

Polarization in heavy ion collisions: a theoretical review

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Abstract. In these proceedings I discuss the recent progress in the theory of spin polarization in relativistic fluids. To date, a number of studies have begun to examine the impact of the shear tensor on the local spin polarization and whether this contribution can restore agreement between the measurements and the predictions obtained from a polarization induced by the gradients of the plasma. I present the derivation of the spin polarization vector of a fermion at local thermal equilibrium and I discuss the role of pseudo-gauge transformations and of dissipative effects. I list what we can learn from the polarization measured at lower energies. Finally, I discuss possible applications of spin polarization measurements in relativistic heavy ion collisions.

1 Introduction

Particles in a rotating medium tends to align their spin in the direction of the total angular momentum due to the spin-rotation coupling. It was then expected that fermions produced in relativistic heavy-ion collisions will have a global spin polarization proportional to the vorticity of the fluid [1–3]. Based on this idea, one obtains that the spin polarization vector of a fermion is given by a Cooper-Frye like formula [4]:

$$S^\mu(p) = -\frac{1}{8m} \epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma d\Sigma \cdot p n_F (1 - n_F) \varpi_{\rho\sigma}}{\int_\Sigma d\Sigma \cdot p n_F}, \quad (1)$$

where

$$\varpi_{\mu\nu} = -\frac{1}{2} (\partial_\mu \beta_\nu - \partial_\nu \beta_\mu) \quad (2)$$

is the thermal vorticity tensor and $n_F = (e^{\beta \cdot p - \mu/T} + 1)^{-1}$ is the Fermi distribution function with $\beta^\mu = u^\mu/T$ the four-temperature vector and μ a chemical potential. The predictions for global polarization obtained from this formula were confirmed in 2017 with Λ hyperons produced in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV [5, 6], see for instance [7] for a review.

However, models based on the vorticity induced spin polarization (1) are not able to reproduce later measurements of local Λ spin polarization, that is as a function of the particle momentum [8–10]. This is known as local polarization sign puzzle because the models predicted the opposite sign for the longitudinal (along the beam axis) component of the spin polarization vector. Once it was established that this discrepancy can not be explained with

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feed-down corrections [11, 12], the sign puzzle was a strong motivation for theoretical investigation as something relevant was clearly missing in the theoretical description. Further investigations, that I review in next section, indeed revealed that spin polarization can also be induced by the shear flow of the fluid and, for heavy-ion collisions, this turned out to be a relevant contribution.

2 Spin polarization at local thermal equilibrium

In this section I review the derivation of the spin polarization vector of a fermion in a fluid that reached the local thermal equilibrium. The spin polarization vector is obtained from the particle branch of the Wigner function using (see the review [13] for details)

$$S_{\omega}^{\mu}(p) = \frac{1}{2} \frac{\int_{\Sigma_{FO}} d\Sigma \cdot p \operatorname{tr}_4 [\gamma^{\mu} \gamma^5 W_{+}(x, p)]}{\int_{\Sigma_{FO}} d\Sigma \cdot p \operatorname{tr}_4 [W_{+}(x, p)]}, \quad (3)$$

where, for heavy-ion collision applications, the hypersurface Σ_{FO} is the decoupling hypersurface, where the system can be described with the quasi-free hadronic effective fields. Hence, Λ particles being quasi-free, we can use the Wigner function of a free Dirac field:

$$W(x, k)_{AB} = \frac{1}{(2\pi)^4} \int d^4y e^{-ik \cdot y} \langle : \bar{\Psi}_B(x + y/2) \Psi_A(x - y/2) : \rangle, \quad (4)$$

where the brackets denotes the thermal average with the statistical operator $\hat{\rho}$: $\langle \hat{X} \rangle = \operatorname{tr}(\hat{\rho} \hat{X})$.

