

Decay parameters and the axial-vector form factors for strangeness changing as well as strangeness conserving semileptonic decays of the hyperons

Harleen Dahiya^{1,*}, Aarti Girdhar², and Monika Randhawa³,

¹Department of Physics, Dr. B.R. Ambedkar National Institute of Technology, Jalandhar, 144027, India

²Regional Centre for Accelerator-based Particle Physics, Harish-Chandra Research Institute, Chatnaag Road Jhansi, Allahabad 211019, India

³University Institute of Engineering and Technology, Panjab University, Chandigarh, 160014, India

Abstract. We have calculated the axial-vector form factors for the hyperon semileptonic decays $B_i \rightarrow B_f l \bar{\nu}$ in the chiral constituent quark model (χ CQM). The decays considered here are the strangeness changing as well as strangeness conserving semileptonic decays of the hyperons. The conventional dipole form of parametrization has been used to analyse the Q^2 dependence of the axial-vector form factors ($g_i^{B_i B_f}(Q^2)$) as well their decay constants.

1 Introduction

Hyperon semileptonic decays provide a way not only to test the Standard Model but also to understand the intriguing interplay between the weak and strong interactions [1–3]. They provide a tool to test the New Physics through the measurement of form factors and branching fractions of various decays. Experiments to study hyperon decays provide us data on form factors, branching fractions, angular correlations and decay rates for the various decays. In spite of the fact that branching fractions of hyperon decays are of the order of 10^{-4} or smaller, they still provide a crucial channel to study the behavior of strong interactions at low energies.

The axial-vector, $g_{i=1,2,3}$ form factors carry information about the structural properties of hadrons. They provide crucial information on the interesting interplay of the weak interaction (low Q^2) and strong interactions (high Q^2) in addition to being the tools to probe the low energy dynamics of the hadrons. The baryonic weak axial-vector form factors are in particular important parameters that relate the measurements of deep inelastic scattering (DIS) parameters to the spin polarization of the quarks. The experimentally revealed data from the earlier experiments [4] for the vector and axial-vector coupling parameters were analyzed by assuming SU(3) flavor symmetry, however, the experiments performed later showed that the SU(3) symmetry is manifestly broken clearly bringing forward the importance of the SU(3) symmetry breaking. This was first announced by the measurement for the $\Sigma^- \rightarrow ne^- \bar{\nu}_e$ decay giving $|(g_1(0) - 0.133g_2(0))/f_1(0)| = 0.327 \pm 0.007 \pm 0.019$ giving $\frac{g_1(0)}{f_1(0)} = -0.20 \pm 0.08$ and $\frac{g_2(0)}{f_1(0)} = -0.56 \pm 0.37$ [5]. There was a significant difference when compared to SU(3) symmetry results ($\frac{g_1(0)}{f_1(0)} = -0.328 \pm 0.019$ and $\frac{g_2(0)}{f_1(0)} = 0$). The latest experimental data available for $\Sigma^- \rightarrow ne^- \bar{\nu}_e$ decay is $|(g_1(0) - 0.237g_2(0))/f_1(0)| = 0.340 \pm 0.017$ [6].

*e-mail: dahiya@nitj.ac.in

The $\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}_e$ decay measurement and the corresponding form factors were reported by KTeV (Fermilab E799) experiment [7]. They presented $\frac{g_1(0)}{f_1(0)} = 1.32_{-0.17stat}^{+0.21} \pm 0.05_{syst}$ assuming SU(3) symmetry. The data on the same process was provided by the NA48/1 Collaboration [8] with improved statistics. They presented $\frac{g_1(0)}{f_1(0)} = 1.20 \pm 0.05$. The PDG results [6] for $\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}_e$ decay with SU(3) symmetry breaking assumption are $\frac{g_1(0)}{f_1(0)} = 1.22 \pm 0.05$ and $\frac{g_2(0)}{f_1(0)} = -1.7_{-2.0stat}^{+2.1} \pm 0.5_{syst}$. The process corresponding to the isospin partner $\Xi^- \rightarrow \Sigma^0 e^- \bar{\nu}_e$ measured $\frac{g_1(0)}{f_1(0)} = 1.25_{-0.16stat}^{+0.14} \pm 0.05_{syst}$ [9]. The results of $\Lambda \rightarrow pe^- \bar{\nu}_e$ and $\Xi^- \rightarrow \Lambda e^- \bar{\nu}_e$ were also presented whose latest experimental results for $\frac{g_1(0)}{f_1(0)}$ are -0.718 ± 0.015 and -0.25 ± 0.05 respectively [6]. The decays $\Sigma^- \rightarrow \Lambda e^- \bar{\nu}_e$ and $\Lambda \rightarrow pe^- \bar{\nu}_e$ were also presented in Ref. [10, 11]. Further, latest data on hyperons is expected to come from the NA62 experiment at CERN [12], the $p\bar{p}$ collider at PANDA [13] and at FAIR/GSI or J-PARC [14].

