Finite angle effects in jet quenching

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Abstract. The medium that forms in a heavy-ion collision modifies the properties of jets traversing it. These modifications give substantial information about the nature of the medium and, therefore, they are one of the main focuses of the heavy-ion program at LHC. The influence of the medium into highly energetic partons depends on correlators of Wilson lines, which have been studied in perturbation theory and in several phenomenological models. Here, we focus on the antenna configuration, the study of the evolution of the large energy partons resulting from a collinear splitting inside of the medium. In the eikonal limit, the interaction of each daughter parton with the medium can be parametrized by a light-like Wilson line in a given direction. We consider the case in which the angle between these two directions θ is small but such that $L\theta$ is of the order of the medium resolution scale, where L is the size of the medium. We find that finite angle effects give sub-leading corrections to jet broadening. The physical effect in the antenna configuration is to make the decoherence effect larger when finite angle corrections are taken into account.

1 Introduction

A consequence of asymptotic freedom is that at very large temperatures and densities quarks and gluons are not confined within hadrons anymore. This new state of matter is called the quark-gluon plasma (QGP). The only way that we can create this state, that was present in the early universe, on earth is by colliding relativistic heavy ions. However, it is very challenging to learn about the properties of the QGP by observing the particles produced in these collisions. The reason is that the QGP exists only during a time much shorter than what it takes the particles to hit the detectors. Therefore, we can only measure the indirect influence of the QGP.

We need well calibrated probes to be able to characterize the QGP. A particularly useful type of probes are those known as hard probes. These are particles whose production need a very large energy and, therefore, they can only be produced at the beginning of the collision in a hard process before the particles have time to thermalize. However, they are affected in a substantial way by the QGP that is formed afterwards. Then, by comparing the results in heavy ion collisions for these probes with the naive extrapolation of proton-proton data we can gain a lot of information about the medium.

In this work we are going to discuss a phenomenon called jet quenching (see [1] for a review). A jet is a collimated set of particles with a large energy and a small opening angle.

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They are useful in the context of Quantum Chromodynamics (QCD) because the are defined in such a way that they are minimally sensitive to infrared non-perturbative physics. Jets are quenched (lose energy) when traversing a hot medium. Therefore, measuring jet quenching we can characterize the opacity of the medium and, from this, we can infer its properties. They are many models that can be used to describe jet quenching. However, in most of them it exists a parameter, called $\hat{q}$, that controls the strength of the medium-jet interaction.

In recent years, there is an increasing interest in jet substructure, both in proton-proton and in heavy-ion collisions. Several techniques appeared that allow to re-cluster jets within a jet. In the context of heavy-ion collisions, this is interesting because it allows to test the interaction with the medium at different scales. However, to make the most of these techniques we need to understand how jet quenching depends on the opening angle of the jet constituents.

The angular structure of a jet is very well understood in the absence of a medium. We find the so-called angular ordering, meaning that each subsequent splitting has a smaller opening angle. This implies a high degree of coherence within the different splittings in the vacuum. The question then is what happens when the high energy partons split inside of a medium. A setting widely used in the literature to address these issues is the antenna configuration [2–7]. It consists of a pair of quark-antiquark pairs that are created inside of the medium as a result of the splitting of a high energy particle. It has been found that, when the splitting angle is small, the medium can not resolve the products of the splitting and it interacts with jet as if the splitting never happened. For example, for a case of a photon splitting into a quark-antiquark pair with a small opening angle, medium interaction is highly suppressed. The opposite happens when the angle is large, from the point of view of the medium the two daughter partons are very far away from each other and they radiate energy as if they were uncorrelated particles. Note that this implies that there is no angular-ordering in this case.

Previously discussed results were obtained as a leading-order computation in the small angle limit. In this work, we want to study what happens when the angle is a little bit larger. What we mean by this is the following. In most studies, it is assumed that the daughter partons that form the antenna travel approximately among the same light-like direction as the mother parton. In our case, we are going to consider that each daughter parton follows a different light-like direction. Then, we are going to compute the leading-order corrections that this introduces. This work is organized as follows, in section 2 we discuss the general setting of the problem. Then, in section 3 we give some details about the computation. Finally, the conclusions are given in section 4. More details will be given in an upcoming publication [8].

