

Anomalies in the heavy sector

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Abstract. Heavy flavour physics is well described by the Standard Model with the exception of some measurements which could be signalling new physics. We briefly review status and prospects of $R(K)$ and $R(D^{(*)})$ anomalies and also address the issue of the long standing tension in the $|V_{cb}|$ and $|V_{ub}|$ exclusive/inclusive determinations.

1 Introduction

In a general scenario of optimal agreement within the Standard Model (SM), the heavy flavour physics sector exhibits some measurements which present non-significant but intriguing tensions with SM predictions. Here we briefly address the discrepancy between measured ratios in semileptonic B decays and the corresponding SM predictions, which may hint at lepton flavour non-universality or lepton flavour violation.

Semileptonic B decays are also the processes of election when it comes to a precise determination of the parameters $|V_{xb}|$ of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Their precise measurement allows a powerful check of the unitarity of the CKM matrix. However, there is long-standing tension among the $|V_{xb}|$ values, depending on whether they are extracted using exclusive or inclusive semi-leptonic B decays. We briefly review recent progress on $|V_{xb}|$ determinations, prompted by the larger and larger samples of B mesons collected at the B -factories and at LHCb, and by concomitant progress in theoretical computations¹.

2 $R(D)$ and $R(D^*)$

Exclusive semi-tauonic B decays were first observed by the Belle Collaboration in 2007 [6]. Subsequent analyses by Babar and Belle [7–9] measured branching fractions above, although consistent with, the SM predictions. The ratio of branching fractions (the denominator is the average for $l \in \{e, \mu\}$)

$$R(D^{(*)}) \equiv \frac{\mathcal{B}(B \rightarrow D^{(*)} \tau \nu_\tau)}{\mathcal{B}(B \rightarrow D^{(*)} l \nu_l)} \quad (1)$$

is typically used instead of the absolute branching fraction of $B \rightarrow D^{(*)} \tau \nu_\tau$ decays to cancel uncertainties common to the numerator and the denominator. These include the CKM matrix

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¹For brief overviews see for example [1–5] and references therein.

element $|V_{cb}|$ and several theoretical uncertainties on hadronic form factors and experimental reconstruction effects. Since the strong force does not couple directly to leptons, its effect on the semileptonic decays in different lepton families is identical, aside from the difference in the phase space. The ratio between the branching fractions of these decays is therefore predicted with $O(1\%)$ precision. In 2012–2013 Babar has measured $R(D^{(*)})$ by using its full data sample [10, 11], and reported a significant excess over the SM expectation, confirmed in 2015 by LHCb [12]. In 2016 such excess has been confirmed also by the Belle collaboration, which has performed the first measurement of $R(D^*)$ using the semileptonic tagging method, giving [13]

$$R(D^*) = 0.302 \pm 0.030 \pm 0.011 \quad (2)$$

where the first error is statistic and the second one is systematic. By selecting the previous measurements in the time-span 2012–2016 [9–13], in 2016 the HFLAV Collaboration found the average [14]

$$R(D) = 0.397 \pm 0.040 \pm 0.028 \quad R(D^*) = 0.316 \pm 0.016 \pm 0.010 \quad (3)$$

where the first uncertainty is statistical and the second is systematic. $R(D)$ and $R(D^*)$ exceed the SM value $R(D)^{SM} = 0.300 \pm 0.008$, given by the HPQCD collaboration [15], and the SM value $R(D^*)^{SM} = 0.252 \pm 0.003$ [16] by 1.9σ and 3.3σ , respectively. The combined analysis of $R(D)$ and $R(D^*)$, taking into account measurement correlations, finds that the deviation is 4σ from the SM prediction.

Fast forward to 2022, the HFLAV analysis has added to the previous results from Babar, Belle and LHCb [9–12] the most recent results from Belle [17–19] and LHCb [20–22]. The HFLAV determination finds lower average values and smaller errors, namely [23]

$$R(D) = 0.358 \pm 0.025 \pm 0.012 \quad R(D^*) = 0.285 \pm 0.010 \pm 0.008 \quad (4)$$

New SM calculations have become available since the 2016. The results are compatible with the previous ones, although generally more accurate or robust. The HFLAV has performed their arithmetic average resulting in [23] $R(D)^{SM} = 0.298 \pm 0.004$ and $R(D^*)^{SM} = 0.254 \pm 0.005$. The experimental averages (4) exceed these SM values by 2.16σ and 2.26σ , respectively. The combined analysis of $R(D)$ and $R(D^*)$ finds that the deviation is 3.2σ from the SM prediction.