For a relativistic system whose underlying microscopic theory is a quantum field theory, the statistical operator describing the thermal states can be obtained using the Zubarev method [14]. By maximizing the entropy of the system at the moment of thermalization and imposing constraints on the momenta and energy densities, one obtains the covariant statistical operator at the time of decoupling

$$\hat{\rho} = \frac{1}{Z} \exp \left[- \int_{\Sigma_{FO}} d\Sigma_{\mu} (\widehat{T}_B^{\mu\nu} \beta_{\nu} - \zeta \widehat{j}^{\mu}) + \int_{\Theta} d\Theta (\widehat{T}_B^{\mu\nu} \nabla_{\mu} \beta_{\nu} - \widehat{j}^{\mu} \nabla_{\mu} \zeta) \right], \quad (5)$$

where $\widehat{T}_B^{\mu\nu}$ is the Belinfante stress-energy tensor (SET) operator, \widehat{j} is a conserved current, and $\zeta = \mu/T$. One can show that the second term in the exponent gives rise to dissipative effects, while the first term gives the non-dissipative local thermal equilibrium. I postpone the discussion of the dissipative part in a later section and here I focus on the local equilibrium statistical operator:

$$\hat{\rho} \simeq \hat{\rho}_{LE} = \frac{1}{Z} \exp \left[- \int_{\Sigma_{FO}} d\Sigma_{\mu} (\widehat{T}_B^{\mu\nu} \beta_{\nu} - \zeta \widehat{j}^{\mu}) \right]. \quad (6)$$

The derivation of the spin polarization at local thermal equilibrium from Eq. (3) reduces to the evaluation of the Wigner function at local equilibrium

$$W(x, k)_{LE} = \frac{1}{Z} \operatorname{tr} \left(\exp \left[- \int_{\Sigma_{FO}} d\Sigma_{\mu}(y) (\widehat{T}_B^{\mu\nu}(y) \beta_{\nu}(y) - \zeta(y) \widehat{j}^{\mu}(y)) \right] \widehat{W}(x, k) \right). \quad (7)$$

For a fluid in the hydrodynamic regime, a good approximation of (7) is obtained expanding the thermodynamic fields as a series of gradients around the point x and using the linear response theory to obtain the Wigner function mean value. Keeping the first order in derivatives of the four-temperature and of the chemical potential

$$\beta_{\nu}(y) \simeq \beta_{\nu}(x) + \partial_{\lambda} \beta_{\nu}(x) (y - x)^{\lambda}, \quad \zeta(y) \simeq \zeta(x) + \partial_{\lambda} \zeta(x) (y - x)^{\lambda}, \quad (8)$$

one finds

$$\widehat{\rho}_{LE} \simeq \frac{1}{Z} \exp \left[-\beta_v(x) \widehat{P}^v + \frac{1}{2} \varpi_{\mu\nu}(x) \widehat{J}_x^{\mu\nu} - \frac{1}{2} \xi_{\mu\nu}(x) \widehat{Q}_x^{\mu\nu} + \partial_\lambda \zeta(x) \int d\Sigma_\mu (y-x)^\lambda \widehat{j}^\mu(y) + \dots \right], \quad (9)$$

where

$$\widehat{J}_x^{\mu\nu} = \int d\Sigma_\lambda \left[(y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y) \right], \quad (10)$$

is the conserved angular momentum operator (the generator of boosts and rotations) and

$$\widehat{Q}_x^{\mu\nu} = \int d\Sigma_\lambda \left[(y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y) \right], \quad (11)$$

is a non-conserved symmetric quadrupole like operator. Notice that $\widehat{J}^{\mu\nu}$ couples with the thermal vorticity ϖ , while $\widehat{Q}^{\mu\nu}$ couples with the thermal shear tensor defined as

$$\xi_{\mu\nu} = \frac{1}{2} \left(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu \right). \quad (12)$$

The spin polarization vector obtained from (3) using the linear response theory on the operators appearing in (9) is

$$S^\mu(p) = -\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma d\Sigma \cdot p n_F (1 - n_F) \left[\varpi_{\rho\sigma} + 2 \hat{t}_\rho \frac{p^\lambda}{\varepsilon} \xi_{\lambda\sigma} - \frac{\hat{t}_\rho \partial_\sigma \zeta}{2\varepsilon} \right]}{8m \int_\Sigma d\Sigma \cdot p n_F}, \quad (13)$$

where \hat{t} is the time direction in the laboratory frame. The first term is the vorticity induced polarization (1), the second is the recently found shear induced polarization [15–19], and the last one is the contribution from the gradient of chemical potential obtained in [20, 21], sometimes called spin Hall effect term.

