Trojan Horse Method: a general introduction

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Abstract. Owing the presence of the Coulomb barrier at astrophysically relevant kinetic energies, it is very difficult, or sometimes impossible to measure astrophysical reaction cross sections in laboratories, especially for the presence of the electron screening effect. This is why different indirect techniques are being used along with direct measurements. The Trojan Horse Method (THM) is a unique indirect technique allowing one to measure astrophysical rearrangement reactions down to astrophysical relevant energies. The basic principle and a review of the main applications of the THM are presented.

1 Introduction

Nuclear fusion reactions, that take place in the hot interiors of remote and long-vanished stars over billions of years, are the origin of nearly all the chemical elements in the universe [1]. The detailed understanding of the origin of the chemical elements and their isotopes has combined astrophysics and nuclear physics, and forms what is called nuclear astrophysics. In turn, nuclear reactions are the heart of nuclear astrophysics: they strongly influence the nucleosynthesis in the earliest stages of the universe and in all the objects formed thereafter, and control the associated energy generation (by processes called nuclear fusion or nuclear burning), neutrino luminosity, and evolution of stars. A good knowledge of the rates of these fusion reactions is essential for understanding this broad picture

In a stellar plasma the constituent nuclei are usually in thermal equilibrium at some local temperature T. Occasionally they collide with other nuclei, whereby two different nuclei can emerge from a collision A+x→ c+C. The cross section σ(E) of nuclear rearrangement reaction A(x,c)C is of course governed by the laws of quantum mechanics where, in most cases, the Coulomb and centrifugal barriers arising from nuclear charges and angular momenta in the entrance channel of the reaction strongly inhibit the penetration of one nucleus into another. This barrier penetration leads to a steep energy dependence of the cross section. It is the challenge to the experimentalist to make precise σ(E) measurements at the Gamow energy (E_G), i.e. the energy range where the reaction rate in a given temperature range is maximum. Owing to the strong Coulomb suppression, the behavior of the cross section at E_G is usually extrapolated from the higher energies by using the definition of the smoother bare nucleus astrophysical factor S_b(E).

Although the S_b(E)-factor allows for an easier extrapolation, large uncertainties to $\sigma(E_G)$ may be introduced due to for instance the presence of unexpected resonances, or high energy

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tails of sub-threshold resonances. In order to avoid the extrapolation procedure, a number of experimental solutions were proposed in direct measurements for enhancing the signal-to-noise ratio at E_G.
Moreover, the measurements in the laboratory at ultra-low energies suffer from the complication due to the effects of electron screening [2]. This leads to an exponential increase of the laboratory measured cross section $\sigma_s(E)$ [or equivalently of the astrophysical factor $S_s(E)$] with decreasing energy relative to the case of bare nuclei. Then, although it is possible to measure cross sections in the Gamow energy range, the bare nucleus cross section σ_b is extracted by extrapolating the direct data behavior at higher energies where negligible electron screening contribution is expected. In order to decrease uncertainties in the case of charged particle induced reactions, extrapolations should be avoided and therefore experimental techniques were improved (e.g. by means of underground laboratories). After improving measurements (at very low energies), electron screening effects were discovered. Finally to extract from direct (shielded) measurements the bare astrophysical S_b(E)-factor, extrapolations were performed at higher energy. In any case the extrapolation procedure is once more necessary.

2 The Trojan Horse Method

Alternative methods for determining bare nucleus cross sections of astrophysical interest are needed. In this context a number of indirect methods, e.g. the Coulomb dissociation (CD) [3], the Asymptotic Normalization Coefficient method (ANC)[4, 5] and references therein and the Trojan-horse method (THM) were developed [6, 7]. For further information on the development and first principles of the method please refer to [8, 9]. The latter has already been applied several times to reactions connected with fundamental astrophysical problems such as primordial nucleosynthesis [10–13], lithium problem [14–16, 28], light elements depletion [20, 24], AGB [39] and Novae nucleosynthesis [33]. It was also applied to reactions induced by radioactive ion beams [34, 35] and neutrons [37, 47]. THM selects the quasi-free (QF) contribution of an appropriate three-body reaction performed at energies well above the Coulomb barrier to extract a charged particle two-body cross section at energies of astrophysical interest. The idea of the THM [7] is to extract the cross section of an astrophysically relevant two-body reaction

