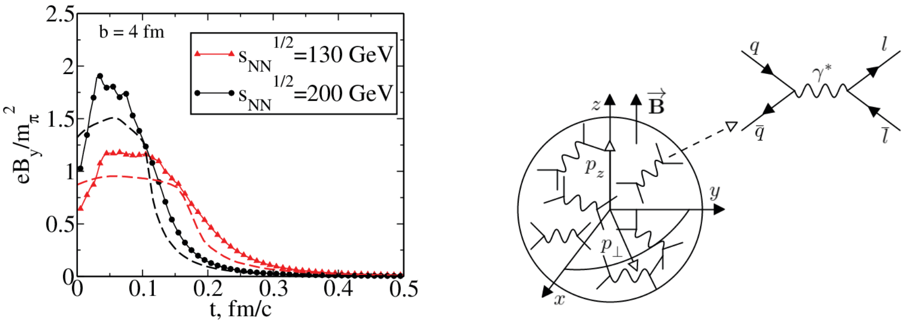

Figure 1. The strength of the magnetic field is plotted as a function of time in the left panel [3]. On the right panel we have a pictorial representation of lepton pairs coming out of a magnetised medium.

The strength is expected to be $\approx m_\pi^2$ at Relativistic Heavy Ion Collider (RHIC) and $\approx 10\, m_\pi^2$ at Large Hadron Collider (LHC). To put things into usual perspective one should note that $m_\pi^2 = 1.96\times10^{-2}$ GeV$^2 \approx 10^{18}$ Gauss. This strength is huge as compared to the magnetic field created in ordinary earth based laboratories ($\approx 10^4 - 10^5$ Gauss) and also much greater than the field strength estimated in magnetar ($\approx 10^{13} - 10^{15}$ Gauss).

The presence of an external field, in principle, can affect different observables related to the QCD medium created in heavy ion collisions. For example, it can affect the production of lepton pairs. This is depicted in the right panel of Fig. 1, where the $x - y$ plane is assumed to be the reaction plane, which is different from the left panel.

So far dilepton rate from a magnetised medium has been calculated in different articles using different techniques. In some of them, the Ritus eigenfunction method has been utilised [4] whereas in others the Schwinger formalism has been exploited [5].

Also, most of the existing calculations use different approximations for the ease of calculations rendering them inapplicable to most possible general scenarios relevant to HICs. These approximations are made either by taking a very strong or weak magnetic field [5, 6] or by considering some components of the dilepton momentum (parallel (p_z) or perpendicular ($p_\perp$) to the direction of eB) to be zero [7].

We, in this work which is also very easy to grasp, try to estimate the rate for arbitrary strength of magnetic field in the most possible general scenario. Similar approach has also been utilised in a recent work [8] where they have also calculated the ellipticity of the dilepton. In our exploration we learn a few interesting things. As opposed to the zero magnetic field case (the so-called Born rate) or the lowest Landau level approximated rate, where only the annihilation process contributes, we observe contributions also arising out of the quark and antiquark decay processes. We found the encouraging result of considerable enhancement of lepton pair production in presence of an arbitrary magnetic field.

In our effort, we further subject the emission of the lepton pairs to an effective model scenario, which also encompasses the known inverse magnetic catalysis (IMC) effect [9], unlike all other previous efforts. We have discussed about the effect of the medium dependent effective quark mass on the dilepton rate (DR) and subsequent occurrence of a gap between the decay and the annihilation processes for certain values of the temperature and the external magnetic field.

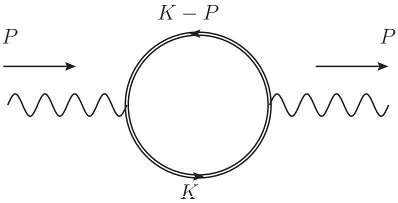

Figure 2. One loop photon self-energy.

2 Methodology

With the knowledge that the lepton pairs are generated from the decay of a virtual photon through the annihilation of quark-antiquark pairs (right panel of Fig. 1), in presence of an external magnetic field it is obvious to assume that both the initial quark pair and the final lepton pair will be affected by the external field. But, since the lepton pairs possess much larger mean free path than the fireball, they come out of it without much interaction as explained in Ref. [8] in detail. Hence, in our current study, we consider the scenario when only the quarks move in a magnetized medium but not the final lepton pairs. In that case, without loss of generality one can neglect the mass of the leptons and the expression of DR can be written as [10],

$$\frac{dN}{d^4Xd^4P} \equiv \frac{dR}{d^4P} = \frac{\alpha_{\rm EM}}{12\pi^4}\frac{1}{P^2}\frac{1}{e^{p_0/T}-1}\sum_{f=u,d} {\rm Im}\,\Pi^{\mu}_{f\,\mu}(P), \tag{1}$$