Theoretically, chiral perturbation theory (ChPT) has been used to understand SU(3) symmetry breaking [15, 16]. Lattice QCD reported the results for $\Sigma^- \rightarrow ne^- \bar{\nu}_e$ decay [17], $1/N_c$ expansion of QCD also reported the results for hyperon decays [18, 19] but still there is a scope of many refinements. Calculations with SU(3) breaking effects have also been done in the framework of $\frac{1}{N_c}$ expansion of QCD by [18],[19] and using mass splitting interactions in [20]. Several new theoretical and experimental tools have been developed, since the previous calculations of hyperon decay parameters in the chiral constituent quark model (χ CQM) [21–27] where an effective interaction Lagrangian approach is used. The Q^2 dependence of axial form factors has also been attempted in the elastic scattering of neutrinos-antineutrinos [28, 29]. Further it has been estimated in the pion electro-production on the proton [30] and Minerva experiment at Fermilab with higher energy [31] but all the observations hint towards the need for added precise data. In light of the above progress, we feel it becomes worthwhile to study the Q^2 dependence of the axial-vector ($g_i^{B_i B_f}(Q^2)$) form factors in the framework of χ CQM, as well of their decay constants using the conventional dipole form of parametrization. This understanding about the axial-vector form factors of the hyperons would without a doubt provide a benchmark to test the vital aspects of nonperturbative QCD and make our model more feasible to determine the CKM matrix element V_{us} and V_{ud} .

In the present communication we intend to use the axial-vector $g_{i=1,2,3}$ form factors for the semileptonic decays $B_i \rightarrow B_f l \bar{\nu}$ estimated in the χ CQM for the case of hyperons and apply the conventional dipole form of parametrization to explain the Q^2 dependence of the axial-vector form factors ($g_i^{B_i B_f}(Q^2)$) as well as their decay constant. The decays we would like to phenomenologically estimate here are the strangeness changing as well as strangeness conserving semileptonic decays of the hyperons.

2 Methodology

The success of χ CQM model in giving ample description of spin polarization function, baryon magnetic moments, quark distribution functions etc. [25, 32] for hadrons encourages us to further explore this framework. We also analyse the Q^2 dependance of the axial vector form factors. Axial vector form factors, used in this work, have been calculated in the framework of χ CQM [26]. We use the semileptonic baryon decays $B_i \rightarrow B_f l \bar{\nu}$ for our work where B_i and B_f are the initial and final baryon states, l is the charged lepton, can be e or μ , but for our case it is e , $\bar{\nu}_l$ is the corresponding antineutrino. The transition matrix element [33, 34] for this process in the momentum space is given by

$$M = \frac{G_F^2}{\sqrt{2}} V_{q_i q_f} \langle B_f(p_f) | J_h^\mu | B_i(p_i) \rangle \times (\bar{u}_l(p_l) \gamma_\mu (1 - \gamma_5) u_\nu(p_{\nu_l})), \quad (1)$$

where $u_{\bar{\nu}_l}(p_\nu)$ and $\bar{u}_l(p_l)$ are the Dirac spinors of the neutrino and the corresponding lepton, respectively. G_F represents the Fermi coupling constant, the CKM element $V_{q_i q_f}$ corresponds to V_{ud} for the strangeness conserving, $\Delta S = 0$ and V_{us} for the strangeness changing, $\Delta S = 1$ processes. The weak hadronic current, J_h^μ can further be expressed in terms of the vector ($V^{\mu,a} = \bar{\mathbf{q}}\gamma^\mu \frac{\lambda^a}{2} \mathbf{q}$) and axial-vector ($A^{\mu,a} = \bar{\mathbf{q}}\gamma^\mu \gamma_5 \frac{\lambda^a}{2} \mathbf{q}$) currents as $J_h^\mu = J_V^\mu - J_A^\mu$ and we have

$$\langle B_f | J_h^\mu | B_i \rangle = \langle B_f | V^{\mu,a} | B_i \rangle - \langle B_f | A^{\mu,a} | B_i \rangle = \langle B_f | \bar{\mathbf{q}}\gamma^\mu \frac{\lambda^a}{2} \mathbf{q} | B_i \rangle - \langle B_f | \bar{\mathbf{q}}\gamma^\mu \gamma_5 \frac{\lambda^a}{2} \mathbf{q} | B_i \rangle. \quad (2)$$