$$A + x \to c + C \tag{1}$$

at low energies from a suitable chosen three-body quasi-free reaction

$$A + a \to c + C + S \tag{2}$$

In this approach S acts as a spectator to the $A + x \to c + C$ binary interaction.This is done with the help of direct processes theory assuming that the Trojan Horse nucleus a has a strong $x \oplus S$ cluster structure [19, 45]. In many applications, this assumption is trivially fulfilled e.g. a = deuteron, x = proton, S = neutron. If the bombarding energy E_A is chosen high enough to overcome the Coulomb barrier in the entrance channel of the three-body reaction, both Coulomb barrier and electron screening effects are negligible. The polar approximation, used in the standard THM prescription has been extensively verified [40, 41] and constitutes a powerful validity test for the method which strenghtens the theoretical approach. We refer to [17, 18] for further and advanced theoretical approach to the method.

We just underline that THM allows to link the three-body cross section which is measured in the laboratory with the half-off-energy shell cross section of the binary process of astrophysical interest. Then after inclusion of the Coulombian effects data are then compared and normalized to direct data, at the higher energies available. After that phase, the reaction rate is calculated according to the standard prescriptions.

2.1 Identification and selection of the three-body reaction of interest

The first step in data-analysis is to identify the events related to the three-body reaction of interest for THM, $A + a \rightarrow c + C + S$ from the other ones occurring in the target. This is accomplished by studying the kinematic locus related to the above reaction and the Q-value spectrum. Coincidences between detectors aiming at c and C are examined and a typical plot of the particle energy detected in each detector is performed (see for instance fig. 4 of [40]). A narrow angular range ($\approx \pm 2^o$) is selected on both detectors and events coming from an appropriate Monte Carlo simulation, taking into account the geometrical properties of the experimental set-up as well as the features of the detectors. If a good agreement shows up, this allows further studies. Using a graphical cut which selects only the events overlapping with the Monte Carlo simulation, the Q-value spectrum is plotted (see figure 6 in [42]). A peak compatible, within the experimental errors, with the theoretical Q-value of the three-body reaction is expected. From now on only the events below the Q-value peak and inside the kinematical graphical cut in the kinematic locus will be used for further data-analysis, in order to be relatively confident that the reaction channel $A + a \rightarrow c + C + S$ is selected.

2.2 Selection of the quasi-free mechanism

The next step after identifying the 3-body process is to investigate the reaction mechanisms involved and to separate the quasi-free (QF) contribution from any other kind of reaction mechanisms as required by the THM prescriptions. This can be done by studying, among all the available observables, the most sensitive to the reaction mechanisms which is, by far, the shape of the momentum distribution, $|\varphi(\mathbf{p_s})|^2$. According to the prescriptions in [43, 44], the momentum distribution of the third and un-detected particle is examined. This gives a major constraint for the presence of the QF mechanism and the possible application of the THM. In order to extract the experimental momentum distribution of the the undetected particle, (the spectator after the QF process is identified and selected) , $|\varphi(\mathbf{p_s})|^2_{exp}$, the energy sharing method can be applied to each pair of coincidence detectors, selecting energy intervals, ΔE_{cm}. Keeping in mind the factorization of Eq. (3), see below, since $[(d\sigma/d\Omega)_{cm}]^{HOES}$ is nearly constant in an adequate energy interval, one can get the shape of the momentum distribution of the undetected neutron directly from the coincidence yield divided by the kinematical factor, as calculated from a suitable Monte Carlo simulation. The obtained typical momentum distribution is reported for instance in [44]. It is also compared with the theoretical distribution calculated from the Hulthen function (dashed line) with parameters taken from [45]. We can see how within the experimental errors the theoretical curve reproduces the experimental data, thus confirming the hypothesis that the neutron is acting as a spectator and that the process under investigation is a quasi-free mechanism. We only considered the s-wave since other contribution, i.e. the d-wave, were shown to be negligible [45]. According to the prescription adopted in [9] and in the standard THM approaches, only data in a limited $|p_s|$ range were chosen according to the chosen Trojan horse nucleus and used in the further analysis.