where ${\rm Im}\,\Pi^{\mu\nu}_f$ is imaginary part of the photon self energy $\Pi^{\mu\nu}_f$ (see Fig. 2) for flavour f. $\alpha_{\rm EM}$ is the electromagnetic (EM) fine structure constant, q_f is the charge of the fermion of flavor f with $q_u = 2/3$ and $q_d = -1/3$, the four-momentum $P = (p_0, \mathbf{p})$ which is also related to the invariant mass (M) of the lepton pairs as $P^2 = p_0^2 - p^2 = M^2$. We have considered the two lightest flavor quarks (up and down; $N_f = 2$) in the present calculation.

We can use the most general fermionic propagator in a magnetized medium to write down the one loop electromagnetic (EM) polarization tensor or the photon self-energy tensor (for a given flavour f) as

$$\Pi^{\mu\nu}_f(P) = -iN_c q_f^2 \int \frac{d^4K}{(2\pi)^4} {\rm Tr}\left[\gamma^\mu S^B_f(K)\gamma^\nu S^B_f(K-P)\right], \tag{2}$$

where, S^B_f is the Schwinger propagator [11] for arbitrary eB. N_c or the number of color is a resultant of a color trace and hence Tr represents only the Dirac traces.

Eq. 1 is our master equation that we use to estimate the rate of lepton pairs coming out of a magnetised medium.

3 Result

3.1 Effect of magnetic field on the rate

On the left panel of Fig. 3, we display the rate calculated at non-zero eB and compare with that calculated at zero eB (the Born rate). The rate is plotted for $eB = 5\,m_\pi^2$ with $T = 0.15$ GeV and $p_\perp = 0$. There is a clear enhancement in the rate due to the magnetic field at the lower invariant mass range.

On the right panel, we have decomposed the rate into the contributions coming from different processes. The decay of the quark/antiquark which comes into existence because

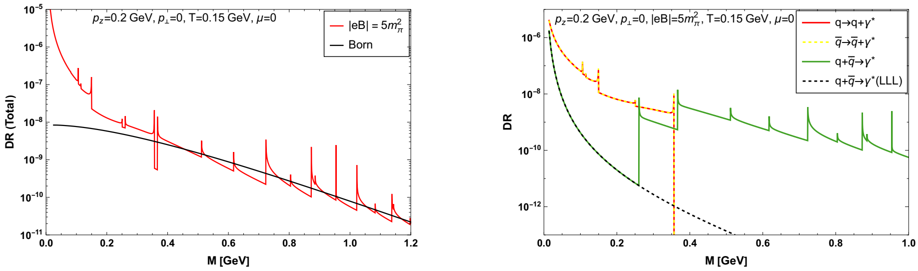

Figure 3. In the left panel we have the plot of DR as a function of invariant mass for $eB = 5\,m_\pi^2$ and compared with the Born rate. In the right panel the contribution coming from different processes are shown separately along with the LLL approximated rate [5].

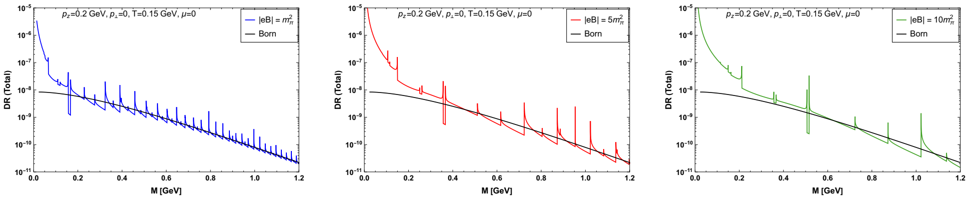

Figure 4. Rate plotted as a function of invariant mass for different values of eB. The figures are obtained for zero values of $p_\perp$.

of the contribution from the Landau cut is a solely magnetic field dependent phenomenon. For zero magnetic field they are off and we have contribution only from the unitary cut or the annihilation process. The decay processes dominate the lower side of the invariant mass, whereas the higher invariant mass range is dominated by the annihilation process. In our opinion, these decays of quark and antiquark in presence of eB are the reasons of the observed enhancement in the rate in the low invariant mass region.

In Fig. 4, we have the rates for different values of magnetic field, starting from m_π^2 to $10\,m_\pi^2$ to understand the effect of increasing magnetic field on the rate. It is obvious from the figure that with higher strength of eB not only the rate gets enhanced but also the enhancement over Born rate moves towards a higher invariant mass range. One obvious question comes into the mind while looking at these plots of rate is that regarding the appearance of the "saw-tooth" like behaviour. The reason of that being the existence of the discretised Landau levels (LL) in the perpendicular direction to the magnetic field. Because of The LL's the dispersion relation possess multiple thresholds and cause those numerous singular spikes.

