The confined phase of the D0-brane matrix model and appearance of M-theory

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Abstract. We discuss the confined phase in the D0-brane matrix model and its interpretation in terms of gravity using gauge/gravity duality based on [1]. In particular, at very low energies we expect the system to describe the M-theory region and not type IIA supergravity and we provide numerical evidence for this.

1 Introduction

The gauge/gravity duality is an interesting conjecture that relates gauge field theories with gravitational theories. Emanating from string theory, the usual strategy is to present it in terms of Dp-branes [2] and the original example is for $p = 3$ [3]. When $p = 0$, the gauge theory is matrix quantum mechanics, a sub-sector of the full $\mathcal{N} = 4$ super Yang-Mills theories. This sub-sector, formulated in terms of D0-branes has a dual description in type IIA superstring theory and an analytically known solution in low energies is the black zero-brane geometry, a solution of Einstein's equations in ten-dimensional supergravity.

In addition, M-theory plays an important role, underlying the web of dualities between the different superstring theories. Type IIA superstring theory is usually defined as M-theory compactified on $\mathbf{S}^1$ [4]. The size of the circle is controlled by the asymptotic value of the dilaton field and it is directly related to the string coupling g_s. As a result, when we flow to low energies (strongly coupled string theory) the radius of the compact circle grows. This is precisely the reason why perturbative superstring theory is doomed to fail to probe this region, the M-theory limit. The authors of [5] proposed the quantum mechanical matrix model, as a non-perturbative definition of M-theory. The fundamental ingredients of M-theory are the supergraviton, the membrane and the fivebrane [6]. Using the matrix model [5], and specifically numerical techniques it might be possible to shed some light on the above conjectures. Results until now indicate that indeed these matrix models can capture some gravitational information [1, 7–12].

However, the parameter region in which the quantum mechanical matrix model is expected to be dual to M-theory, that is very low energies, has not been studied in the past. Doing so was believed to be a very challenging task because stringy corrections to the effective string coupling constant become large at temperature $T \sim N^{-10/21}$. The latter is a parametrically low temperature in the large-N limit.

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The confined phase is a feature of M-theory and not of the analytically known gravitational dual (the black zero-brane of type IIA) of the quantum mechanical matrix model. The latter is deconfined for all non-zero temperatures. In this note, we are providing evidence for a stable confined phase at low temperatures surviving in the large N limit.

2 Theoretical analysis

The model under consideration is a matrix model consisting of nine bosonic (X) and sixteen fermionic (ψ) matrices and is described by the action S_{BFSS} [5]

$$S_{\text{BFSS}} = \frac{N}{\lambda}\int_0^\beta dt\, \text{Tr}\left\{\frac{1}{2}(D_t X_M)^2 - \frac{1}{4}[X_M, X_N]^2 + \frac{1}{2}\bar{\psi}^\alpha\gamma^{10}D_t\psi_\alpha - \bar{\psi}^\alpha\gamma^M_{\alpha\beta}[X_M,\psi^\beta]\right\}. \quad (1)$$

The indices run as $I = 1,\cdots,9$, $\alpha = 1,\cdots,16$ while the covariant derivative acts on observables (O) as $D_t O = \partial_t O - i[A_t, O]$. The equations of motion for the gauge field A_t implement a Gauss constraint

$$\mathcal{G} := \frac{iN}{2\lambda}(2[\dot{X}_M, X_M] + [\bar{\psi}_\alpha, \psi_\alpha])\,|\mathbf{\Psi}_{\text{phys}}\rangle = 0 \quad , \quad \dot{X} \equiv \partial_t X, \quad (2)$$

which sets all states $|\mathbf{\Psi}_{\text{phys}}\rangle$ to be singlets of $SU(N)$.