The actual recorded luminosity at Belle II is 428 fb^{-1} , to be compared with Belle (988 fb^{-1}) and BaBar (513 fb^{-1}); the ultimate goal is to reach 50 ab^{-1} . Investigating the anomaly through precision measurements of $R(D^{(*)})$ is a chief goal of Belle II. At Belle II a better understanding of backgrounds tails under the signal and a reduction of the uncertainty to 3% for $R(D^*)$ and 5% for R_D is expected at 5 ab^{-1} [24].

Belle II has the unique potential to measure the ratio $R(X)$ of the inclusive rate $B \rightarrow X\tau\nu$ to the lower-mass lepton counterparts. This ratio would probe both electron and muon modes with a precise consistency check whose phenomenological interpretation is independent from LQCD uncertainties that affect the other observables. However, this is a challenging measurement attempted and never completed at previous B-factory experiments. The main experimental challenge is to control the significant systematic uncertainties associated with background composition [24]. Belle II is preparing and this year it has measured the ratio $R(X_{e/\mu})$ of the inclusive rate $B \rightarrow X_e\nu$ to $B \rightarrow X_\mu\nu$, which is in good agreement with the SM [25].

The B-factories have performed the most precise measurements of $R(D)$ and $R(D^*)$ to date thanks to their ability to significantly constrain the kinematics of these neutrinos by leveraging their knowledge of the e^+e^- collision energy. LHCb, however, is expected to

surpass the B-factories precision and reach uncertainties down to the percent level from the analysis of the enormous data samples expected from the operation of the Upgrades I and II detectors, provided that the relevant systematic uncertainties can be properly controlled [26].

Other ratios are currently under investigation. LHCb has found the value [27] $R(J/\Psi) = 0.71 \pm 0.17 \pm 0.18$, which is 2.1σ from the LQCD value [28] $R(J/\Psi)^{\text{HPQCD}} = 0.2582 \pm 0.0038$. Another LHCb measurement gives [29]

$$R(\Lambda_c) \equiv \frac{\mathcal{B}(\Lambda_b \rightarrow \Lambda_c \tau \nu_\tau)}{\mathcal{B}(\Lambda_c \rightarrow \Lambda_c \mu \nu_\mu)} = 0.242 \pm 0.026 \pm 0.040 \pm 0.059 \quad (5)$$

where the external branching fraction uncertainty from the channel $\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu$ contributes to the last term. The last ratio seems can be related to the values of $R(D)$ and $R(D^*)$ [30].

3 $R(K)$ and $R(K^*)$

While $R(B)$ is defined as the ratio of branching fractions of decays that occur at tree level in the SM at the lowest perturbative order, the observable $R(K)$ and $R(K^*)$ are defined as the ratios of branching fractions of rare decays. In general, for semileptonic decays of B hadrons, where $B = B^+, B^0, B_s^0$ or Λ_b^0 , the ratio $R(H)$ is defined in the dilepton mass-squared range $q_{\min}^2 < q^2 < q_{\max}^2$ as

$$R(H) = \frac{\int_{q_{\min}^2}^{q_{\max}^2} \frac{d\mathcal{B}(B \rightarrow H \mu^+ \mu^-)}{dq^2} dq^2}{\int_{q_{\min}^2}^{q_{\max}^2} \frac{d\mathcal{B}(B \rightarrow H e^+ e^-)}{dq^2} dq^2} \quad (6)$$

where H can be an hadron as K or a combination of particles such as a proton and charged kaon, pK^- .