Once the experimental full width at half maximum (FWHM) Γ is obtained it has to be compared with the asymptotic theoretical value (e.g. for deuteron about 58 MeV/c) in order to highlight the presence of possible distortions. If those are present they should be taken into account as reported in [43, 44]. After this test we can stress the role of the spectator to the QF process, which constitutes a solid base for the further THM application to the studied three-body reaction for retrieving information on the binary reaction bare nucleus cross section of interest at astrophysical energies. For the following analysis only data with p_s <30 MeV/c are considered as arising from quasi-free mechanisms. This, of course, depends on the chosen

Trojan Horse nucleus; the above value of p_s <30 MeV/c is typical in most cases for a l =0 relative inter cluster motion as for ^{2}H, ^{6}Li, ^{3}He.

Reaction mechanisms other than the quasi-free one should be considered as a "noise" and removed from the dataset eligible for THM application. This is done for example for the sequential mechanisms, usually occurring in the target, where an intermediate compound nucleus is formed. Examples are shown in [28]. Once the quasi-free mechanism is selected the bulk of data available for THM analysis is consistently reduced. For typical cases around 90% of data are usually rejected because they are not arising from a quasi-free process.

2.3 Extraction of the binary reaction cross section

In the standard THM analysis, the two body cross section is derived by dividing the experimental three-body one by the product of the kinematic factor modulated by the momentum distribution of the spectator inside the Trojan Horse nucleus [8], i.e.

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\mathrm{HOES}} \propto \frac{\mathrm{d}^3\sigma}{\mathrm{dE}_{\alpha_1}\mathrm{d}\Omega_{\alpha_1}\mathrm{d}\Omega_{\alpha_2}} / \left(\mathrm{KF}\cdot|\varphi_{\mathrm{exp}}(\mathbf{p_s})|^2\right) \tag{3}$$

Usually the factors $KF \cdot |\varphi_{\mathrm{exp}}(\mathbf{p_s})|^2$ are calculated by means of a Monte Carlo simulation, taking into account the geometrical position of the detectors. The width of the momentum distribution is set to the experimentally measured value in order to account for the distortion effects arising at low transferred momenta as discussed in [43].

The extracted $[d\sigma/d\Omega]^{\mathrm{HOES}}$ as a function of E_{cm}, corrected for the penetration factor (usually described in terms of the Regular and Irregular Coulomb functions) has to be compared to direct data in order to perform normalization at higher available energy. This allows to calculate and compare also the astrophysical S(E)-factor, which is extracted from the cross section according to the usual definition. This has been done in a big number of cases, many times both above and below the Coulomb barrier thus showing the strength of the method. After this necessary step the reaction-rate can be calculated according to the standard expression available in literature. Multiple examples of this procedure and some results are given here for reference [22].

Despite several outstanding achievements like the application to radioactive ion beams or to neutron emitting or neutron induced reactions ([36, 50]), the method has still some points that should be improved, like for the necessity of normalization and the possibility of absolute measurements, the possible application to (p,γ) and (α, γ) reactions and so on. The possibility of measuring absolute cross sections is, for example, under consideration for the next applications in order to avoid the normalization procedure to direct data.

A summary of the recent results obtained by means of the THM method is summarized in table 1 together with references. Table 1 shows success for (p, α), (α, p), (α, n), (p, n), (n, α) and (n, p) reactions of astrophysical interest studied with the THM method, both with stable as well unstable beams. Moreover, our study of ^{12}C(^{12}C,α)^{20}Ne and ^{12}C(^{12}C,p)^{23}Na [27] has demonstrated that we can extend the method to heavier ions [32].

The THM is complementary to direct measurements which are needed in the higher energy range where indirect data must be normalized to available data in literature. A numbers of astrophysical scenarios were thus explored helping to shed light in many cosmic problems. We underline that in many cases the THM will be a powerful aid to nuclear astrophysics knowledge especially in the investigation of the interaction of neutron-induced reactions with unstable particles.