Here, λ^a are the Gell-Mann matrices of SU(3) relating to the flavor structure of the 3 light quarks. For the strangeness conserving, $\Delta S = 0$ transitions, we have $a = 1 \pm i2$ and for the strangeness changing, $\Delta S = 1$ transitions, we have $a = 4 \pm i5$.

The matrix elements for the axial-vector current is given in terms of the dimensionless axial-vector functions $g_i^{B_i B_f}(Q^2)$ ($i = 1, 2, 3$) expressed as [35, 36]

$$\langle B_f | \bar{\mathbf{q}}\gamma^\mu \gamma_5 \frac{\lambda^a}{2} \mathbf{q} | B_i \rangle = \bar{u}_f(p_f) \left(g_1^{B_i B_f}(Q^2) \gamma^\mu + \frac{g_2^{B_i B_f}(Q^2) i \sigma^{\mu\nu} q_\nu}{M_{B_i} + M_{B_f}} + \frac{g_3^{B_i B_f}(Q^2) q^\mu}{M_{B_i} + M_{B_f}} \right) \gamma^5 \frac{\lambda^a}{2} u_i(p_i). \quad (3)$$

Here M_{B_i} (M_{B_f}) are the masses of the initial (final) baryon states, respectively. The four momenta transfer is given as $Q^2 = -q^2$, where $q \equiv p_i - p_f$. The functions $g_i^{B_i B_f}(Q^2)$ ($i = 1, 2, 3$) are the axial-vector form factors and it is necessary that they are real by G -parity. More specifically, g_1 , g_2 and g_3 are the axial-vector, induced pseudoscalar or weak electricity and the induced pseudoscalar scalar form factors respectively. Since the current with form factor g_2 transforms with the opposite sign under G -parity [37], it is referred to as second-class current, whereas the others as first-class currents. According to the Ademollo-Gatto theorem, g_1 can be expressed in terms of the Clebsch-Gordan coefficients which give the axial-vector coupling constants respectively when the momentum transfer tends to zero. Further, at the quark level only the first class currents corresponding to the $d \rightarrow u$ transitions occur, the magnitude of second class currents is very small.

In order to determine the axial-vector functions at $Q^2 = 0$, it is often convenient to introduce the Sachs-type form factors directly related to the time and space components of the axial-vector currents [35, 38]. The axial-vector functions can then be expressed in terms of Sachs form factor $G_{A,0}^{B_i B_f}$ (calculated from the time component $\langle B_f | A^{0,a} | B_i \rangle$) and form factors $G_{A,S}^{B_i B_f}$ and $G_{A,T}^{B_i B_f}$ (calculated from the space component $\langle B_f | A^{i,a} | B_i \rangle$).

Starting from the QCD Lagrangian describing the dynamics of light quarks (u , d , and s), the chiral symmetry is believed to be spontaneously broken around a scale of 1 GeV and a set of massless particles identified with the observed Goldstone bosons (GBs) exist (π , K , η mesons). The effective interaction Lagrangian in this region can then be expressed as

$$\mathcal{L}_{\text{int}} = c_8 \bar{\psi} \Phi \psi + c_1 \bar{\psi} \frac{\eta'}{\sqrt{3}} \psi = c_8 \bar{\psi} \left(\Phi + \zeta \frac{\eta'}{\sqrt{3}} I \right) \psi = c_8 \bar{\psi} (\Phi') \psi, \quad (4)$$

where $\zeta = c_1/c_8$, c_1 is the coupling constant for the singlet GB. The fluctuation process describing the effective Lagrangian in the χ CQM [21] is

$$q^\pm \rightarrow \text{GB} + q'^\mp \rightarrow (q\bar{q}') + q'^\mp, \quad (5)$$

where $q\bar{q}' + q'$ constitute the sea quarks [22, 23, 25]. In the presence of a pseudoscalar field Φ' , the χ CQM axial-vector current is given as

$$J_{A,q\bar{q}'}^\mu = g_a \bar{q}' \gamma^\mu \gamma_5 q - f_\Phi \partial^\mu \Phi', \quad (6)$$

where g_a is the quark axial-vector current coupling constant and can be taken 1 as argued by Weinberg [37], f_{Φ} is the pseudoscalar decay constant.