2.1 Thermal shear and solution of the sign puzzle

The effect of thermal shear in spin polarization was studied in ref. [16, 19, 22–27]. These analyses revealed the following facts. The thermal shear tensor at the freeze-out hypersurface can be larger than the thermal vorticity. Because of its momentum dependence (see Eq. (13)), the thermal shear does not contribute to global polarization but it was found to give a large contribution to local spin polarization. Predictions including the thermal shear are closer to the experimental data but they are still lacking a good quantitative agreement.

In order to reproduce the experimental data, no new effects are needed. It is sufficient to improve the approximation used to derive the spin polarization spectrum (13). Since the available experimental data for local spin polarization was taken at 200 GeV and at 5 TeV, at such high energies it is expected that the hadronization happens at the same temperature. In that case the statistical operator is given by the Isothermal Local Equilibrium operator:

$$\widehat{\rho}_{ILE} = \frac{1}{Z} \exp \left[- \int_{\Sigma_{FO}} d\Sigma_\mu \widehat{T}_B^{\mu\nu} \left(\frac{u_\nu}{T_{FO}} \right) \right] = \frac{1}{Z} \exp \left[- \frac{1}{T_{FO}} \int_{\Sigma_{FO}} d\Sigma_\mu \widehat{T}_B^{\mu\nu} u_\nu \right], \quad (14)$$

because the temperature T_{FO} is constant along the decoupling hypersurface. In this setting, expanding the temperature in a Taylor series together with β as in Eq. (8) will only introduce unnecessary mistakes. The appropriate spin polarization spectrum at high energies involves only the fluid velocity gradients [16]:

$$S_{ILE}^\mu(p) = -\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma d\Sigma \cdot p n_F (1 - n_F) \left[\omega_{\rho\sigma} + 2 \hat{t}_\rho \frac{p^\lambda}{\varepsilon} \Xi_{\lambda\sigma} \right]}{8m T_{FO} \int_\Sigma d\Sigma \cdot p n_F}, \quad (15)$$

where

$$\omega_{\rho\sigma} = \frac{1}{2}(\partial_\sigma u_\rho - \partial_\rho u_\sigma), \quad \Xi_{\rho\sigma} = \frac{1}{2}(\partial_\sigma u_\rho + \partial_\rho u_\sigma) \quad (16)$$

denote the kinematic vorticity and shear tensors. The predictions of (15) are in quantitative agreement with the data taken at 200 GeV [16]. Preliminary results presented at this conference [28] showed an improvement of the agreement when including the feed-down effects and that the predictions of (15) are also in quantitative agreement with the local spin polarization measured by ALICE [10]. Future analyses of high energy spin polarization should use the isothermal settings.

What also emerged from these analyses is that the local spin polarization is sensitive to the properties of the plasma, which ultimately determines the gradients of the thermo-hydrodynamic fields in (13). This feature allows to use spin polarization as a probe of the quark gluon plasma properties.

2.2 Low energies

With new data of spin polarization at very low energy [29], we can further test the models and see whether they can reproduce the maximum of spin polarization [30–34] expected to occurs around $\sqrt{s_{NN}} = 7$ GeV. This energy range is also important for polarization in the FAIR and NICA facilities. At lower energies we also observe a larger deviation between the polarization of Λ and $\bar{\Lambda}$, which allows to probe the gradients of baryonic chemical potential [20, 21] and the effect of the magnetic field [35]. Physics of rotating matter at these energies was also discussed in this conference [36].

2.3 Spin tensor and pseudo-gauge dependence

For the first time, the importance of the spin degrees of freedom in the determination of the spin polarization vector, opens the possibility to have a direct experimental test of a local description of spin, i.e. a spin tensor. Indeed one can include a spin tensor in hydrodynamic equations [37–43] and let it evolve with the rest. In the same way that the temperature is a thermodynamic potential for energy, the thermodynamic quantity associated with spin is called spin potential: $\Omega_{\mu\nu}$, and it might be present in spin polarization.