The 't Hooft coupling λ is dimensionful with dimensions of (energy)^3, which makes the theory non-conformal. Instead, we have a $(0+1)-$dimensional matrix quantum mechanical model whose vacuum, via the gauge gravity duality, corresponds to a black zero-brane geometry in type IIA supergravity [2]. The latter is given via the metric

$$\begin{aligned}
\frac{ds^2}{\alpha'} &= -H(r)^{-1/2}f(r)dt^2 + H(r)^{1/2}\left(\frac{dr^2}{f(r)} + r^2 d\Omega_8^2\right)\,, \quad (3)\\
H(r) &= \frac{240\pi^5\lambda}{r^7}\,,\quad \lambda = g^2_{YM}N\,,\\
e^\phi &= \frac{(2\pi)^2}{240\pi^5}\frac{1}{N}\left(\frac{240\pi^5\lambda}{r^3}\right)^{\frac{7}{4}}\,,\\
f(r) &= 1 - \left(\frac{r_0}{r}\right)^7\,.
\end{aligned}$$

The location of the horizon r_0 is expressed by the Hawking temperature T as

$$T = \frac{7}{4\pi\sqrt{240\pi^5\lambda}}r_0^{\frac{5}{2}}\,. \quad (4)$$

Using black hole thermodynamics we can find the energy and entropy of this geometry [2]

$$E = 7.41N^2\lambda^{-\frac{3}{5}}T^{\frac{14}{5}}, \quad (5)$$
$$S = 11.52N^2\lambda^{-\frac{3}{5}}T^{\frac{9}{5}}. \quad (6)$$

We can trust this solution in the region for which temperatures are bounded by

$$1 \ll \frac{\lambda}{T^3} \ll N^{\frac{10}{7}}. \quad (7)$$

The lower bound comes from demanding the curvature of $\mathbf{S}^8$ to be small, while the upper bound emanates from demanding the dilaton on the horizon to be small. For these temperatures, we are in the type IIA supergravity region (see Fig. 2).

The coordinate r here is thought of as an energy scale, and its dimension is respectively (energy). The holographic idea in the matrix model is the following: diagonal elements of matrices (X_I) correspond to locations of D0-branes while non-diagonal entries characterise open strings stretching between the D0-branes[1] [13]. Thus, intuitively one can think that the

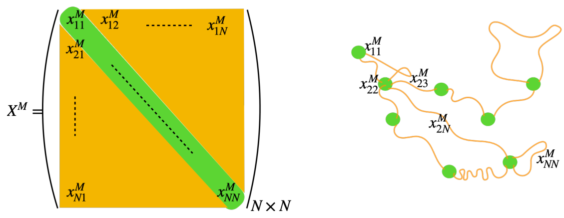

Figure 1. The holographic picture of the matrix model. One matrix gives information for the D0-branes in one spatial dimension and we have nine of them. This way we are constructing nine-spatial dimensions where the D0-branes live and dynamics are generated upon integration over the time coordinate.

minimum of the BFSS potential in (1) corresponds to $X_M = 0 = \psi_a$, which means that the D0-branes are coincident and there are no open strings. These N coincident D0-branes create the geometry (3) in type IIA supergravity.

When we want to study very low energies (resp. very low temperatures) we have to uplift the geometry to eleven dimensions. That is because Type IIA string theory is *defined* as M-theory compactified on a circle $\mathbf{S}^1$ [4]. This circle has radius $R_{11} = l_s g_s$, with l_s and g_s being the string length and string coupling respectively. The size of the eleventh circle is related to the dilaton (ϕ) on the horizon via the geometry (3) as

$$R_{11} = l_s e^{\langle\phi\rangle} \sim l_s \frac{g_{\text{eff}}^{\frac{7}{4}}}{N}, \tag{8}$$

with g_{eff} being an effective *dimensionless* coupling constructed as the combination

$$g_{\text{eff}} := \frac{\lambda}{r^3}. \tag{9}$$

Note that since r has units of energy, it is sufficient to express everything in terms of energy (E) or temperature (T). In what follows we will use g_{eff} for all such scenarios without ambiguity and the relevant parameter will depend on the context.[2] As we lower the temperatures we expect a Gregory-Laflamme [14] transition to take place. This happens precisely when the entropy is of order of N [15] and from (5) we get $T_2 \sim N^{-\frac{5}{9}}$. Around this transition, we expect a confinement to deconfinement transition at T_c. In a first order scenario as in [1, 11] it holds that $T_2 \leq T_c \leq T_1$ where T_1 is the Hagedorn transition [16] (see Fig. 3). This transition is signalled by jumps of the Polyakov loop and the energy [17]. The definition