Finally, in Fig. 5 we obtain the rate for non-zero values of transverse momentum $p_\perp$. The left panel shows the dilepton rate estimated for $p_\perp = 0.15$ GeV and given values of eB and T. It is also compared with the corresponding born rate. The right panel takes a closer view on the effect of $p_\perp$ in the total DR by comparing three different $p_\perp$ values. A mild decrease in the rate with increasing $p_\perp$ is observed. Physically it can be thought of with a picture that favours the emission of lepton pairs along the direction of the magnetic field as opposed to the perpendicular direction.

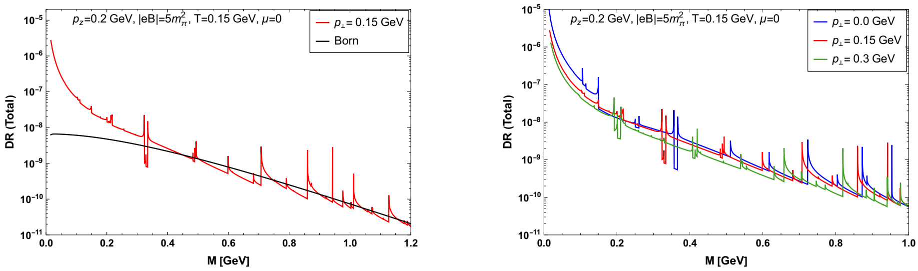

Figure 5. Plot of rate as a function of invariant mass for different values of the transverse momentum.

3.2 In the ambit of an effective model

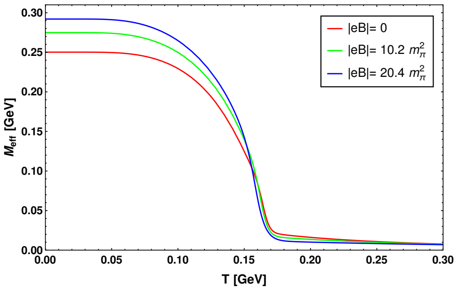

Figure 6. Plot of constituent mass as a function of T for different values of eB.

In this section, we explore the effect of quasi-quarks on the rate. The quasi-quark picture is accommodated through an effective mean field model, namely the Nambu— Jona-Lasinio model [12]. The quark masses are dynamically generated in presence of a background magnetic field. The corresponding Lagrangian is,

$$\mathcal{L}^B_{\rm NJL} = \bar{\psi}(i\not{D} - m_0)\psi + \frac{G_S}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}. \tag{3}$$

From the observation made by the lattice QCD [9] we know that around the region of crossover temperature, T_{CO} the condensate decreases with the increase of magnetic field. This is termed as IMC. We incorporate the IMC effect in the effective model by using a medium dependent coupling constant $G_S(eB, T)$ [13]:

$$G_S(eB, T) = c(eB)\left[1 - \frac{1}{1 + e^{\beta(eB)[T_a(eB)-T]}}\right] + s(eB), \tag{4}$$

where $c(eB)$, $\beta(eB)$, $T_a(eB)$ and $s(eB)$ depend only on the magnitude of B.

The plot of a medium dependent effective mass as a function of temperature for different values of eB is shown in Fig. 6. At low temperature (below T_{CO}) we observe the expected magnetic catalysis (MC) behaviour and around the transition region we get the IMC effect.

In Fig. 7, the total rate has been compared between the two scenarios —in ordinary finite temperature field theory (FTFT) and in the ambit of an effective model, i.e., without and with the effective quark mass M$_{\rm eff}$, respectively. The magnetic field and temperature we used for this figure are $10.2\, m_\pi^2$ and 0.1 GeV, respectively.

We observe that the rate calculated in the effective model is suppressed as compared to that in usual FTFT. Another major difference between the two is the presence of a gap

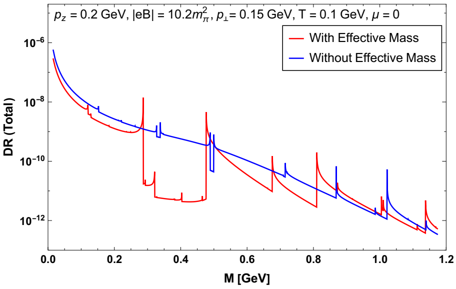

Figure 7. The comparison of rates calculated using basic thermal field theory and in the ambit of an effective model (NJL).

between the rates arising from the decays and annihilation in the effective model scenario. The essential reason of the difference is the mass difference of the quarks between the two scenarios. The quarks simply possess the current masses in the usual FTFT calculation, whereas in the effective model they acquire the effective mass values which depend on the external parameters including the temperature and magnetic field (see Fig. 6). The mass difference is $\sim 10^2$ at the temperature that we considered, which causes the gap in the rate calculated using effective model. In our opinion, this is an interesting observation.