LHCb has provided the most precise measurements of the branching fractions of the $B^+ \rightarrow K^{(*)+} \mu^+ \mu^-$ and $B^0 \rightarrow K^{(*)0} \mu^+ \mu^-$ decays [31–33]. All these measurements are below the SM prediction. At LHCb, $R(K)$ has been measured to be [34]

$$R(K^+) = 0.745_{-0.074}^{+0.090} \pm 0.036 \quad (7)$$

where the first error is statistical and the second one is systematic. The measurement was performed across the dilepton invariant-mass-squared range $[1,6] \text{ GeV}^2$. This result is 2.6σ deviations away from the SM prediction $R(K)^{\text{SM}} = 1.0003 \pm 0.0001$ [35]. The impact of radiative corrections has been estimated not to exceed a few % [36]. The result in Eq. (7) has been superseded in 2021 by an LHCb analysis which uses essentially identical techniques but an additional 4 fb^{-1} of data collected in 2017 and 2018 [37]. Hence, the entire amount of data was recorded during the years 2011, 2012 and 2015–2018, in which the centre-of-mass energy of the collisions was 7, 8 and 13 TeV, and correspond to an integrated luminosity of 9 fb^{-1} . The analysis [37] gives, in the dilepton invariant-mass-squared range $[1,6] \text{ GeV}^2$, the value

$$R(K^+) = 0.846_{-0.039-0.012}^{+0.042+0.013} \quad (8)$$

where the first uncertainty is statistical and the second systematic. Combining the uncertainties they obtain $R(K^+) = 0.846_{-0.041}^{+0.044}$. There are previous measurements by Babar [38] and Belle [39], but the LHCb measurement is the most precise to date and is consistent with the SM expectation at the level of 3.1 standard deviations.

The other ratio measurements from LHCb include $R(K^{*0})$ with the Run 1 data [40] in two regions of the dilepton invariant mass squared; the value found for the interval $[0.045, 1.1] \text{ GeV}^2$ is

$$R(K^{*0}) = 0.66_{-0.07}^{+0.11} \pm 0.03 \quad (9)$$

and the value found for the interval $[1.1, 6.0] \text{ GeV}^2$ is

$$R(K^{*0}) = 0.69^{+0.11}_{-0.07} \pm 0.05 \quad (10)$$

These results, which represent the most precise measurements of $R(K^{*0})$ to date, are compatible with the Standard Model expectations at the level of 2.1-2.3 and 2.4-2.5 standard deviations (depending on the theoretical predictions) in the two q^2 regions. Previous results from Babar [38] and Belle [41] are also available. LHCb current results are reported in figure 1, where also the relevant dilepton invariant mass squared interval is indicated.

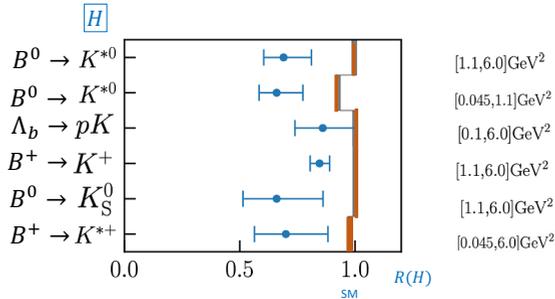


Figure 1. Current LHCb values for semileptonic ratios $R(H)$, where the decaying particle is also indicated. The first two results are taken from [40], the baryon decay has been analyzed in [42], $R(K^+)$ in [37], and the last two measurements in [43].

4 Data interpretation

The alleged breaking of lepton-flavour universality suggested by most of the data is quite large, and several theoretical models have been tested against the experimental results. A model independent approach, where the SM is considered as an effective low-energy theory, is mostly used. The SM Effective Theory (SMEFT) is the effective theory above the electroweak (EW) scale, incorporating the full SM content, with non renormalizable operators which are gauge-invariant under the SM gauge group. The main assumption inherent to the SMEFT framework is that new physics interactions have been integrated out at some high scale. The SMEFT aims at characterizing experimental deviations from the predictions of the Standard Model and pointing towards the structures of its possible extensions above the EW scale. The Weak Effective Theory (WET) is the effective theory valid below the EW scale which includes all SM particles except the gauge bosons, the Higgs and the t quark. In contrast to the SMEFT case, the dimension-six operators in WET are not invariant under the full SM gauge group, but only under $SU(3)_c \times U(1)_{em}$, as this EFT is defined below the EW scale where $SU(2)_L \times U(1)_Y$ is already broken. For a given set of SMEFT dimension-six operators with the corresponding Wilson coefficients specified at the higher scale, one has to perform the renormalization group evolution of the SMEFT Wilson coefficients down to the EW scale, match the given set of SMEFT operators onto the WET ones at the EW scale and finally perform the RGE of the WET Wilson coefficients from the EW down to the dynamical scale of the B meson. This way, one can then bridge the gap between the SMEFT Lagrangian and the low-energy measurements in B physics.

