

# Anomalous enhancement of dilepton production due to soft modes in dense quark matter

Toru Nishimura<sup>1,2,\*</sup>, Masakiyo Kitazawa<sup>2,3,1</sup>, and Teiji Kunihiro<sup>2</sup>

<sup>1</sup>Department of Physics, Osaka University, 560-0043, Toyonaka, Osaka, Japan

<sup>2</sup>Yukawa Institute for Theoretical Physics, Kyoto University, 606-8502, Kyoto, Japan

<sup>3</sup>J-PARC Branch, KEK Theory Center, Institute of Particle and Nuclear Studies, KEK, 203-1, 319-1106, Shirakata, Tokai, Ibaraki, Japan

**Abstract.** We explore how the dilepton production rate is modified near the critical temperature of color superconductivity and QCD critical point by the soft modes inherently associated with the phase transitions of second-order. It is shown that the soft modes affect the photon self-energy significantly through so called the Aslamasov-Larkin, Maki-Thompson and density of states terms, which are known responsible for the paraconductivity in the metallic superconductivity, and cause an anomalous enhancement of the production rate in the low energy/momentum region.

## 1 Introduction

Rich physics is expected to exist in high baryon-density matter at finite temperature, and may be hopefully revealed by experimental programs in relativistic heavy-ion collisions (HIC) such as the beam energy scan program at RHIC, HADES and NA61/SHINE, as well as the future plans at FAIR, NICA and J-PARC-HI. In this report, we show a promising observability of the color superconducting (CSC) [1] phase transition and QCD critical point (CP) in these experiments through an anomalous enhancement of the dilepton production rate (DPR) caused by the soft modes inherently associated with these phase transitions of second-order; the CSC phase transition is assumed to be second-order [4]. We calculate the effects of the soft modes on the DPR by extending the theory of the paraconductivity in metals [5], and show that the soft modes lead to an anomalous enhancement of the DPR at low energy region near each phase transition.

## 2 Formalism

To explore the temperature and density region around the 2-flavor color superconductivity (2SC) and QCD CP, which is expected to be realized at relatively low densities, we employ the 2-flavor NJL model

$$\mathcal{L} = \bar{\psi}i(\gamma^\mu\partial_\mu - m)\psi + G_S[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] + G_C(\bar{\psi}i\gamma_5\tau_2\lambda_A\psi^C)(\bar{\psi}^C i\gamma_5\tau_2\lambda_A\psi), \quad (1)$$

where  $\tau_2$  ( $\gamma_{A=2,5,7}$ ) is the antisymmetric component of the Pauli (Gell-Mann) matrices for the flavor  $SU(2)_f$  (color  $SU(3)_c$ ), and  $\psi^C(x) \equiv C\bar{\psi}^T(x)$  with  $C = i\gamma_2\gamma_0$ . We consider the chiral

\*e-mail: nishimura@kern.phys.sci.osaka-u.ac.jp

limit for the analysis of the DPR near the critical temperature ( $T_c$ ) of the 2SC, while we set to  $m = 4$  MeV for investigating the QCD CP. The scalar coupling constant  $G_S = 5.01$  MeV<sup>-2</sup> and the three-momentum cutoff  $\Lambda = 650$  MeV are used, which leads to the pion decay constant  $f_\pi = 93$  MeV and the chiral condensate  $\langle \bar{\psi}\psi \rangle = (-250 \text{ MeV})^3$ , while the diquark coupling  $G_C$  is treated as a free parameter.

The DPR is related to the retarded photon self-energy  $\Pi^{R\mu\nu}(k)$  as

$$\frac{d^4\Gamma}{d^4k} = -\frac{\alpha}{12\pi^4} \frac{1}{k^2} \frac{1}{e^{\omega/T} - 1} g_{\mu\nu} \text{Im}\Pi^{R\mu\nu}(k), \quad (2)$$

where  $k = (\mathbf{k}, \omega)$  is the four momentum of the photon and  $\alpha$  is the fine structure constant. In this study, we calculate the modification of  $\tilde{\Pi}^{\mu\nu}(k)$  due to the soft modes of the 2SC and QCD CP. In this section, we first calculate  $\tilde{\Pi}^{\mu\nu}(k)$  including the diquark soft modes of 2SC [4] in Sec. 2.1, and then deal with the soft modes of the QCD CP that are the mesonic modes in Sec. 2.2.

