

Coherent pion photoproduction on spin-zero nuclei

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Abstract. We study the possibility of building a universal model describing pion scattering and photoproduction on spin-zero nuclei within the same framework. We show our prediction for photoproduction cross section based on parameters taken from our fit to pion-¹²C scattering data and demonstrate the importance of the second-order effects arising from the multiple scattering theory.

1 Introduction

In heavy nuclei, the distribution of neutrons extends out further than the proton distribution forming a so-called “neutron skin”. An accurate experimental determination of the difference between the neutron and proton rms-radii for heavy nuclei would provide a unique constraint on the symmetry energy of the nuclear Equation Of State, which strongly depends on poorly constrained three-body forces [1]. Photons have an advantage over other nuclear probes for this purpose since they can interact with the whole volume of the nucleus. Consistently, coherent neutral pion photoproduction on nuclei is sensitive to the distribution of nucleons. The information on the neutron distribution can be extracted by comparing the diffraction pattern of the measured photoproduction cross section with theoretical calculations. The method of coherent pion photoproduction provides an efficient tool to study the neutron skin however requires a reliable theoretical model [2, 3].

The observed cross section is strongly affected by various many-body effects, which must be accounted for in the model calculations. [4, 5]. In the distorted wave impulse approximation framework, we develop a model that describes both pion scattering and photoproduction. The many-body medium effects are incorporated in the complex effective Δ self-energy Σ_{Δ} , shifting the Δ mass and width in scattering and photoproduction amplitudes on a single nucleon. To reliably account for the pion-nucleus final-state interaction, we design the effective second-order potential for the pion photoproduction on spin-zero nuclei, including the final-state charge exchange. It is shown that the second-order correction to the photoproduction potential plays an essential role in low-energy and Δ -resonance regions. Moreover, it is demonstrated that the inclusion of the second-order term allows us to significantly improve our prediction for photoproduction cross section on ¹²C, which is calculated using the Σ_{Δ} parameter taken from the fit to 80 – 180 MeV pion-¹²C scattering data.

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2 Elementary pion photoproduction

The relativistically invariant pion photoproduction amplitude on a nucleon in the pion-nucleon c.m. frame has the form [6, 7]

$$f^\gamma = i\tilde{\sigma} \cdot \epsilon_\lambda F_1 + (\sigma \cdot \hat{k})\sigma \cdot (\hat{q} \times \epsilon_\lambda)F_2 + i(\sigma \cdot \hat{q})(\hat{k} \cdot \epsilon_\lambda)F_3 + i(\sigma \cdot \hat{k})(\hat{k} \cdot \epsilon_\lambda)F_4, \quad (1)$$

where ϵ_λ is the photon polarisation vector, σ is the nucleon spin operator, $\hat{k} = \mathbf{k}/|\mathbf{k}|$, $\hat{q} = \mathbf{q}/|\mathbf{q}|$, $\tilde{\sigma} = \sigma - (\sigma \cdot \hat{q})\hat{q}$ and $\hat{k} = \hat{k} - (\hat{k} \cdot \hat{q})\hat{q}$. The four independent CGLN amplitudes F_1, \dots, F_4 are complex functions of the total energy W and the pion scattering angle θ .

Only the second term in Eq. (1) contains the non-spin-flip part of the elementary amplitude. As a result, only the term proportional to F_2 CGLN amplitude will contribute after spin-averaging for a spin-zero nucleus. For this reason, we restrict our further consideration only to the F_2 CGLN amplitude, which can be expanded in terms of magnetic multipole amplitudes $M_{l\pm}$ as

$$F_2 = \sum_{l \geq 1} [(l+1)M_{l+} + lM_{l-}] P'_l, \quad (2)$$

where P'_l is the derivatives of the Legendre polynomials of argument $x = \cos \theta$.