References

[1] E.M.Burbidge, G.R.Burbidge, W.A.Fowler and F.Hoyle, Rev. Mod. Phys. **29**, (1957) 547
[2] H.J.Assenbaum, K.Langanke and C.Rolfs, Z. Phys.A **327**, (1987) 461
[3] G.Baur, C.A. Bertulani & H Rebel , Nucl. Phys. A **458**, (1986) 188
[4] A.M.Mukhamedzhanov and R.E.Tribble, Phys.Rev. C **59**, (1999) 3418
[5] G.G. Kiss, M. La Cognata, C. Spitaleri et al., (2020) Phys. Lett. B, 135606
[6] G.Baur, Phys. Lett. B **178**, (1986) 135
[7] C.Spitaleri 1991, in *Problems of Fundamental Modern Physics, II* : Proceedings, Ed. by R. Cherubini,P. Dalpiaz, and B. Minetti (World Sci., 1991), p. 21
[8] Spitaleri, C., La Cognata, M., Lamia, L., Mukhamedzhanov, A. M., & Pizzone, R. G. 2016, European Physical Journal A, **52**, 77
[9] Spitaleri, C.; La Cognata, M.; Lamia, L.; Pizzone, R. G.; Tumino, A.; *Eur. Phys. Journ. A* **2019**, *55*, 161.
[10] A. Rinollo et al., Nucl. Phys. A **758**, (2005)146c
[11] A. Tumino et al., Phys. Lett. B **705**, (2011) 545
[12] R.G. Pizzone et al., Astrophys. J. **786**, (2014) 112
[13] C. Li, et al., Phys. Rev. C **92** (2015) 025805
[14] Aliotta, M., Spitaleri, C., Lattuada, M., et al. 2000, European Physical Journal A, **9**, 435
[15] R.G. Pizzone et al., A. & A. **438**, (2005) 779
[16] L. Lamia et al., Astrophys. J. **768**, (2013) 65
[17] A. Tumino, et al., Ann. Rev. Nucl. Part. Sc. **71**, 345 (2021)
[18] A. Tumino et al., in: Handbook of Nuclear Physics (Springer, Singapore, 2022)
[19] R.G. Pizzone, C. Spampinato et al., Eur. Phys. J. , **A56**, 199 (2020)
[20] C. Spitaleri et al., Phys. Rev. C **95**, (2017) 035801
[21] A. Caciolli et al., Eur. Phys. Jour. A **52**, 136, 2016
[22] S. Romano, et al., Eur. Phys. Jour. A **27**, (2006) 221
[23] Wen, Q.-G., Li, C.-B., Zhou, S.-H., et al. 2008, Phys. Rev. C **78**, 035805
[24] L. Lamia et al., Astrophys. J. **811**, (2015) 99
[25] M. La Cognata et al., Eur. Phys. J. A**27**, (2006) 249
[26] La Cognata, M., Spitaleri, C., Mukhamedzhanov, A., et al. 2010, The Astrophysical Journal, **708**, 796
[27] Indelicato, I., La Cognata, M., Spitaleri, C., et al. 2017, The Astrophysical Journal, **845**, 19
[28] L.Lamia, et al., J. Phys. G 39, (2012) 015106
[29] Pizzone, R. G., D'Agata, G., La Cognata, M., et al. 2017, The Astrophysical Journal, **836**, 57
[30] M.Lattuada et al., Ap. J. **562**, (2001) 1076
[31] D'Agata, G.; Pizzone, R. G. et al. 2018, The Astrophysical Journal, **860**, 61
[32] A. Tumino, C. Spitaleri, M. La Cognata, S. Cherubini, G. L. Guardo, M. Gulino, S. Hayakawa, I. Indelicato, L. Lamia, H. Petrascu, R. G. Pizzone et al., 2018, Nature, **557**, 687-690
[33] Sergi, M. L., Spitaleri, C., La Cognata, M., et al. 2015, Phys. Rev. C, **91**, 065803
[34] S. Cherubini et al., Phys. Rev. **C 92**, (2015) 015805
[35] Pizzone, R. G., Roeder, B. T., McCleskey, M., et al. 2016, European Physical Journal A, **52**, 24

[36] La Cognata, M., Pizzone, R. G., José, J., et al. 2017, The Astrophysical Journal, **846**, 65
[37] Guardo, G. L., Spitaleri, C., Lamia, L., et al. 2017, Phys. Rev. C, **95**, 025807
[38] La Cognata, M.; Spitaleri, C.; Trippella, O.; et al., 2012, Phys. Rev. Lett.,**109**, 232701
[39] S. Palmerini et al., Ap. J., **764**, (2013) 128
[40] R.G. Pizzone et al. 2011, Phys. Rev. C, **83**, 045801
[41] R.G. Pizzone et al. 2013, Phys. Rev. C **87**, 025805
[42] A.Tumino et al. 2008,Phys. Rev. C **78**, (2008) 064001
[43] R.G. Pizzone et al., Phys. Rev. C **71**, (2005) 058801
[44] R.G. Pizzone et al., Phys. Rev. C **80**, (2009) 025807
[45] L.Lamia,M. La Cognata, C. Spitaleri, B. Irgaziev, R.G. Pizzone, Phys. Rev. C **85**, (2012) 025805
[46] Pizzone R.G. et al., Eur. Phys. J. A, **56** 283 (2020)
[47] M. Gulino et al., Phys. Rev. C **87**, (2013) 012801
[48] La Cognata, et al., 2005, Phys. Rev. C, **72**, 065802
[49] L. Lamia et al., Ap. J. **850**, 175, (2017)
[50] L. Lamia, M. Mazzocco, R.G. Pizzone, et al. , Astrophys. J., **879**, 23 (2019)
[51] S.Hayakawa, M. La Cognata, L. Lamia et al., Astroph. J,**915**, L13, (2021)
[52] S. Palmerini, M. La Cognata, F. Hammache et al., 2021, European Physical Journal A, **136**, 898
[53] M. La Cognata, S. Palmerini, P. Adsley et al., 2022, Astrophys. J., **941**, 96