2 Setting of the problem

We focus on the simple scenario of a pair of quark-antiquark created from a photon that radiate a high energy gluon after traversing the medium. This process is represented in fig. 1. It is a simple setting but it is already sensitive to the coherence effects that we wish to highlight. To discuss this problem, we use approximations that are common in the literature:

- The large N_c limit.
- We take the classical limit. In this context, we mean by classical that the gauge fields are equal in the amplitude and in the complex-conjugated amplitude. This is a good approximation for soft fields at finite temperature and on-shell particles at tree level.
- The eikonal approximation. This means that collisions of the high energy partons with medium particles do not deviate them from their original trajectory along a light-like line.
- In this work, or main interest is the medium influence of partons traversing a static high temperature QCD brick. For this reason, we consider that chromoelectric and chromomagnetic fields outside of the medium vanish. Note that this does not imply that the gauge field

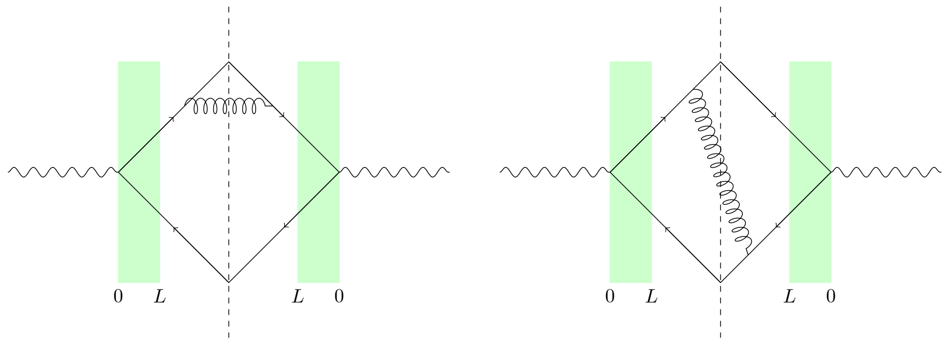

Figure 1. Representation of the contributions to the cross-section of the process studied in this work. Only high energy particles are represented. The green-shaded area represents the extent of the medium. On the left (right) of the dashed line we represent the amplitude (complex-conjugated amplitude).

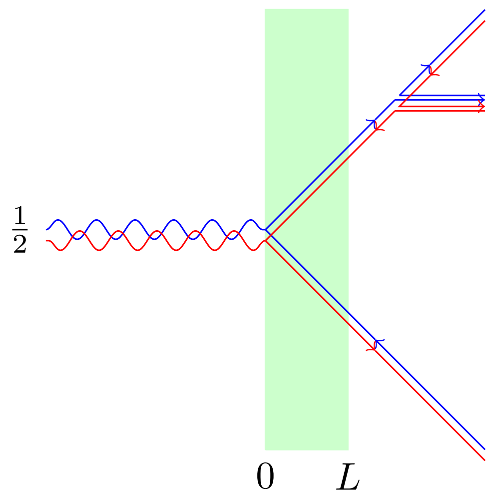

Figure 2. Result of applying the large N_c and the classical limit to the self-energy diagram. Blue lines represent the original amplitude while red elements comes from the folding of the complex-conjugated amplitude

itself vanishes outside of the medium. However, it does imply that the field is a pure gauge outside of the QCD brick.

First, let us focus on the diagram on the left-hand side of fig. 1, that we call the self-energy diagram. We recall that in the eikonal limit a high energy quark (gluon) can be substituted by a light-like fundamental (adjoin) Wilson line. In this sense, in the large N_c limit we can substitute the gluon by the exchange of a quark-antiquark pair separated by a very small distance and multiply the diagram by an overall $\frac{1}{2}$ factor. This is equivalent to applying the Fierz identity to leading order in the large N_c counting. Moreover, note that in the classical limit a quark line in the complex conjugate amplitude can be *folded* into an antiquark line in the amplitude such that we get a cancellation. The combined effect of the large N_c plus the classical approximation for the self-energy diagram is illustrated in fig. 2. A blue line cancels with an equivalent red line with the arrows pointing out to the opposite direction. Therefore, all Wilson lines cancel in this process and we can conclude that there are no thermal effects.

The situation is completely different for the right-hand side diagram of fig. 1, that we call the interference diagram. Applying the same approximations (large N_c and classical limits) and using the property that the field is a pure gauge outside of the medium, we can see that

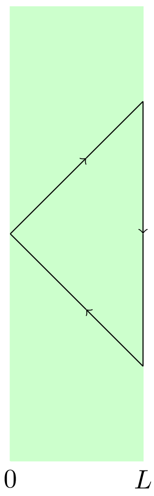

Figure 3. Diagram whose square contains all the relevant thermal modifications that are needed to describe the right-hand diagram of fig. 1

all the thermal effects are equivalently included in the modulus square of the diagram of fig. 3. In summary, the contribution from the interference term is proportional to Δ_{med} and that contains all medium effects.