New physics effects in semileptonic decays into light quarks are stringently constrained. For instance the ratio [44]

$$\frac{\mathcal{B}(B^0 \rightarrow D^{*-} e^+ \nu_e)}{\mathcal{B}(B^0 \rightarrow D^{*-} \mu^+ \nu_\mu)} = 1.01 \pm 0.01 \pm 0.03 \quad (11)$$

is in agreement with the SM. Hence new physics effects are generally assumed in semitauonic B decays. The relevant terms of the WET Lagrangian for the underlying $b \rightarrow c \tau \nu$ transition are [45–47]

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{cb} \left[(1 + C_V^L) O_V^L + C_S^R O_S^R + C_S^L O_S^L + C_T O_T \right] + \text{h.c.} \quad (12)$$

where G_F is the Fermi constant, V_{cb} is an element of the CKM matrix and the dimension-six four-fermion operators are

$$\begin{aligned} O_V^L &= (\bar{c} \gamma^\mu P_L b) (\bar{\tau} \gamma_\mu P_L \nu_\tau), & O_S^R &= (\bar{c} P_R b) (\bar{\tau} P_L \nu_\tau), \\ O_T &= (\bar{c} \sigma^{\mu\nu} P_L b) (\bar{\tau} \sigma_{\mu\nu} P_L \nu_\tau), & O_S^L &= (\bar{c} P_L b) (\bar{\tau} P_L \nu_\tau). \end{aligned}$$

Non-zero Wilson coefficients relate to the possible tree-level NP mediators: (i) exchange of an heavy W' boson with left-handed couplings. Simplified models (spin-1 colorless weak triplet, two Higgs doublet models, a spin-0 or spin-1 leptoquark) are in tension with existing $\tau^+ \tau^-$ LHC results [48]. These constraints do not apply to the low mass region in the W' and vector leptoquark models; (ii) exchange of a charged Higgs boson. This scenario is in tension with the LHC mono- τ data and it can be related to (large) branching ratios for $B_c \rightarrow \tau \nu$ decays. (iii) leptoquark scenarios. They generally evade the LHC mono- τ tests, but they are subject to LHC constraints from their pair-production and t-channel mediated di-lepton processes. Other constraints may come from characteristics of the models, as for instance induced CP violations or the presence of colour-octet resonances, often introduced together in UV-complete models. In general, and in all the above models, more severe constraints can appear in concrete UV completions.

The $b \rightarrow s \ell^+ \ell^-$ transitions are described by WET Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb}^* V_{ts} \frac{e^2}{16\pi^2} \sum_i (C_i O_i + C_i' O_i') + \text{h.c.}, \quad (13)$$

where the operators most sensitive to new physics are:

$$O_7^{(\prime)} = \frac{m_b}{e} (\bar{s} \sigma_{\mu\nu} P_{R(L)} b) F^{\mu\nu} \quad O_{9,10}^{(\prime)} = (\bar{s} b)_{V\mp A} (\bar{\ell} \ell)_{V,A} \quad (14)$$

and their corresponding Wilson coefficients have contributions from the SM processes as well as from new physics ones. Several global fits have been performed in the literature (see for instance [49–55]). They can be fully data driven, which means that no assumption is done about charming penguins, or partly or fully model dependent, assuming assume LCSR results for charming penguins. There is a general agreement among the fits, which show that even the simple one-dimensional new physics scenarios, such as $C_9^{\text{NP}\mu} \simeq -0.73$ or $C_9^{\text{NP}\mu} = -C_{10}^{\text{NP}\mu} \simeq -0.39$, can lead to a significant improvement of the quality of the fit, compared to the SM-only hypothesis. Mainstream models are loop-induced new physics, as well as tree-level new physics contributions, mediated by Z' gauge bosons or leptoquarks. At a variance with the $b \rightarrow c$ anomalies, the new physics scale is found of order 40 TeV, so one cannot check direct production of new particles at the (HL-)LHC. There is a limited reach for searches for