The definition of spin tensor is not unique in special relativity but is given up to a transformation called pseudo-gauge transformation. For instance, from the Belinfante pseudo-gauge with SET $\widehat{T}_B^{\mu\nu}$ and vanishing spin tensor, a generic pseudo-gauge Φ is obtained with:

$$\begin{aligned} \widehat{T}_\Phi^{\mu\nu} &= \widehat{T}_B^{\mu\nu} + \frac{1}{2}\nabla_\lambda (\widehat{\Phi}^{\lambda,\mu\nu} - \widehat{\Phi}^{\mu,\lambda\nu} - \widehat{\Phi}^{\nu,\lambda\mu}), \\ \widehat{S}_\Phi^{\lambda,\mu\nu} &= -\widehat{\Phi}^{\lambda,\mu\nu} + \nabla_\rho \widehat{Z}^{\mu\nu,\lambda\rho}, \end{aligned} \quad (17)$$

where $\widehat{\Phi}^{\lambda,\mu\nu} = -\widehat{\Phi}^{\lambda,\nu\mu}$ and $\widehat{Z}^{\mu\nu,\lambda\rho} = -\widehat{Z}^{\nu\mu,\lambda\rho} = -\widehat{Z}^{\mu\nu,\rho\lambda}$. This raises the question whether a pseudo-gauge transformation affects the results for spin polarization. It was indeed realized that the local equilibrium form of statistical operator is different for different pseudo-gauges [44]:

$$\widehat{\rho}_{\text{LTE}}^\Phi = \frac{1}{Z} \exp \left\{ - \int d\Sigma_\mu \left[\widehat{T}_B^{\mu\nu} \beta_\nu - \frac{1}{2} (\omega_{\lambda\nu} - \Omega_{\lambda\nu}) \widehat{\Phi}^{\mu,\lambda\nu} - \xi_{\lambda\nu} \widehat{\Phi}^{\lambda,\mu\nu} - \frac{1}{2} \Omega_{\lambda\nu} \nabla_\rho \widehat{Z}^{\lambda\nu,\mu\rho} - \widehat{j}^\mu \zeta \right] \right\}. \quad (18)$$

Starting from this, the different predictions of the spin polarization spectrum for several pseudo-gauge were obtained [45]. This difference is not only related to the contribution of

the spin potential but also of the thermal shear. This feature is not unique of the statistical mechanics approach and the role of pseudo-gauge was studied in different approaches [46–50]. This is an unique opportunity to study the physical meaning of pseudo-gauge transformations and possibly to shed light on the role of spin potential in gravitational phenomena.

3 Dissipative effects and relativistic kinetic theory with spin

The current data and the inclusion of the thermal shear support the local equilibrium picture with a small contribution of dissipative effects. However, one should check how much the dissipative phenomena affect the spin polarization. These corrections in the spin polarization formula have been found for the massless [51] and massive [52] Dirac field, but so far there are no analyses on the impact of these effects on the spin polarization of Λ particles in heavy-ion collisions. If the dissipative effects will be found to be relevant, this would imply a separation of scales between the interaction time and the time in which the spin degrees of freedom equilibrates [39, 40, 53]. In this scenario the inclusion of a spin tensor in hydrodynamics mentioned above will be crucial.

An estimate of the magnitude of the dissipative effect might not be easy, as it was realized that, in addition to usual transport coefficients, in a hydrodynamic with spin additional transport coefficients related to spin emerge [39, 40, 52, 53]. The values of these new transport coefficients for the QGP can be fixed by fitting the data of spin polarization, but in this way one inevitably loses the predictive power of the current theory.

A powerful approach to study dissipative effects and the role of spin in hydrodynamics is the relativistic kinetic theory. The Wigner-function formalism in particular is convenient to preserve all the covariant properties of the system and to find results as an expansion in \hbar . Motivated by spin polarization, this approach recently experienced a rich and fast development [54–59].