If isospin conservation is assumed, the isospin structure of any photoproduction amplitude can be written as

$$M_{l\pm} = \delta_{\alpha 3} M_{l\pm}^+ + \frac{1}{2} [\tau_\alpha, \tau_3] M_{l\pm}^- + \tau_\alpha M_{l\pm}^0, \quad (3)$$

where α is the pion isospin index and τ_α are the nucleon isospin matrices. The isoscalar amplitudes $M_{l\pm}^0$ correspond to isospin-1/2 states. Also, we can introduce two isovector amplitudes, $M_{l\pm}^{1/2}$ and $M_{l\pm}^{3/2}$, for the πN system with total isospin 1/2 and 3/2, respectively:

$$M_{l\pm}^{1/2} = M_{l\pm}^+ + 2M_{l\pm}^- \quad \text{and} \quad M_{l\pm}^{3/2} = M_{l\pm}^+ - M_{l\pm}^-. \quad (4)$$

As a result, the isospin averaged F_2 becomes

$$\bar{F}_2^+ \equiv \frac{1}{2} (F_2^{+(p)} + F_2^{+(n)}) = \sum_{l \geq 1} [(l+1)M_{l+}^+ + lM_{l-}^+] P'_l. \quad (5)$$

The $l = 1$ contribution is dominant, and using Eq. (4), it can be decomposed as

$$\bar{F}_2^+ = \frac{4}{3} M_{1+}^{3/2} + \frac{2}{3} M_{1+}^{1/2} + M_{1-}^+ + \sum_{l \geq 2} [(l+1)M_{l+}^+ + lM_{l-}^+] P'_l. \quad (6)$$

In this way, we have explicitly separated the magnetic multipole $M_{1+}^{3/2}$. The main reason for doing so is the fact that the spin-isospin- $\frac{3}{2}$ channel contains the dominant part of the photoproduction amplitude, which corresponds to the photoexcitation of $\Delta(1232)$.

3 Elementary amplitudes in nuclear medium

The interaction of the Δ -isobar with the surrounding nucleons significantly modifies the pion scattering and photoproduction in the dominant spin-isospin- $\frac{3}{2}$ channel [4, 8]. This means that both the amplitudes are not only functions of the reaction energy and angle but also acquire a dependence on nuclear density $\rho(r)$. However, even if the exact form of this dependence were known, its inclusion in the momentum space approach would not be trivial. For this reason, we consider the in-medium interactions approximately, renormalizing the Δ propagator by the complex effective self-energy Σ_Δ .

For the scattering process, we adopt the *relativistic Δ -isobar model* by Oset, Toki, and Weise [8], which successfully reproduces the p -wave pion-nucleon phase shifts at low and intermediate energies, especially the resonant P_{33} channel. The model is based on the K -matrix formalism, and as our first step in modifying it, we explicitly separate the direct Δ contribution to the p -wave spin-isospin- $\frac{3}{2}$ K -matrix, $K_{33}^{1(\Delta)}$. Then the free propagator is replaced to get $K_{33}^{1(\Delta)}(\Sigma_\Delta)$ inside the nucleus:

$$K_{33}^{1(\Delta)} = \frac{1}{k_{0,2\text{cm}}} \frac{m_\Delta \Gamma_\Delta}{m_\Delta^2 - W^2} \quad \Longrightarrow \quad K_{33}^{1(\Delta)}(\Sigma_\Delta) = \frac{1}{k_{0,2\text{cm}}} \frac{m_\Delta}{m_\Delta + W} \frac{\Gamma_\Delta}{m_\Delta + \Sigma_\Delta - W}, \quad (7)$$

where m_Δ and Γ_Δ are the Δ mass and decay width. As the last step, $K_{33}^{1(\Delta)}(\Sigma_\Delta)$ is used to calculate the modified scattering amplitude $f_{33}^1(\Sigma_\Delta)$ in the regular way.

Our starting point for calculating photoproduction is the MAID2007 [9] analysis, providing even particular contributions to the free amplitude. The resonant part of the $M_{1+}^{3/2}$ amplitude, $M_{1+}^{3/2(\Delta)}$, is extracted directly from MAID and is modified in the same way :

$$M_{1+}^{3/2(\Delta)} \quad \Longrightarrow \quad M_{1+}^{3/2(\Delta)}(\Sigma_\Delta) = M_{1+}^{3/2(\Delta)} \frac{W - m_\Delta + i\Gamma_\Delta/2}{W - m_\Delta - \Sigma_\Delta + i\Gamma_\Delta/2}. \quad (8)$$

