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Assessing critical point signatures through proton intermittency in NA61/SHINE

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Abstract. The search for experimental signatures of the critical point (CP) of strongly interacting matter is one of the main objectives of the NA61/SHINE experiment at CERN SPS. One such candidate observable is local fluctuations of the proton density in transverse space, constituting an order parameter of the chiral phase transition, and expected to scale according to a universal power-law in the vicinity of the CP. Its scaling can be probed through an intermittency analysis of the proton second scaled factorial moments (SSFMs) in transverse momentum space. The first such analysis [1] revealed power-law behavior in NA49 Si+Si collisions at 158A GeV/c, with a fitted exponent value consistent with the critical prediction, within errors.

In the present work, we discuss known challenges posed by standard intermittency analysis in the face of low to moderate event statistics, and propose a novel technique that solves them. We perform a scan of Monte Carlo models simulating alternative critical and background scenarios, and weigh them against experimental results; we explore the method's efficacy on simulated data. Once adapted to the available NA61/SHINE systems, the method will allow us to obtain reliable confidence intervals for the intermittency index (power-law exponent) ϕ_2 compatible with the experimental data.

1 Introduction

NA61/SHINE is a fixed target particle and high-energy nuclear physics experiment at CERN SPS [2], colliding a variety of beams on hydrogen and nuclear targets. One of the physics goals of its strong interactions programme is to search for experimental signatures of the critical point (CP) of strongly interacting matter. To this purpose, a scan is performed of the phase diagram of strongly interacting matter, varying system size (p+p, Be+Be, Ar+Sc, Xe+La, Pb+Pb) and collision energy (13A – 150A GeV/c), thus probing different freeze-out conditions in temperature T and baryochemical potential μ_B (Fig. 1).

A characteristic feature of a second order phase transition (expected to occur at the CP) is the divergence of the correlation length, leading to a scale-invariant system effectively described by a universality class. Of particular interest are local fluctuations of the order parameter of the QCD chiral phase transition, the chiral condensate $\sigma(\mathbf{x}) = \langle \bar{q}(\mathbf{x}) q(\mathbf{x}) \rangle$. At finite baryochemical potential, the critical fluctuations of the chiral condensate are transferred to the net-baryon density [3]. For a critical system, we expect proton density fluctuations to

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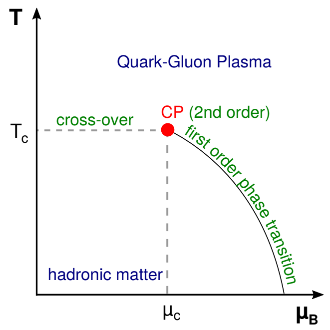

Figure 1. Hypothetical sketch of the phase diagram of strongly interacting matter with critical point, drawn as a function of baryochemical potential μ_B and temperature T.

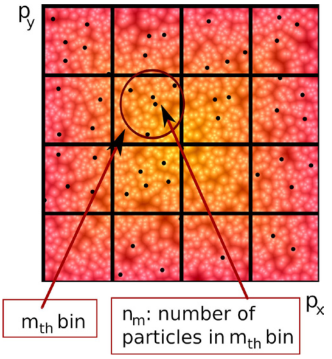

Figure 2. Counting proton pairs in a transverse momentum space partitioning of $M \times M$ equal size bins. *[Image by I. Sputowska]*

be self-similar [4], obeying power laws with critical exponents determined by the 3D Ising universality class [5–7]. Such fluctuations correspond to a power-law scaling of the proton density-density correlation function, which can be detected in transverse momentum space within the framework of an intermittency analysis [7, 8] of proton scaled factorial moments (SFMs). A detailed analysis can be found in Ref. [1], where we study various heavy nuclei collision datasets recorded in the NA49 experiment at the maximum energy (158A GeV/c, $\sqrt{s_{NN}} \approx 17$ GeV) of the SPS (CERN).

2 Methodology

Intermittency analysis examines how the Second Scaled Factorial Moments (SSFM) $F_2(M)$ of proton transverse momenta scale with the number of 2D bins M^2 at mid-rapidity (Fig. 2):

$$F_2(M) \equiv \left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i(n_i-1) \right\rangle \Big/ \left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \right\rangle^2 , \tag{1}$$

where n_i is the number of protons in the i-th bin, and $\langle \ldots \rangle$ denotes average over events.