In this regard, I would like to draw attention to an often overlooked assumption needed to solve the Wigner equation in kinetic theory with collisions:

$$\left[\gamma \cdot \left(p + i \frac{\hbar}{2} \partial \right) - m \right] W_{\alpha\beta} = \hbar C_{\alpha\beta}. \quad (19)$$

Wigner equation itself is not enough to determine all the properties of the system. For instance, changing $\widehat{\Phi}$ in (18) results in different Wigner functions all solving Eq. (19). Indeed the determination of the non equilibrium, dissipative Wigner function from (19) requires the input of the equilibrium Wigner function. For a system in absence of vorticity, the equilibrium form is well known. But the general form of global equilibrium Wigner function in the presence of vorticity was obtained only recently [60, 61]. Usually the ansatz for the equilibrium form used in kinetic theory agrees with the correct form only at first order in thermal vorticity. For the current applications is enough, but one has to keep it in mind when deriving higher order results.

4 Applications of spin polarization in heavy ion collisions

Spin is a new element in heavy-ion collisions and as a commonly accepted theoretical framework is being built, several ideas on how to use it as a diagnostic tool for the QGP have been proposed. What makes spin polarization a good probe to the QGP properties is its dependence on the gradients of the thermo-hydrodynamics fields (fluid velocity, temperature and baryonic chemical potential).

For instance, reproducing the vorticity structure of the plasma requires knowledge of the properties and evolution of the QGP. It was then proposed to use spin correlations of two Λ hyperons to probe the vorticity structure of the fluid [23, 62, 63]. In this way one acquires more constraints on the properties of the QGP.

Correlations of helicity, that is the projection of spin along the momentum of the particle, of two hyperons in the same event can also reveal the presence of local parity violation in hot QCD matter [64–66]. Local parity violations should create an imbalance of right-handed and left-handed quarks in the plasma, described with an axial chemical potential μ_A . Currently the chiral magnetic effect is used to reveal the presence of μ_A [67]. But the axial imbalance should also add a contribution to spin polarization (13) as [65]

$$S^\mu(p) = S_{\text{hydro}}^\mu(p) + \frac{g_h}{2} \frac{\int_\Sigma d\Sigma \cdot p (\mu_A/T) n_F (1 - n_F) \varepsilon p^\mu - m^2 \hat{p}^\mu}{\int_\Sigma d\Sigma \cdot p n_F} \frac{m\varepsilon}{m\varepsilon}, \quad (20)$$

resulting into a peculiar (parity breaking) contribution to helicity correlations.

Spin polarization can be used to study anything that produce large enough gradients. This is the case of a jet crossing the plasma. The vorticity field generated by it contains information about the energy lost by the jet and can be studied with spin polarization measurements in reference to the jet axis [68].

The results derived so far for spin polarization do not include interactions. Vorticity induced polarization is a consequence of spin-rotation coupling $g_\Omega \vec{S} \cdot \vec{\Omega}$, and Einstein Equivalence Principle (EEP) prevents spin-rotation coupling to receive radiative corrections: $g_\Omega \equiv 1$ [69]. However, the breaking of Lorentz covariance at finite temperature also breaks the EEP and this allows interactions to affect the spin-rotation coupling [70]. For strong interactions the leading order correction is expected to be

$$g_\Omega = 1 - \frac{N_c^2 - 1}{2} \frac{1}{6} \frac{g^2 T^2}{m^2}, \quad T \ll m \quad (21)$$

which, in a typical event, results in a reduction of about 40% in the spin polarization of the strange quark. Whether we can detect this consequence of the breaking of EEP in the energy dependence of Λ polarization is still under investigation.

5 Conclusions

Spin polarization is a new topic in heavy ion collisions that just started to show its potential. It can be used to answer many questions left unanswered in relativistic hydrodynamics and to probe the properties of hot QCD matter. When all the relevant gradients are included and more accurate approximations adopted, the description of the QGP as a fluid close to local equilibrium with small viscous corrections is receiving confirmation as a solid model in heavy ion collisions. Right now, theory and experiments in this field are rapidly expanding and spin will be used as a tool to investigate fundamental physics in the QGP and beyond.

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