The background amplitudes in MAID are complex functions defined according to K -matrix theory,

$$M_{1+}^{3/2(B)} = v_{1+}^{3/2(B)} \left[1 + ikf_{33}^1 \right], \quad (9)$$

and in the nuclear medium we replace the free f_{33}^1 with $f_{33}^1(\Sigma_\Delta)$. The Fermi-Watson theorem should still be valid, however, now the final phase coincide with the corresponding modified scattering amplitude:

$$M_{1+}^{3/2}(\Sigma_\Delta) = \left| M_{1+}^{3/2}(\Sigma_\Delta) \right| \exp \left[i \arg f_{33}^1(\Sigma_\Delta) \right]. \quad (10)$$

Finally, collecting all the modifications we arrive at

$$M_{1+}^{3/2}(\Sigma_\Delta) = \left| v_{\gamma\pi}^{B,\alpha} \left[1 + ikf_{33}^1(\Sigma_\Delta) \right] + M_{1+}^{3/2(\Delta)}(\Sigma_\Delta) \right| \exp \left[i \arg f_{33}^1(\Sigma_\Delta) \right]. \quad (11)$$

A comparison of free and in-medium $M_{1+}^{3/2}$ magnetic multipoles is shown in Fig. 1 for $\Sigma_\Delta = 11.3 - 27.5i$ MeV, which was obtained from our multi-energy fit to π^\pm - ^{12}C scattering data at 80-180 MeV lab kinetic energy.

4 Distorted wave impulse approximation and the effective photoproduction potential

In multiple scattering theory, the T -matrix for pion photoproduction on nuclei can be presented as

$$\hat{T}^\gamma = \sum_i \hat{\tau}_i^\gamma + \sum_i \sum_{j \neq i} \hat{\tau}_j \hat{G} \hat{\tau}_i^\gamma + \sum_i \sum_{j \neq i} \sum_{k \neq j} \hat{\tau}_k \hat{G} \hat{\tau}_j \hat{G} \hat{\tau}_i^\gamma + \dots, \quad (12)$$

where $\hat{\tau}_i^\gamma$ and $\hat{\tau}_i$ are transition amplitudes for pion scattering and photoproduction inside the nucleus, which include scattering to all orders on a single bound nucleon. Extending the KMT multiple scattering formalism [10] to the case of nuclear pion photoproduction, Eq. (12)

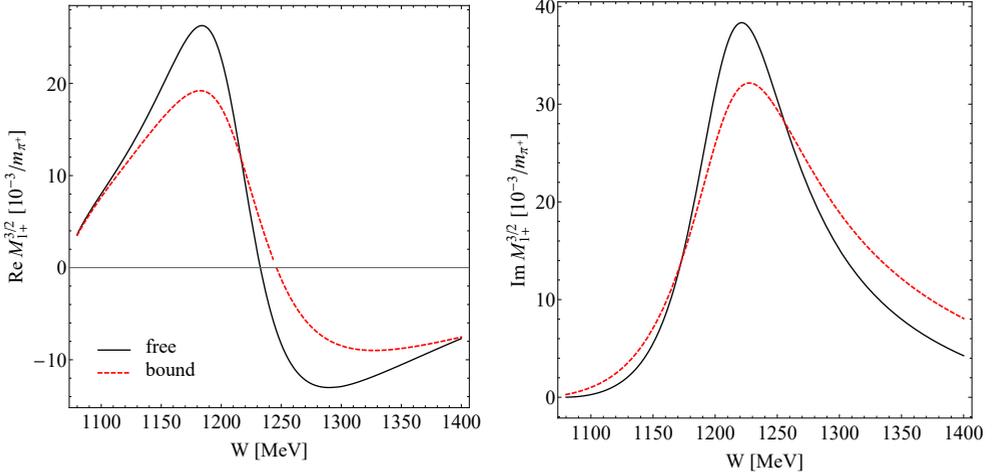


Figure 1. The real (right panel) and imaginary (left panel) parts of the $M_{1+}^{3/2}$ magnetic multipole amplitude. The solid black curves correspond to the unmodified free-nucleon amplitude, and the dashed red curves represent the amplitude in the nuclear medium, modified by the effective Δ self-energy $\Sigma_{\Delta} = 11.3 - 27.5 i$ MeV.

can be subdivided into a system of Lippmann-Schwinger-like equations with effective pion photoproduction and scattering potentials:

$$\hat{T}^{\gamma} = \hat{U}^{\gamma} + \frac{A-1}{A} \hat{T} \hat{G} \hat{P}_0 \hat{U}^{\gamma}, \quad (13a)$$

$$\hat{T} = \hat{U} + \frac{A-1}{A} \hat{U} \hat{G} \hat{P}_0 \hat{T}. \quad (13b)$$