## 2.1 Modification of $\Pi^{\mu\nu}$ by diquark soft modes

It is known that the diquark fluctuations form a collective mode with a significant strength above but near  $T_c$  of the 2SC [2, 3]. The collective mode associated with the second-order nature of phase transition is called the soft mode. The propagator of the diquark soft modes in the random-phase approximation (RPA) in the imaginary-time formalism is given by

$$\tilde{\Xi}_C(k) = \frac{1}{G_C^{-1} + Q_C(k)}, \quad Q_C(k) = \begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \end{array} = -8 \int_p \text{Tr}[\mathcal{G}_0(k-p)\mathcal{G}_0(p)], \quad (3)$$

where  $Q_C(k) = Q_C(\mathbf{k}, i\nu_n)$  is the one-loop  $qq$  correlation function with  $\mathcal{G}_0(p) = \mathcal{G}_0(\mathbf{p}, i\omega_m) = 1/[(i\omega_m + \mu)\gamma_0 - \mathbf{p} \cdot \boldsymbol{\gamma}]$  the free quark propagator,  $\omega_m$  ( $\nu_n$ ) the Matsubara frequency for fermions (bosons), and Tr the trace over the Dirac indices. The retarded propagator is obtained by the analytic continuation  $\Xi_C^R(\mathbf{k}, \omega) = \tilde{\Xi}_C(\mathbf{k}, i\nu_n \rightarrow \omega + i\eta)$ .

We remark that  $\Xi_C^R(\mathbf{k}, \omega)$  satisfies the Thouless criterion, that is  $[\Xi_C^R(\mathbf{0}, 0)]^{-1} = 0$  at  $T = T_c$  for the second-order phase transition. The criterion shows that  $\Xi_C^R(\mathbf{k}, \omega)$  has a pole at the origin at  $T = T_c$ , and hence the diquark fluctuations become soft near  $T_c$  [2]. This fact also allows us to adopt the time-dependent Ginzburg-Landau (TDGL) approximation

$$[\Xi_C^R(\mathbf{k}, \omega)]^{-1} = A(\mathbf{k}) + C(\mathbf{k})\omega, \quad (4)$$

near but above  $T_c$ , where  $A(\mathbf{k}) = G_C^{-1} + Q_C^R(\mathbf{k}, 0)$  and  $C(\mathbf{k}) = \partial Q_C^R(\mathbf{k}, \omega)/\partial\omega|_{\omega=0}$ . It is known that Eq. (4) reproduces  $\Xi_C^R(\mathbf{k}, \omega)$  over wide ranges of  $\omega$ ,  $\mathbf{k}^2$  and  $T(> T_c)$  [2].

To construct  $\tilde{\Pi}^{\mu\nu}(k)$  that involves the soft mode, we start from the one-loop diagram of  $\tilde{\Xi}_C(\mathbf{k}, i\nu_n)$ , which is the lowest contribution of the soft modes to the thermodynamic potential [4]. The photon self-energy is then constructed by attaching electromagnetic vertices at two points of quark lines in the thermodynamic potential. One then obtains four types of diagrams shown in Fig. 1. These diagrams are called the Aslamasov-Larkin (AL) (Fig. 1(a)), Maki-Thompson (MT) (Fig. 1(b)) and density of states (DOS) (Fig. 1(c, d)) [5] terms, respectively, in the theory of metallic superconductivity. Each contribution to the photon self-energy,  $\tilde{\Pi}_{\text{AL}}^{\mu\nu}(k)$ ,  $\tilde{\Pi}_{\text{MT}}^{\mu\nu}(k)$  and  $\tilde{\Pi}_{\text{DOS}}^{\mu\nu}(k)$ , respectively, is given by

$$\tilde{\Pi}_{\text{AL}}^{\mu\nu}(k) = 3 \int_q \tilde{\Gamma}^\mu(q, q+k) \tilde{\Xi}_C(q+k) \tilde{\Gamma}^\nu(q+k, q) \tilde{\Xi}_C(q), \quad (5)$$

$$\tilde{\Pi}_{\text{MT(DOS)}}^{\mu\nu}(k) = 3 \int_q \tilde{\Xi}_C(q) \mathcal{R}_{\text{MT(DOS)}}^{\mu\nu}(q, k), \quad (6)$$

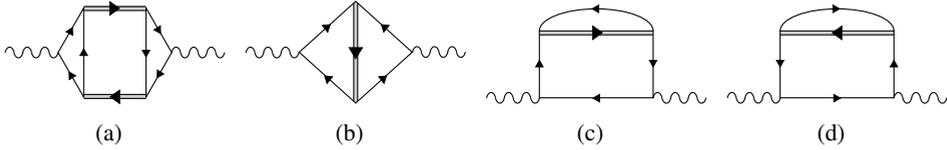


Figure 1: Diagrammatic representations of the Aslamasov-Larkin (a), Maki-Thompson (b) and density of states (c,d) terms with the diquark soft mode. The wavy and double lines are photons and the diquark soft modes, respectively.