$$\Delta_{med} = 1 - \frac{1}{N_c^2}\langle \mathbf{Tr} e^{ig\oint dl\cdot A}\rangle^2 , \tag{1}$$

where the contour is shown in fig. 3. The quark (antiquark) goes around a light-cone direction that we call n_1 (n_2). This has to be compared with previous results in which the role of Δ_{med} was played Δ_{med}^{old}

$$\Delta_{med}^{old} = 1 - \frac{1}{N_c^2}\langle \mathbf{Tr} U_1(L^+,0)U_2^\dagger(L^+,0)\rangle^2 , \tag{2}$$

with

$$U_i(x^+,0) = \mathcal{P}_+ \exp\left[ig\int_0^{x^+} d\tau n\cdot A\left(n\tau + \frac{k_{i,\perp}\tau}{\sqrt{2}E_i}\right)\right] , \tag{3}$$

where n is the light-cone direction of the parent photon. Although Δ_{med} and Δ_{med}^{old} are equal at leading order in perturbation theory, they have important differences. First, we now take into account components of the gauge field perpendicular to n both in the quark and the antiquark lines. Second, there is an additional vertical lines that goes along the border of the medium. We can trace back the origin of this Wilson line to the effect that the gauge field have on the large energy particles once they escape the medium. We remind that these fields are a pure gauge outside of the medium, therefore, we could regard this effect as similar to the Aharonov-Bohm effect. Finally, Δ_{med} is manifestly gauge invariant.

Let us now discuss some aspects related to the eikonal approximation. We assume that the quark (antiquark) moves eikonally with direction n_1 (n_2). They are related by the formula

$$n_1 n_2 = \frac{1}{2}(1-\cos\theta) \sim \frac{\theta^2}{4} , \tag{4}$$

where $\theta \ll 1$. It is useful to define

$$n = \frac{n_1+n_2}{2} , \tag{5}$$

and

$$\delta n = n_1 - n_2 . \tag{6}$$

We note that n is almost a light-cone vector, since $n^2 \sim \theta^2$. This vector is almost identical to the direction of the parent photon and, therefore, when no confusion is possible we name it with the same letter. Note also that $n\delta n = 0$, therefore δn is a transverse vector with modulus of order θ.

3 Computation

First, let us review the computation in the small angle limit in perturbation theory. If $W = \mathbf{Tr}e^{ig\oint dl\cdot A}$, then at leading order we get

$$\log\langle W\rangle = -\frac{1}{4\sqrt{2}}\int_0^{L^+} d\tau_s \hat{q}(\delta n_\perp \tau_s)(\delta n_\perp \tau_s)^2\,, \tag{7}$$

where L^+ is the size of the medium in the light-cone + direction and $\hat{q}$ was computed in [9]. $\hat{q}$ has dimensions of energy to the cube. We consider the general case in which $\delta n_\perp L^+ T$ is of order 1. This means that the resolution scale of the medium is of the order of quark-antiquark dipole. Since in perturbation theory thermal effects only enter at one loop [10], we could naive expect that $\hat{q}$ is of order αT^3. However, this is not the case. The reason is that in one loop perturbation theory thermal effects only enter through on-shell particles [10] and it happens that they do not contribute to $\hat{q}$ in the strict small angle limit. Therefore, the leading contributions to $\hat{q}$ come from the following regions:

- Particles with energy of order T at two loop. Giving a contribution to $\hat{q}$ of order $\alpha^2 T^3$.
- Particles with energy of order gT. Naive perturbation theory breaks for these particles and we must use the Hard Thermal Loop (HTL) resummation (for a review see [10]). Then we get a one loop contribution to $\hat{q}$ of order $\alpha T(gT)^2$. Note that in HTL computations the classical contribution is enhanced by a factor $1/g$ due to Bose-enhancement.

In conclusion, we get that $\log\langle W\rangle$ is of order $\alpha^2 T^3 L^3 \theta^2 \sim \alpha^2 TL$. Let us note that, as discussed in [9], the error induce by considering that the HTL expression is valid for all energies is very small. As a consequence, it is a common practice in the literature to always use the HTL result as it simplifies the computations considerably.

Now, let us discuss what happens when we relax the condition that the angle is small. In practice, we mean by this that n_1 and n_2 are not equal. We can divide the type of corrections that we expect in two categories:

- Sub-leading contributions from energy regions that already contribute in the strict small angle limit.
- Contributions from regions that can be ignored in the small angle limit but that must be taken into account when we relax this condition.

It happens that the leading correction is of the second type. Its origin is the influence of on-shell particles with energies of order T that are ignored at leading order because they are not enhanced by powers of LT, like off-shell particles. We can predict that these corrections to $\log\langle W\rangle$ are of size α_s. Note that any correction from the regions that were already considered at leading order would be at most of size $\alpha_s^2 TL\theta \sim \alpha_s^2$ and, therefore, sub-leading.