Note, Eq. (13b) resembles the standard Lippmann-Schwinger equation, with an additional factor $(A-1)/A$ preventing double counting of pion rescattering on the same nucleon and projector $\hat{P}_0 = |\Psi_0\rangle\langle\Psi_0|$, which forbids intermediate nuclear excited states. The effective scattering and photoproduction potentials are

$$\hat{U}^{\gamma} = A\hat{\tau}_i^{\gamma} + A(A-1)\hat{\tau}_i\hat{G}\hat{P}_0\hat{\tau}_j^{\gamma} + A(A-1)^2\hat{\tau}_i\hat{G}\hat{P}_0\hat{\tau}_j\hat{G}\hat{P}_0\hat{\tau}_k^{\gamma} + \dots, \quad (14a)$$

$$\hat{U} = A\hat{\tau}_i + A(A-1)\hat{\tau}_i\hat{G}\hat{P}_0\hat{\tau}_j + A(A-1)^2\hat{\tau}_i\hat{G}\hat{P}_0\hat{\tau}_j\hat{G}\hat{P}_0\hat{\tau}_k + \dots \quad (14b)$$

where $\hat{P}_0 = \hat{1} - \hat{P}_0$ is the projector on nuclear excited states.

In general, $\hat{\tau}_i^{\gamma}$ and $\hat{\tau}_i$ are $(A+1)$ -body operators dependent on the reaction energy $E(k_0) = \omega_{\pi}(k_0) + E_A(k_0)$, where k_0 is the pion on-shell momentum, $\omega_{\pi}(k)(E_A(k))$ is the pion (nucleus) energy, and the pion-nucleus c.m. frame is chosen. We use the *impulse approximation*, assuming $\hat{\tau}_i^{\gamma}(E) \approx \hat{\tau}^{\gamma}(W)$ and $\hat{\tau}_i(E) \approx \hat{i}(W)$ with the following choice of the 2-body reaction energy:

$$W = \sqrt{(\omega_{\pi,\gamma}(k) + E_N(p))^2 - (\mathbf{k} + \mathbf{p})^2}, \quad (15)$$

where \mathbf{p} is the target nucleon momentum and Eq. (15) is evaluated for $|\mathbf{k}| = k_0$. The transition matrix element for free scattering (photoproduction) in the pion-nucleus c.m. frames is related to the elastic scattering (photoproduction) amplitude as

$$\langle \pi(\mathbf{k}'), N(\mathbf{p}') | \hat{i}^{\gamma}(W) | \gamma(\mathbf{k}), N(\mathbf{p}) \rangle = - \frac{2\pi}{\sqrt{\omega_{\pi N}(\mathbf{k}', \mathbf{p}') \omega_{\gamma N}(\mathbf{k}, \mathbf{p})}} f^{(\gamma)}(2\pi)^3 \delta(\mathbf{k}' + \mathbf{p}' - \mathbf{k} - \mathbf{p}), \quad (16)$$

$$\omega_{\pi(\gamma)A}(\mathbf{k}, \mathbf{p}) = \omega_{\pi(\gamma)}(k)E_N(p)/W(\mathbf{k}, \mathbf{p}).$$

Finally, the pion photoproduction amplitudes within the distorted wave impulse approximation in momentum space can be presented as [11]

$$F^\gamma(k_0, \mathbf{k}', k\lambda) = V^\gamma(k_0, \mathbf{k}', k\lambda) - \frac{A-1}{A} \frac{1}{(2\pi)^2} \int \frac{d\mathbf{k}''}{\omega_{\pi A}(k'')} \frac{F(k_0, \mathbf{k}', k'')V^\gamma(k_0, \mathbf{k}'', k\lambda)}{E(k_0) - E(k'') + i\varepsilon}. \quad (17)$$

with

$$F(k_0, \mathbf{k}', k) \equiv -\frac{\sqrt{\omega_{\pi A}(k')\omega_{\pi A}(k)}}{2\pi} \langle 0, \mathbf{k}' | \hat{T} | 0, \mathbf{k} \rangle, \quad (18a)$$

$$F^\gamma(k_0, \mathbf{k}', k\lambda) \equiv -\frac{\sqrt{\omega_{\pi A}(k')\omega_{\gamma A}(k)}}{2\pi} \langle 0, \mathbf{k}' | \hat{T}^\gamma | 0, \mathbf{k} \lambda \rangle, \quad (18b)$$

and $\omega_{\pi A}(k) = \omega_\pi(k)E_A(k)/E(k)$.