where  $\tilde{\Gamma}^\mu(q, q+k)$  and  $\mathcal{R}^{\mu\nu}(q, k) = \mathcal{R}_{\text{MT}}^{\mu\nu}(q, k) + \mathcal{R}_{\text{DOS}}^{\mu\nu}(q, k)$  satisfy the following Ward-Takahashi (WT) identities

$$k_\mu \tilde{\Gamma}^\mu(q, q+k) = (e_u + e_d)[Q_C(q+k) - Q_C(q)], \quad (7)$$

$$k_\mu \mathcal{R}^{\mu\nu}(q, k) = (e_u + e_d)[\tilde{\Gamma}^\nu(q-k, q) - \tilde{\Gamma}^\nu(q, q+k)], \quad (8)$$

where  $e_u = 2|e|/3$  ( $e_d = -|e|/3$ ) is the electric charge of up (down) quarks. The total photon self-energy is then given by  $\tilde{\Pi}^{\mu\nu}(k) = \tilde{\Pi}_{\text{free}}^{\mu\nu}(k) + \tilde{\Pi}_{\text{AL}}^{\mu\nu}(k) + \tilde{\Pi}_{\text{MT}}^{\mu\nu}(k) + \tilde{\Pi}_{\text{DOS}}^{\mu\nu}(k)$ . One can explicitly check that this photon self-energy satisfies the WT identity using Eqs. (7) and (8).

To obtain the DPR with Eq. (2), the calculation of the spatial components of  $\Pi^{R\mu\nu}(k)$  is sufficient since  $\Pi^{R00}(k)$  is expressed in terms of the longitudinal part from the WT identity as  $\Pi^{R00}(k) = \mathbf{k}^2 \Pi^{R11}(k)/k_0^2$  with  $k = (k_0, |\mathbf{k}|, 0, 0)$ . For the vertex functions (7) and (8), we approximate  $\tilde{\Gamma}^\mu(q, q+k)$  and  $\mathcal{R}^{\mu\nu}(q, k)$  using Eq. (4). For  $\tilde{\Gamma}^i(q, q+k)$ , we employ an ansatz

$$\tilde{\Gamma}^i(q, q+k) = -(e_u + e_d) \frac{Q_C(\mathbf{q} + \mathbf{k}, 0) - Q_C(\mathbf{q}, 0)}{|\mathbf{q} + \mathbf{k}|^2 - |\mathbf{q}|^2} (2q+k)^i, \quad (9)$$

which is a real number and satisfies the WT identity (7) with Eq. (4). One can also obtain  $\mathcal{R}^{ij}(q, k)$  in the same manner, and finds that it is a real number as well. Using this vertex and Eq. (4), the imaginary part of  $\Pi_{\text{MT}}^{Rij}(q) + \Pi_{\text{DOS}}^{Rij}(q)$  vanishes. Therefore, the MT and DOS terms do not contribute to the calculation of the DPR. This result is in accordance with the case of the metallic superconductivity [5].

## 2.2 Modification of $\Pi^{\mu\nu}$ by mesonic soft modes near QCD CP

The photon self-energies including the mesonic soft modes of the QCD CP can also be calculated with a similar manner. The propagator of the mesonic mode is evaluated by the RPA as

$$\tilde{\Xi}_S(k) = \frac{1}{G_S^{-1} + Q_S(k)}, \quad Q_S(k) = \text{loop diagram} = 12 \int_p \text{Tr}[\mathcal{G}_0(p-k)\mathcal{G}_0(p)], \quad (10)$$

where  $Q_S(k)$  is the one-loop  $q\bar{q}$  correlation function. As described in Ref. [6],  $\Xi_S^R(k)$  has a discontinuity at the light cone, and its strength is limited to the space-like region. This property is contrasted with the diquark soft mode (3), which does not have the discontinuity at the light cone, while the diquark soft mode also has a significant strength in the space-like region. The photon self-energy with the mesonic modes is given by

$$\tilde{\Pi}_{\text{AL}}^{\mu\nu}(k) = \sum_f \int_q \tilde{\Gamma}_f^\mu(q, q+k) \tilde{\Xi}_S(q+k) \tilde{\Gamma}_f^\nu(q+k, q) \tilde{\Xi}_S(q), \quad (11)$$

$$\tilde{\Pi}_{\text{MT(DOS)}}^{\mu\nu}(k) = \sum_f \int_q \tilde{\Xi}_S(q) \mathcal{R}_f^{\mu\nu}{}_{\text{MT(DOS)}}(q, k), \quad (12)$$

where  $f = u, d$  are the indices of the flavors.  $\tilde{\Gamma}_f^\mu(q, q+k)$  and  $\mathcal{R}_f^{\mu\nu}(q, k) = \mathcal{R}_f^{\mu\nu}{}_{\text{MT}}(q, k) + \mathcal{R}_f^{\mu\nu}{}_{\text{DOS}}(q, k)$  satisfy the WT identity, similarly to Eqs. (7) and (8).