[1]This becomes apparent when we assign the matrix indices $i, j = 1, \cdots, N$ to the Chan-Paton indices of string theory.

[2]For the particular case of simulations, since we are in the canonical ensemble, the tuning parameter having units of energy is the temperature.

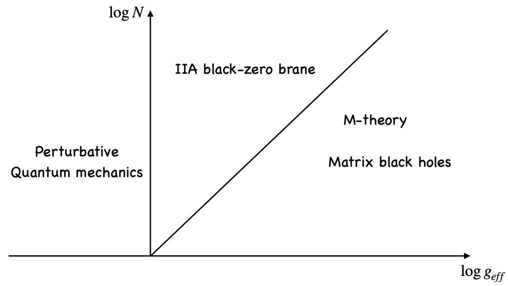

Figure 2. A cartoon for the phases of the matrix model in the large N limit based on [2]. The horizontal axis can be expressed via $g_{\text{eff}} = \frac{\lambda}{E^3}$ in the microcanonical ensemble and via $g_{\text{eff}} = \frac{\lambda}{T^3}$ in the canonical ensemble. The type IIA black-zero brane phase is valid for temperatures in the range (7). When this upper bound is exceeded, we transition to the M-theory region.

of the Polyakov loop is

$$P \equiv \frac{1}{N} \operatorname{Tr} \left(\mathcal{P} \exp \left(i \int_0^\beta A_t dt \right) \right), \tag{10}$$

with $\mathcal{P}$ denoting the path ordering, β being the inverse temperature and A_t the gauge field. Its lattice counterpart is given via

$$P \equiv \frac{1}{N} \sum_{j=1}^{N} e^{i\alpha_j} \ , \ 0 \le \alpha_j \le \pi \ . \tag{11}$$

When the Polyakov loop (P) is zero and the energy is of order N^0 we have a confined phase, while for the deconfined phase we have $P \simeq \frac{1}{2}$ and $E \sim N^2$. For non-zero temperature (T) the gravitational system is always deconfined (see e.g eq. (5)).

In [17] the phase diagrams of AdS_5 and $\mathcal{N} = 4$ SYM theories were studied showing four different saddles dominating the partition function for different energies in the microcanonical ensemble. For high energies the only available saddle is the big black hole filling the whole $AdS_5 \times S^5$ spacetime. As we lower the energy there is an intermediate unstable phase corresponding to a small (Schwarzschild) black hole localised along S^5 separating the big black hole from the graviton gas in AdS_5 for extremely low energies. The latter is the only saddle at these low energies.

We argue that a similar scenario happens for the matrix model and the system transitions from a black zero-brane in type IIA to a graviton gas in eleven-dimensional flat spacetime. The unstable phase in between is considered as the Schwarzschild black hole localised on S^1, the compact circle of M-theory. The smallest possible black hole for low energies comes by considering a Schwarzschild black hole with mass the Planck mass resulting in a (minimum deconfined) temperature $T_1 \sim N^{\frac{1}{3}}$. Therefore, at very low temperatures, we expect a wide separation between the two phases. The same argument for AdS_5 leads to a scaling[3] $T_1 \sim N^{\frac{2}{17}}$.