Neglecting the contribution from the coupling to the breakup channels, we can keep only the first term of Eq. (14a), which results in the following first-order photoproduction potential [4, 11]

$$V^{\gamma(1)}(k_0, \mathbf{k}', k\lambda) = \sqrt{\frac{\omega_{\pi A}(k')\omega_{\gamma A}(k)}{\omega_{\pi N}(k', \mathbf{p}')\omega_{\gamma N}(k, \mathbf{p})}} \bar{F}_2^+(W, \theta^*) [\hat{\mathbf{k}}' \times \hat{\mathbf{k}}] \cdot \boldsymbol{\epsilon}_\lambda \rho(q), \quad (19)$$

where $\rho(q)$ is the nuclear form factor, normalized to $\rho(0) = A$.

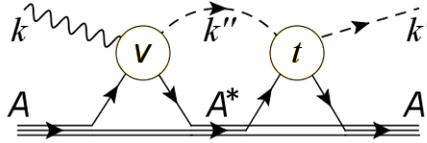


Figure 2. Diagram representation of the second order term in the pion photoproduction potential. A and A* correspond to the nucleus in the ground and excited states, respectively.

The second terms in Eqs. (13a) and (13b) describe photoproduction and scattering on one nucleon (including the final state interaction with the corresponding nucleon), after which the nucleus makes transitions into an excited state, propagation, and then scattering to all orders on a second nucleon (see Fig. 2). This contribution is known to be extremely important for the pion-nucleus scattering process [12]. For this reason, we consider the second-order correction to the photoproduction potential, Eq. (14a). The resulting potential then becomes

$$V^\gamma(k_0, \mathbf{k}', k\lambda) \approx V^{\gamma(1)}(k_0, \mathbf{k}', k\lambda) + V^{\gamma(2)}(k_0, \mathbf{k}', k\lambda) \quad (20)$$

with

$$V^{\gamma(2)}(k_0, \mathbf{k}', k\lambda) = - \int \frac{d\mathbf{k}''}{(2\pi)^3} \frac{\Omega(k_0, \mathbf{k}, \mathbf{k}', k'')}{E(k_0) - E(k'') + i\varepsilon} [\hat{\mathbf{k}}'' \times \hat{\mathbf{k}}] \cdot \boldsymbol{\epsilon}_\lambda \left[f^+(k', k'') \bar{F}_2^+(k'', \mathbf{k}) C(k' - k'', k'' - \mathbf{k}) + 2f^-(k', k'') \bar{F}_2^-(k'', \mathbf{k}) D(k' - k'', k'' - \mathbf{k}) \right]. \quad (21)$$

where $\bar{F}_2^\pm = \sum_{l \geq 1} [(l+1)M_{l+}^+ + lM_{l-}^+] P_l'$ and f^\pm are the components of the scattering amplitude, which are introduced in the similar way as \bar{F}_2^\pm (see Appendix 8 in [13]). $\Omega(k_0, \mathbf{k}, \mathbf{k}', k'')$

is a kinematic factor derived in the same way as the square root in front of the right-hand side of Eq. (19).

The properties of nucleon distribution enter the second-order potential via the correlation functions C and D based on the Slater determinant form of the total nuclear wave function, see [14] for the definitions. Within the harmonic oscillator shell model, the correlation functions for ^{12}C acquire the form:

$$D(\mathbf{q}_1, \mathbf{q}_2) = \left(12 - \frac{4}{3}a^2(q_1^2 + q_2^2) - 4\sqrt{\frac{2}{3}}a^2\mathbf{q}_1 \cdot \mathbf{q}_2 + \frac{2}{3}a^4(\mathbf{q}_1 \cdot \mathbf{q}_2)^2 \right) e^{-\frac{1}{4}\frac{A-1}{A}a^2(q_1^2+q_2^2)}, \quad (22a)$$

$$C(\mathbf{q}_1, \mathbf{q}_2) = \left(-4\sqrt{\frac{2}{3}}a^2\mathbf{q}_1 \cdot \mathbf{q}_2 + \frac{2}{3}a^4(\mathbf{q}_1 \cdot \mathbf{q}_2)^2 - \frac{4}{27}a^4q_1^2q_2^2 \right) e^{-\frac{1}{4}\frac{A-1}{A}a^2(q_1^2+q_2^2)}, \quad (22b)$$

with $a = 1.63$ fm found from [15].