Table 1. Two-body reactions recently studied via Trojan Horse Method at the astrophysical energies.

Binary reaction	Indirect reaction	TH nucleus	Reference
^{7}Li(p,α)^{4}He	^{7}Li(d,α α)n	d = (p$\oplus$n)	[16, 30]
^{6}Li(d,α)^{4}He	^{6}Li(^{6}Li,α α)^{4}H	^{6}Li = ($\alpha$$\oplus$d)	[14]
^{6}Li(p,α)^{3}He	^{6}Li(d,α ^{3}He)n	d = (p$\oplus$n)	[28]
^{11}B(p ,α)^{8}Be	^{11}B(d ,^{8}Be α)n	d = (p$\oplus$n)	[28]
^{10}B(p,α)^{7}Be	^{10}B(d,^{7}Be α)n	d = (p$\oplus$n)	[20, 21]
^{9}Be(p,α)^{6}Li	^{9}Be(d,^{6}Li α)n	d = (p$\oplus$n)	[22, 23]
^{2}H(^{3}He,p)^{4}He	^{6}Li(^{3}He,p α)^{4}He	^{3}He = (p$\oplus$n)	[25]
^{2}H(d,p)^{3}H	^{2}H(^{6}Li,t p)^{4}He	^{6}Li = ($\alpha$$\oplus$d)	[10]
^{18}F(p,α)^{15}O	^{18}F(d,α^{15}O)n	d = (p$\oplus$n)	[34–36]
^{15}N(p,α)^{12}C	^{15}N(d,α^{12}C)n	d = (p$\oplus$n)	[25]
^{18}O(p,α)^{15}N	^{18}O(d,α^{15}N)n	d = (p$\oplus$n)	[26]
^{19}F(p,α)^{16}O	^{19}F(d,α^{16}O)n	d = (p$\oplus$n)	[27]
^{19}F(α,p)^{22}Ne	^{19}F(^{6}Li,p^{22}Ne)α	^{6}Li = ($\alpha$$\oplus$d)	[29, 31]
^{12}C(^{12}C,α)^{20}Ne	^{12}C(^{14}N,α d)^{20}Ne	^{14}N=^{12}(C $\oplus$d)	[32]
^{17}O(p,α)^{14}N	^{17}O(d,α^{14}N)n	d = (p$\oplus$n)	[33]
^{17}O(n,α)^{14}C	^{17}O(d,α^{14}C)H	d = (p$\oplus$n)	[37, 47]
^{13}C(α,n)^{16}O	^{13}C(^{6}Li,n d)^{16}O	^{6}Li = ($\alpha$$\oplus$d)	[38]
^{12}C(^{12}C,p)^{23}Na	^{12}C(^{14}N,p d)^{23}Na	^{14}N=^{12}(C $\oplus$d)	[32]
^{3}He(d,p)^{4}He	^{3}He(^{6}Li,p α)^{4}He	^{6}Li = ($\alpha$$\oplus$d)	[48]
^{3}He(n,p)^{3}H	^{3}He(^{2}H,pp)^{3}H	^{2}H = (p$\oplus$n)	[19]
^{7}Be(n,α)^{4}He	^{7}Be(^{2}H,$\alpha\alpha$)n	^{2}H = (p$\oplus$n)	[49–51]
^{27}Al(p,α)^{24}Mg	^{27}Al(^{2}H,α^{24}Mg)n	^{2}H = (p$\oplus$n)	[52, 53]