In order to compare our results with the experiments at higher Q^2 , we need to see how form factors we obtained at $Q^2 = 0$ get modified by their Q^2 dependence. For a low and moderate momentum transfer $Q^2 \leq 1$, the dipole form of parametrization has been conventionally used to analyse the axial-vector form factors

$$g_i^{B_i B_f}(Q^2) = \frac{g_i^{B_i B_f}(0)}{\left(1 + \frac{Q^2}{M_{g_i}^2}\right)^2}, \quad (7)$$

where $g_i^{B_i B_f}(0)$ are the axial-vector coupling constants at zero momentum transfer. Here M_{g_i} are the dipole masses for axial vector. This parameterization in addition to providing correct low energy behaviour also gives correct asymptotic limit $G_A \propto \frac{1}{Q^4}$ and $G_P \propto \frac{1}{Q^6}$ at large momentum transfer. We consider in this work the semileptonic hyperon decays $\Sigma^- \rightarrow n e^- \bar{\nu}_e$, $\Xi^- \rightarrow \Sigma^0 e^- \bar{\nu}_e$, $\Xi^- \rightarrow \Lambda e^- \bar{\nu}_e$, $\Lambda \rightarrow p e^- \bar{\nu}_e$, $\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}_e$ corresponding to the strangeness conserving decays and $n \rightarrow p e^- \bar{\nu}_e$, $\Sigma^- \rightarrow \Sigma^0 e^- \bar{\nu}_e$, $\Sigma^- \rightarrow \Lambda e^- \bar{\nu}_e$, $\Sigma^+ \rightarrow \Lambda e^- \bar{\nu}_e$, $\Xi^- \rightarrow \Xi^0 e^- \bar{\nu}_e$ corresponding to the strangeness changing decays. These form factors are calculated in at low Q^2 using generalized Sachs form factors at $Q^2 \sim 0$ (details can be found in [26] and the references therein).

3 Inputs

In order to numerically calculate the axial-vector form factors, we must fix the symmetry breaking parameters defined in terms of the transition probabilities for the fluctuations of quarks which measure the contribution of the quark sea in a particular baryon. These parameters have been obtained by carrying out a best fit analysis for the case of experimentally well known spin and flavor distribution functions of the octet baryons. This fitting has already been discussed in detail in Ref. [26] and we will use the same set of parameters in the present work.

The other set of parameters used in the calculations of the axial-vector form factors are the baryon masses. The masses corresponding to the initial and final baryons have been given in Tables 2 and 3 for each decay. Further to analyse the running axial-vector form factors Q^2 upto 1 GeV from Eq. (7) we need the value of M_A/M_{g_i} . The values corresponding to the $\Delta S = 0$ decays have been taken from Ref. [19] and for $\Delta S = 1$ decays from Ref. [1]. For ready reference, the values have been summarized in Table 1.

Table 1. Input parameters used for Q^2 dependence.

	$\Delta S = 0$ decay	$\Delta S = 1$ decay
M_V/M_{g_i}	0.84 ± 0.04 GeV	0.97 GeV

4 Axial-Vector Form factors

Using the input parameters discussed above, in Table 2 we present the results for the axial-vector form factors for the strangeness changing decays. Similarly, in Table 3, we present the results for form factors for the strangeness conserving decays. The results in Tables 2 and 3 are valid at $Q^2 = 0$ and include SU(3) symmetry breaking effects. We present the axial-vector (g_1) and the induced pseudotensor (g_2) form factors. As we discussed earlier

the second class currents are expected to have a smaller contribution when compared to first class currents. This is true for all the strangeness changing and strangeness conserving decays presented in Tables 2 and 3. Further, from the results of the second class current g_2 in Table 3, it is clear that the magnitude of g_2 is very small for the case where the initial and final baryons come from the same isospin multiplets. This is because of the small mass difference between the initial and final decay particles when from the same isospin multiplet. The decays $n \rightarrow pe^- \bar{\nu}_e$, $\Sigma^- \rightarrow \Sigma^0 e^- \bar{\nu}_e$ and $\Xi^- \rightarrow \Xi^0 e^- \bar{\nu}_e$ clearly support this finding. For all the other cases, the second class current g_2 also contributes and the contribution is more when the mass difference between the initial and final decay particles is more.