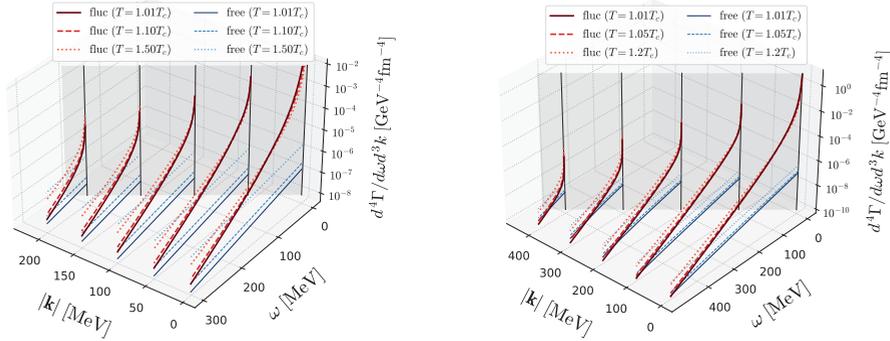


Figure 2: DPRs per unit energy  $\omega$  and momentum  $\mathbf{k}$  above  $T_c$  of the 2SC at  $\mu = 350$  MeV (left) [4] and of the QCD CP at  $\mu = \mu_{CP}$  (right). The thick lines are the contribution of the soft modes, and the thin lines are the results for the free quark gas.

### 3 Numerical results and summary

In the left panel of Fig. 2, we show the DPR per unit energy and momentum  $d^4\Gamma/d^4k$  near the 2SC phase transition for  $T = 1.01T_c, 1.1T_c$  and  $1.5T_c$  at  $\mu = 350$  MeV ( $T_c = 42.94$  MeV) [4]. The thick lines are the contribution from the soft modes, while the thin lines are the ones of the free quark gas. One sees that the DPR from the soft modes is anomalously enhanced in the small  $\omega$  and  $\mathbf{k}$  region in comparison with the free quark gas for  $T \lesssim 1.5T_c$ , and this enhancement becomes more pronounced as  $T$  approaches  $T_c$ . This result is expected through the properties of the soft modes. Shown in the right panel of Fig. 2 is the DPR near the QCD CP for  $T = 1.01T_{CP}, 1.05T_{CP}$  and  $1.2T_{CP}$  at  $\mu = \mu_{CP}$ , where  $(T_{CP}, \mu_{CP}) = (35.07, 322.20)$  MeV is the location of the QCD CP. One finds that the anomalous enhancement of the DPR is also observed similarly to the case of the 2SC.

The low-energy part of the DPR is related to the electrical conductivity  $\sigma$  and relaxation time  $\tau$ , which are obtained from differentiations of  $\rho_L(\mathbf{0}, \omega) = \sum_i \text{Im} \Pi_{AL}^{Rii}(\mathbf{0}, \omega) / \pi$  at  $\omega = 0$ . The analysis of  $\sigma$  and  $\tau$  thus allows us to understand behavior of the DPR near  $T_c$  more quantitatively. We find that these quantities diverge at  $T_c$  with  $\sigma \propto \epsilon^{-1/2}$  and  $\tau \propto \epsilon^{-3/2}$  for the 2SC, while for the case of the QCD CP,  $\sigma \propto \epsilon^{-2/3}$  and  $\tau \propto \epsilon^{-1}$  with  $\epsilon = |T - T_c|/T_c$ , as will be discussed in the forthcoming publication. The origin of the difference of two cases is due to the  $T$ -dependence of  $A(\mathbf{k})$  and the  $\mathbf{k}$ -dependence of  $C(\mathbf{k})$  in Eq. (4).

In this report, we studied how the soft modes of the 2SC and QCD CP affect the DPR near but above  $T_c$ . The modification of the photon self-energy due to the soft modes are investigated through the analysis of the Aslamasov-Larkin, Maki-Thompson and density of states terms with the TDGL approximation for the soft mode propagator and vertices. The enhancement of the DPR found in this study would allow us to detect these signals of the 2SC and QCD CP in the future HIC experiments.

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