Having identified the interesting region, we proceed to the computation. The diagrams that contribute are shown in fig. 4. It happens that in the Coulomb gauge the bottom left diagram is zero. The results that we got are the following. If

$$\log\langle W\rangle = g^2 C_F (W_2^{LO} + W_2^{NLO})\,, \tag{8}$$

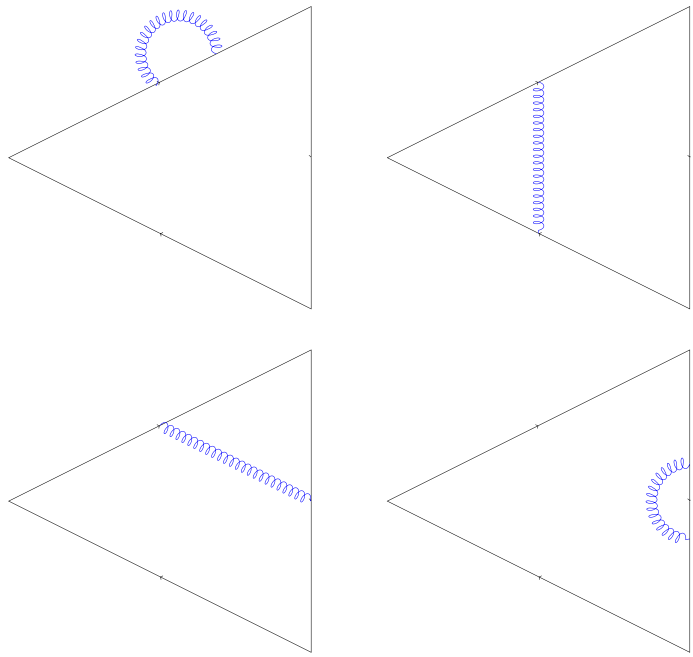

Figure 4. Relevant diagrams in the one-gluon-exchange approximation. The blue wavy line represents a soft thermal gluon exchange.

then W_2^{LO} can be read from eq. (7). We did not arrive to a simple analytic expression for W_2^{NLO}. However, we were able to compute it numerically and to understand its behavior for small and large angles. For large θ

$$W_2^{NLO} \sim -\frac{TL^+\delta n}{4\pi}\left(\log(TL^+\delta n) + \gamma_E\right) , \tag{9}$$

and for small θ

$$W_2^{NLO} \sim -\frac{T^2\delta n^2 L^{+2}}{18} , \tag{10}$$

where $\delta n = \theta/\sqrt{2}$ and $L^+ = \sqrt{2}L$. The numerical results can be found in fig. 5. More details about the computation can be found in a future publication [8].

4 Conclusions

In this work we have computed corrections that need to be considered when the opening angle of the antenna configuration is small but we are interested in sub-leading corrections. At leading order, the high energy partons only couple directly to space-like (off-shell) gluons. In other words, the interaction with medium particles can be encoded in a potential. At next-to-leading order this is no longer the case. As we have seen, we need to consider corrections coming from on-shell gluons. This implies, that at this level of accuracy we can no longer regard the medium as a collection of static scattering centers.

Our results are compatible with the previous literature as long as $\alpha_s LT \gg 1$, which is true for nowadays heavy ion collisions. The leading corrections come from the influence of on-shell gluons. They have less suppression in α_s compared to off-shell gluons (because they get

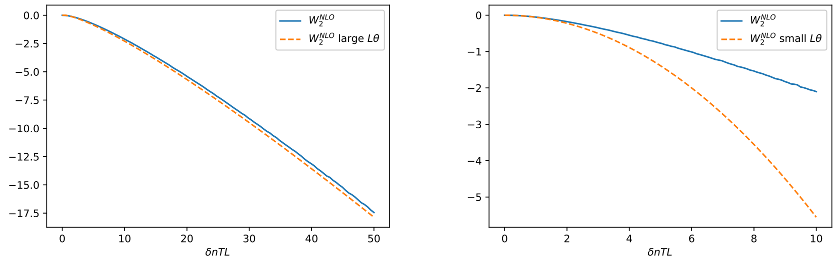

Figure 5. Numerical evaluation of W_2^{NLO}. In the left panel we compare with the large $LT\theta$ limit, while in the right panel we compare with the opposite limit.

thermal corrections already at tree level) but they do not have any phase space enhancement that results in an enhancement due to the size of the medium (LT power).

We note that the interaction with the gauge fields after the medium ends up giving the vertical Wilson line, which can not be ignored at next-to-leading order.

In conclusion, we can consistently add higher order corrections, that matter when θ is not so small, in a manifestly gauge invariant way.

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