For the case of axial-vector form factor g_1 we find from Table 2 and 3

$$g_1(\Lambda \rightarrow p) < g_1(\Xi^- \rightarrow \Lambda) < g_1(\Sigma^- \rightarrow n) < g_1(\Xi^- \rightarrow \Sigma^0) < g_1(\Xi^0 \rightarrow \Sigma^+), \quad (8)$$

$$g_1(\Xi^- \rightarrow \Xi^0) < g_1(\Sigma^+ \rightarrow \Lambda) = g_1(\Sigma^- \rightarrow \Lambda) < g_1(\Sigma^- \rightarrow \Sigma^0) < g_1(n \rightarrow p). \quad (9)$$

For the case of induced pseudotensor or weak electricity form factor g_2 we find from Table 2 and 3

$$g_2(\Lambda \rightarrow p) < g_2(\Sigma^- \rightarrow n) < g_2(\Xi^- \rightarrow \Lambda) < g_2(\Xi^- \rightarrow \Sigma^0) < g_2(\Xi^0 \rightarrow \Sigma^+), \quad (10)$$

$$g_2(\Sigma^- \rightarrow \Lambda) < g_2(\Sigma^+ \rightarrow \Lambda) < g_2(\Sigma^- \rightarrow \Sigma^0) < g_2(\Xi^- \rightarrow \Xi^0) < g_2(n \rightarrow p). \quad (11)$$

5 Q^2 Dependence in Axial-Vector Form Factors

In Figs. 1 and 2, we have presented the dependence of g_1 and g_2 for the strangeness conserving $\Delta S = 0$ and the strangeness changing $\Delta S = 1$ decays for finite $0 \leq Q^2 \leq 1$. It is well known that the sea quarks dominate in the low Q^2 region whereas the valence quarks dominate at high Q^2 . Broadly speaking, it is clear from the plots that the Q^2 dependence is more profound at low values of Q^2 than at higher values. As Q^2 approaches 1, g_1 and g_2 approach 0. This clearly indicates the significance of the quark sea in understanding its underlying dynamics as well as to have a deeper knowledge of the internal structure of baryons in terms of the constituent quarks. The dependence of masses is also clearly reflected in the plots from the values of form factors at $Q^2 = 0$ which directly depend on the baryon and quark masses. The magnitude is large for the cases where the mass difference between the initial and final particles is more. This dependence is not much different for $\Delta S = 0$ and $\Delta S = 1$ particles. The magnitude of the form factors decrease with the increasing value of Q^2 and approach 0. Another important observation in the plots is that higher the value of the form factors at $Q^2 = 0$, faster is the drop in the value with increasing Q^2 . For the form factors having lower magnitude at $Q^2 = 0$, the variation with increasing Q^2 is very small. As we move toward higher Q^2 , the quark sea contribution falls off and the form factors get contribution only from the constituent quarks.

From the plots, for the case of axial-vector form factors $g_1(Q^2)$ presented in Fig. 1, we note that for $\Delta S = 1$ decays the form factors corresponding to $\Sigma^- \rightarrow ne^- \bar{\nu}_e$, $\Xi^- \rightarrow \Lambda e^- \bar{\nu}_e$, $\Xi^- \rightarrow \Sigma^0 e^- \bar{\nu}_e$ and $\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}_e$ fall with increasing Q^2 whereas for $\Lambda \rightarrow pe^- \bar{\nu}_e$ it increases. The value of g_1 at $Q^2 = 0$ for $\Sigma^- \rightarrow ne^- \bar{\nu}_e$ and $\Xi^- \rightarrow \Lambda e^- \bar{\nu}_e$ is small so the values drop slowly. On the other hand, for $\Delta S = 0$ decays, g_1 decreases with Q^2 for all decays. For the induced pseudotensor form factors $g_2(Q^2)$ presented in Fig. 2, for the $\Delta S = 1$ decays the form factors are very small for $\Sigma^- \rightarrow ne^- \bar{\nu}_e$ and $\Xi^- \rightarrow \Lambda e^- \bar{\nu}_e$ so the variation is negligible. For $\Lambda \rightarrow pe^- \bar{\nu}_e$, the form factor increase with increasing Q^2 whereas for $\Xi^- \rightarrow \Sigma^0 e^- \bar{\nu}_e$ and $\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}_e$ it decreases. For the $\Delta S = 0$ decays, the form factors for $\Sigma^+ \rightarrow \Lambda e^- \bar{\nu}_e$ and $\Sigma^- \rightarrow \Lambda e^- \bar{\nu}_e$ increase with increasing Q^2 whereas others having very small values at $Q^2 = 0$

increase very slowly. The quark sea contributes negligibly in the low Q^2 region as the mass difference between the incoming and outgoing particles is small and the explicit contributions coming from the quark sea cancel each other.