Let us discuss the situation for the matrix model in the microcanonical ensemble. For very low energies the only available saddle is that of the graviton gas in eleven dimensions. As

[3]We may use $l_{p,10d}^9 \sim N^{-2}$ for $R_{AdS} \sim 1$ such that the entropy of the black hole $S_{\text{BH}} \simeq \left(l_{p,10d}E\right)^{8/7}$ and the entropy of the graviton gas $S_{\text{gas}} \simeq (R_{AdS}E)^{8/7}$ is of the same order at $T = T_1$. This yields $E \sim N^{20/17}$ and at Hagedorn temperature where the free energy vanishes one gets $T_1 \sim N^{\frac{2}{17}}$.

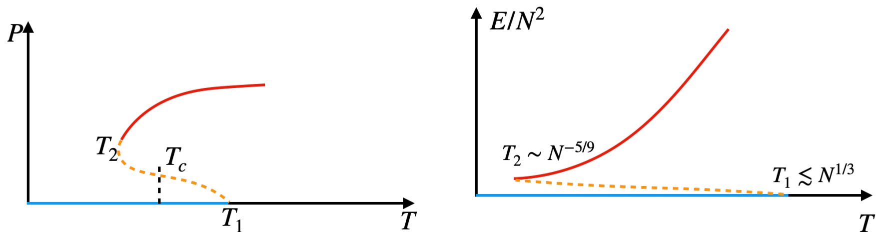

Figure 3. A cartoon of the Polyakov loop (left) and the energy (right) as functions of the temperature. The unstable phase (dashed/orange line) appears only in the microcanonical ensemble and is interpreted as a small Schwarzschild black hole along the compact M-theory circle [1] and is a partially deconfined phase [18, 19]. This phase separates the black zero-brane geometry at high energies (red) and that of the graviton gas at very low energies (blue).

we raise the energy, long strings start added in the spectrum showing a Hagedorn behaviour. If we further increase the energy, the boosted Schwarzschild black hole and the black string along S^1 are added to the spectrum, while for even higher energies the only available saddle is that of the black zero-brane (3). In case the black hole has negative specific heat and is unstable, it cannot appear in the canonical ensemble. In addition, this (black hole) phase separates the black zero brane geometry at temperatures $T > T_2 \sim N^{-5/9}$ and the graviton gas phase at temperatures $T < T_1 \sim N^{1/3}$. A scaling of this latter transition temperature with positive powers of N is perhaps a universal characteristic of SYM theories [15]. We summarize pictorially this in Fig. 3.

3 Simulation strategy

The aforementioned model can be put in a computer quite efficiently. After all, we have to deal with just one dimension, so the worldline is discretized in L steps. The discretization action has the form reported in [1, 12]. We use a canonical ensemble where the energy is fixed by the temperature and the partition function is given as

$$Z = \int [DA_t][DX][D\psi]e^{-S_{\text{BFSS}}}. \tag{12}$$

Naively, one would expect that at very low energies (resp. temperatures) the model suffers from flat directions. Indeed, that is true for the deconfined phase corresponding to the black brane. In the confined phase instead, we observed that the system behaves as a bound state where all the eigenvalues are bounded together and are stable. This is a physical phenomenon since for the deconfined case (corresponding to the black hole phase) the flat directions correspond to the emission of eigenvalues (i.e D0-branes) to infinity with probability $\Pi \sim e^{-N}$, a higher dimensional analogue of particle emission. For the confined case nothing is emitted and the bound state is stable.

We performed simulations at low temperatures using matrix sizes $N = 10, 12, 16$ and lattice points $L = 30, 36, 48, 72$. A confined state has the characteristics that the Polyakov loop fluctuates around $P \simeq 0$ as it is shown in the Monte Carlo histories in Fig. 4.