deviations from the SM in high p_T di-muon tails. The couplings are stringently constrained by B_s^0 meson mixing data. One has also to remark that vector leptoquarks can provide a unique solution for both $b \rightarrow c$ and $b \rightarrow s$ anomalies. Last but not least, the discrepancies with SM predictions have also generated alternative data analyses, as for example the Dispersive Matrix method, claiming the values $R(D) = 0.296 \pm 0.008$ and $R(D^*) = 0.261 \pm 0.020$, which differ by about 1.4σ from the latest experimental determinations [56].

5 $|V_{xb}|$ determination

Improving our knowledge of $|V_{cb}|$ and $|V_{ub}|$ and reducing their uncertainty are crucial to the so-called $|V_{cb}|$ and $|V_{ub}|$ puzzles, the longstanding tension between the values obtained from exclusive and inclusive determinations. The latest global analysis of the inclusive $B \rightarrow X_c \ell \nu$ by HFLAV [57] gives $|V_{cb}^{\text{incl}}| = (42.19 \pm 0.78) \times 10^{-3}$. It has been done in the framework of kinetic scheme, where $|V_{cb}|$ is extracted together with the b and c quark masses and 4 non-perturbative parameters (namely μ_π^2 , μ_G^2 , ρ_D^3 and ρ_{LS}^3). Details on the extraction can be found for instance in Ref. [5]. More recent results are generally consistent with the HFLAV analysis, as [58] $|V_{cb}^{\text{incl}}| = (42.16 \pm 0.50) \times 10^{-3}$, or the first determination using moments of the dilepton invariant mass [59] $|V_{cb}^{\text{incl}}| = (41.69 \pm 0.63) \times 10^{-3}$. One can compare these results with the latest FLAG determination for exclusive decays [60] $|V_{cb}^{\text{excl}}| = (39.36 \pm 0.68) \times 10^{-3}$. In the case of Ref. [58], for instance, one finds a 2.6σ difference. At a variance with previous determinations, in the dispersion matrix approach [61] the $|V_{cb}|$ inclusive/exclusive values are compatible within the 1σ level. Novel approaches have been recently put forward, that allow one to address inclusive decays in LQCD, see for instance Ref. [62] and references therein.

In order to extract $|V_{ub}|$ from semileptonic $B \rightarrow X_u \ell \nu$ decays one has to reduce the $b \rightarrow c$ semileptonic background through experimental cuts. Such cuts enhance the relevance of the so-called threshold region in the phase space, jeopardizing the use of the heavy quark expansion. In order to face this problem, that is absent in the inclusive determination of $|V_{cb}|$, different theoretical schemes have been devised, which are tailored to analyze data in the threshold region, but differ in their treatment of perturbative corrections and the parametrization of non-perturbative effects. The value of $|V_{ub}|$ has been extracted by BaBar [63, 64], Belle [65–67] and HFLAV [68] collaborations. It is based on the measured partial branching fractions value with the state-of-the-art theory predictions on decay rate in four different theoretical different approaches: ADFR [69–71], BLNP [72–74], DGE [75] and GGOU [76]. The arithmetic average of the most precise determinations for the phase-space region $E_\ell^B > 1$ gives (Belle 2022 [77]) $|V_{ub}^{\text{incl}}| = (4.10 \pm 0.09 \pm 0.22 \pm 0.15) \times 10^{-3}$, where the uncertainties are statistical, systematic and from the theory calculation, respectively. This can be compared with the world average of exclusive results $|V_{ub}^{\text{excl}}| = (3.67 \pm 0.09 \pm 0.12) \times 10^{-3}$. The 2022 Belle inclusive value is smaller than the previous inclusive ones, reducing the discrepancy with the exclusive measurement of about 2-3 to 1.3 standard deviations. It is also compatible with the value expected from CKM unitarity from a global fit [78] $|V_{ub}| = (3.62_{-0.08}^{+0.11}) \times 10^{-3}$ within 1.6 standard deviations.

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