Table 2. The decay constants $g_1(Q^2 = 0)$ and $g_2(Q^2 = 0)$ for the strangeness changing decays.

Decay	$M_i(\text{GeV})$	$M_f(\text{GeV})$	g_1	g_2
$\Sigma^- \rightarrow ne^- \bar{\nu}_e$	1.197	0.939	0.314	0.017
$\Xi^- \rightarrow \Sigma^0 e^- \bar{\nu}_e$	1.321	1.192	0.898	0.310
$\Xi^- \rightarrow \Lambda e^- \bar{\nu}_e$	1.321	1.116	0.262	0.047
$\Lambda \rightarrow pe^- \bar{\nu}_e$	1.116	0.938	-0.909	-0.170
$\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}_e$	1.315	1.189	1.27	0.446

Table 3. The decay constants $g_1(Q^2 = 0)$ and $g_2(Q^2 = 0)$ for the strangeness conserving decays.

Decay	$M_i(\text{GeV})$	$M_f(\text{GeV})$	g_1	g_2
$n \rightarrow pe^- \bar{\nu}_e$	0.939	0.938	1.270	-0.004
$\Sigma^- \rightarrow \Sigma^0 e^- \bar{\nu}_e$	1.197	1.192	0.676	-0.010
$\Sigma^- \rightarrow \Lambda e^- \bar{\nu}_e$	1.197	1.116	0.646	-0.152
$\Sigma^+ \rightarrow \Lambda e^- \bar{\nu}_e$	1.189	1.116	0.646	-0.136
$\Xi^- \rightarrow \Xi^0 e^- \bar{\nu}_e$	1.322	1.315	0.314	-0.007

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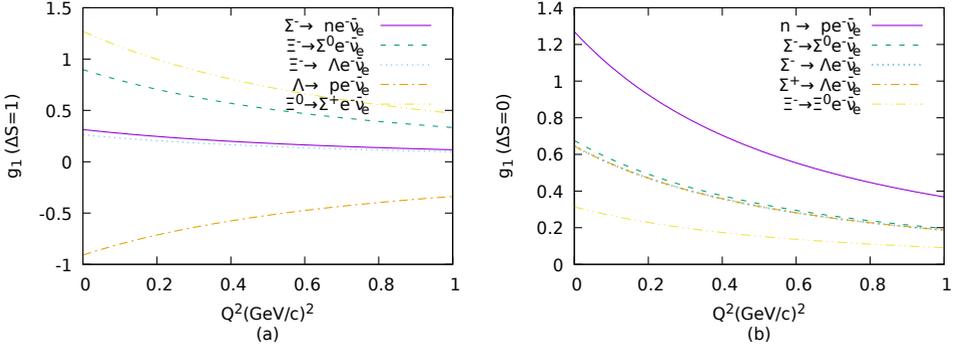


Figure 1. Variation of $g_1(Q^2)$ with Q^2 for $\Delta S = 1$ and $\Delta S = 0$ decays.

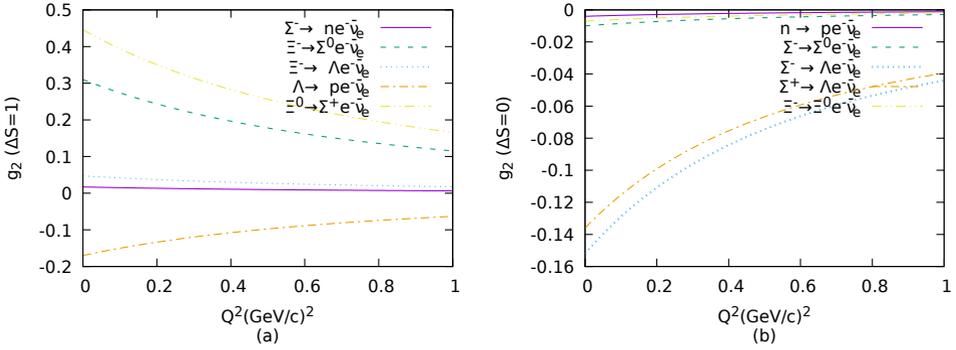


Figure 2. Variation of $g_2(Q^2)$ with Q^2 for $\Delta S = 1$ and $\Delta S = 0$ decays.