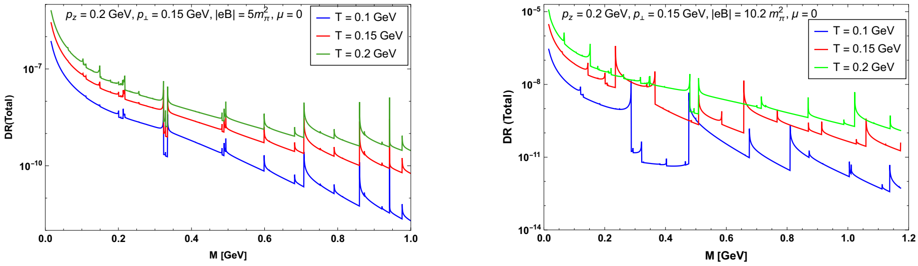

Figure 8. Dilepton rate plotted as a function of invariant mass for three different values of temperatures: in ordinary FTFT scenario (left panel) and in an effective model environment (right panel).

From the above discussion, it is easy to visualise that if we are at a temperature well below T_{CO} ($\approx 100\,$MeV) increase in eB will simply increase the rate along with a wider gap between decay and annihilation rates. On the other hand, for a given value of eB, let's say $10.2m_\pi^2$, if we change the temperature the resulting effects are shown in the right panel of Fig. 8. There we have plotted the DR for three different values of temperatures in the ambit of effective model. This plot is analogous to the plot given in the left panel of that same Fig. 8 but with some considerable differences. The three temperatures are chosen to represent three different regions of the QCD phase diagram — below the transition (0.1 GeV), around the transition (0.15 GeV) and above the transition (0.2 GeV).

For the plot in the left panel (Fig. 8), the current quarks masses are taken into account and differences among the three coloured lines are only in terms of the temperature. On the other hand, in the right panel, three different temperatures also give three distinct values of the effective mass which plays important role in the rate as already explained.

For a given T with similar eB, the rate gets suppressed in presence of an effective mass as compared to the FTFT case. Apart from suppressing the rate in the effective model for a given T, the overall effect of temperature remains the same which is to increase the rate with its increasing values. The other important feature being the appearance of the gap in the rate between the decay processes and the annihilation. With the increase of T the effective mass decreases and so does the width of the gap and at some certain value of T it disappears.

4 Conclusion

The lepton pair coming out of a strongly interacting medium has always been valued as a penetrating probe of the medium. Such emission of lepton pairs from a magnetised hot and dense QCD medium has been investigated in this present exploration.

In our effort, we have relaxed different approximations that were previously considered for the ease of calculation. The strength of the magnetic field was either taken to be strong or weak. For the arbitrary case of magnetic field, different components of the lepton momentum were taken to be zero.

We apply Schwinger method [11] in the imaginary time formalism to estimate the rate in presence of an arbitrary strength of the magnetic field. Different components of the lepton momentum are incorporated with the possibility of them having arbitrary values.

The most encouraging observation from our exploration is that there is an enhancement in the rate in the presence of the magnetic field, particularly at the lower end of the invariant mass. As the strength of the magnetic field is increased the enhancement gets bigger and appears for a larger range of invariant mass.

Because of the presence of the discrete Landau levels (LL), the dispersion relation possess multiple thresholds and cause numerous singular spikes, which we termed as “saw-tooth”. These sharp “saw-tooth” might become smooth in an interacting plasma when strong interactions of quarks are taken into account [14]

As permitted by our generalised framework, we also study the effect of nonzero perpendicular component of the momentum on the DR. We observe that for a given value of magnetic field, the nonzero $p_\perp$ suppresses the DR as compared to zero $p_\perp$ and the suppression is all along the range of the invariant mass. This we interpret as the unwillingness of the leptons in coming out in a direction perpendicular to the magnetic field.

We have also calculated the rate in the ambit of an effective model, namely the NJL. We have discussed about the effect of the medium dependent effective quark mass on the DR and subsequent occurrence of a gap between the decay and the annihilation processes for certain values of the temperature and the external magnetic field. The interesting behaviour of the gap between the decay and annihilation rates at temperatures lower than the phase transition temperature has been explored in detail. The nature of the gap also depends on the values of $p_\perp$, the detail of which can be found in the Ref. [15].

As an outlook, it will be interesting to have an estimation of the dilepton spectrum. That will facilitate to have a direct experimental comparision. Efforts in that direction are in progress and will be reported in near future.

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