The obtained structure of the second-order correction, Eq. (21), incorporates the Pauli principle taking into account nucleon isospin. To see this clearly, one can consider the process with the zero-momentum transfer. Only the C correlation function at zero momentum transfer becomes zero. As a result, the first term in Eq. (21), proportional to the isospin-independent part of the scattering amplitude, makes zero contribution. This is in perfect agreement that if there is no isospin exchange in the intermediate state, nucleon remains in the same state, and no correction is needed. In contrast, the pion and nucleon changing their isospin states are described by the second term in Eq. (21). The non-zero correction is needed since the intermediate nucleon state is already occupied.

5 Results and discussion

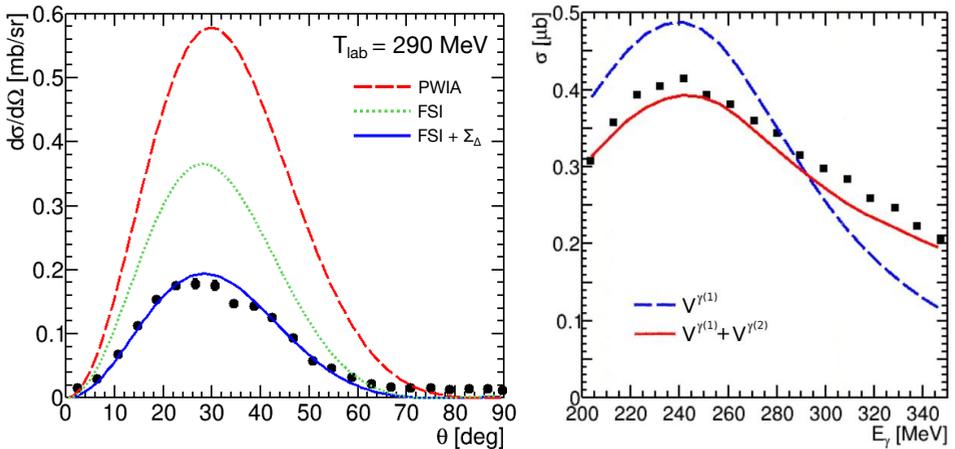


Figure 3. Predicted cross sections for pion photoproduction on ^{12}C . **Left panel:** The differential cross section at photon lab energy 290 MeV. The dashed red curve represents the plain wave impulse approximation, i.e., no medium effects ($\Sigma_\Delta = 0$) and no final state interaction (FSI) ($F^\gamma(k_0, k', k\lambda) = V^\gamma(k_0, k', k\lambda)$) included; the dotted green curve demonstrates the effect of the FSI, i.e., Eq. (17) is calculated with $\Sigma_\Delta = 0$; the solid blue curve is obtained with the FSI and $\Sigma_\Delta = 11.3 - 27.5 i$ MeV. **Right panel:** Energy dependence of the total cross section. The dashed blue and solid red curves are calculated with the first-order, Eq. (19), and the second-order, Eq. (21), photoproduction potentials, respectively.

We have briefly presented our model for pion photoproduction on spin-zero nuclei. The key feature of this model is the description of pion scattering and photoproduction within the same approaches with the common set of model parameters. To examine our predictions for photoproduction cross sections, we fix the free parameter of our model, Σ_{Δ} , to the value obtained by fitting the available data on pion- ^{12}C scattering in the 80-180 MeV pion lab kinetic energy range. The different tests of the model are shown in Fig. 3. The left panel of Fig. 3 demonstrates the known importance of the final state interaction and single-nucleon photoproduction amplitude modification by in-medium effects [4]. However, as is demonstrated by the right panel, the second-order correction to the photoproduction potential, Eq. (21) is also significant, especially in the low-energy region. This result should be expected because the same is true for pion-nucleus scattering. Even more important, the inclusion of the second-order photoproduction potential significantly improves the agreement between the prediction and experimental data, making the Σ_{Δ} the universal parameter for scattering and photoproduction.

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