To make contact with continuous and classical physics we performed a continuous and large N extrapolation respectively. The results show zero energy and a vanishing Polyakov loop in this limit (see Fig. 5), giving confidence about the results and the interpretation we motivated previously. We find a stable confined phase in the large N limit and following the

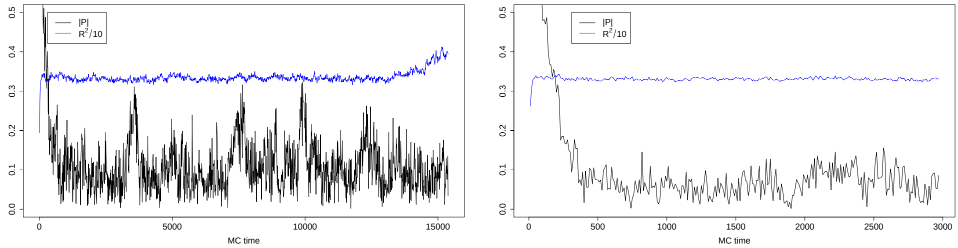

Figure 4. Monte Carlo histories from cold starts ($X1 = X2 = \cdots = X9 = 0$) for BFSS, $T = 0.2, L = 48, N = 10$ (left) and $N = 16$ (right). For $N = 10$, the onset of the run-away behavior (i.e., the increase of $R^2/10$) can be seen at a late time. $R^2/10$ denotes the normalized sum of all (nine) scalars $R^2 = \sum_{M=1}^{9} \operatorname{Tr} X_M^2$.

work of [17] we interpret this phase as a graviton gas in eleven dimensions. As can be seen from the gravitational dual (5), for nonzero temperature, the energy cannot be zero, and thus (3) is not a good approximation anymore for the confined phase in these low temperatures. The only possibility to find a valid dual description in this regime comes by uplifting the dual solution to M-theory.

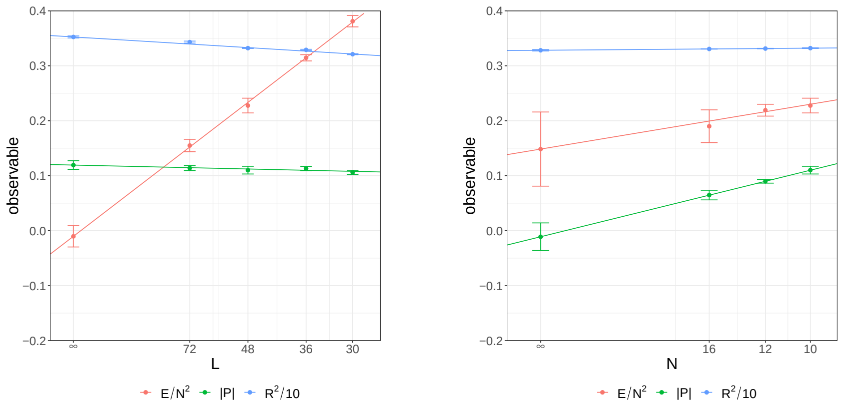

Figure 5. Large N (right) and continuous (left) extrapolations for temperature $T = 0.2$ lead to confinement. Large N extrapolation sets the Polyakov loop to zero while continuous extrapolation sets the energy to zero. $R^2/10$ denotes the normalized sum of all (nine) scalars $R^2 = \sum_{M=1}^{9} \operatorname{Tr} X_M^2$.

4 Conclusions

The confined phase of the D0-brane matrix model (BFSS) has been observed for the first time in the large N and continuum limits at low temperatures. It is a stable phase in the large N limit that, following the motivation of [17], is interpreted as a graviton gas in flat spacetime but in eleven dimensions. A small Schwarzschild black hole localised along S^1 is the unstable phase (maximum of the free energy) that separates the two other stable phases, the supergraviton gas and the black zero-brane geometry (minima of the free energy). Since the Schwarzschild black hole is unstable it cannot appear in the canonical ensemble.

The phase diagram sketched in Fig. 3 is under construction since there are no analytic results available in the interesting region at very low energies. It is worth stressing that in the interesting region we do not have a good understanding of the system, neither analytically nor numerically. More systematic work and studies are required to safely understand the gravity dual of the D0-brane matrix model in the low energy regime. In particular, the small black hole phase has not been studied, but it is indeed crucial to understand its importance on the behaviour of the matrix model in this regime.

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