For a pure critical system, $F_2(M)$ is predicted to follow a power-law [7]:

$$F_2(M) \sim M^{2\cdot\phi_{2,cr}} \quad , \quad \phi_{2,cr}^{(p)} = 5/6. \tag{2}$$

For a noisy system, mixed event moments must be subtracted from the data moments in order to recover the critical component [1]. Thus, we define the correlator $\Delta F_2(M)$:

$$\Delta F_2(M) = F_2^{(d)}(M) - F_2^{(m)}(M). \tag{3}$$

SSFMs statistical errors are estimated by the bootstrap method [9], whereby the original set of events is resampled with replacement.

3 Results

Intermittency analysis of peripheral NA61/SHINE Ar+Sc collisions at 150A GeV/c (Fig. 3) reveals a non-trivial scaling effect; however, large uncertainties in $F_2(M)$ due to low multiplicity and event statistics, as well as M-bin error correlations [9–13] bias power-law fits of $F_2(M)$ and prevent an unbiased estimation of ϕ_2 confidence intervals.

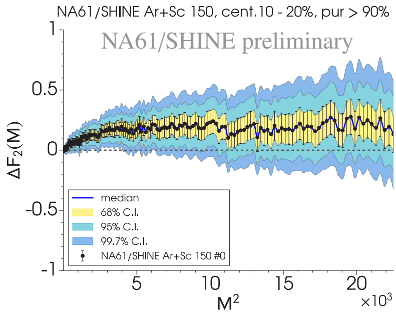

Figure 3. Correlated bin analysis results for $\Delta F_2(M)$ of NA61/SHINE Ar+Sc 10-20% most central collisions @ 150A GeV/c [14].

A better alternative to fitting for a power-law is to model the critical system, along with its background, for a variety of different power-law exponents and critical proton percentages; then, $F_2(M)$ values can be computed and compared to the corresponding experimental moments via a goodness-of-fit function (typically, a χ^2 metric). A scan performed for a large number of different models will hopefully reveal those compatible with the experimental results, thus allowing us to determine ϕ_2 confidence intervals.

We use a version of the Critical Monte Carlo (CMC) algorithm [7] adapted to proton cluster production, in order to simulate pure critical proton events. Subsequently, we "dilute" correlations by randomly replacing critical with uncorrelated protons at a fixed (adjustable) percentage. Fig. 4(a) shows $F_2(M)$ for 1000 iterations of the same critical + background model; Fig. 4(b) shows the correlation matrix between M-bins. Strong correlations between neighboring bins make the $F_2(M)$ distribution hard to understand and handle. We can untangle M-bins using the statistical technique of Principal Component Analysis (PCA), which determines linear combinations of the original M-bins such that their variations are maximal and statistically independent. Just the first few components give a fairly good reconstruction of the empirical $F_2(M)$ distribution.

Fig. 4(c) shows F_2 transformed by PCA, where the first 10 principal components have been kept. By construction, the new bins are uncorrelated , Fig. 4(d). We can now apply our goodness-of-fit function to the F_2 of the model vs the data, after we correspondingly transform the data to PCA-space. Fig. 5 shows the results of repeating this process on a grid of different models, each model compared to the one corresponding to Fig. 4. In this "exclusion plot", we see a narrow band of likely models (including our plug-in), surrounded by an "excluded" region. Event statistics (adjusted to that of Ref. [14]) is unfortunately insufficient to uniquely determine the optimal parameter set.

4 Conclusions

Proton intermittency analysis is a promising tool for detecting the critical point of strongly interacting matter. However, large uncertainties and bin correlations cannot be handled by the conventional analysis method. New techniques of scanning and evaluating Monte Carlo models against the data can handle statistical and systematic uncertainties without sacrificing event statistics.

A full model scan against NA61/SHINE experimental data, through a carefully calibrated Monte Carlo, is in progress; once completed, it will allow the estimation of ϕ_2 and critical component confidence intervals, for the Ar+Sc system as well as other NA61/SHINE available systems.

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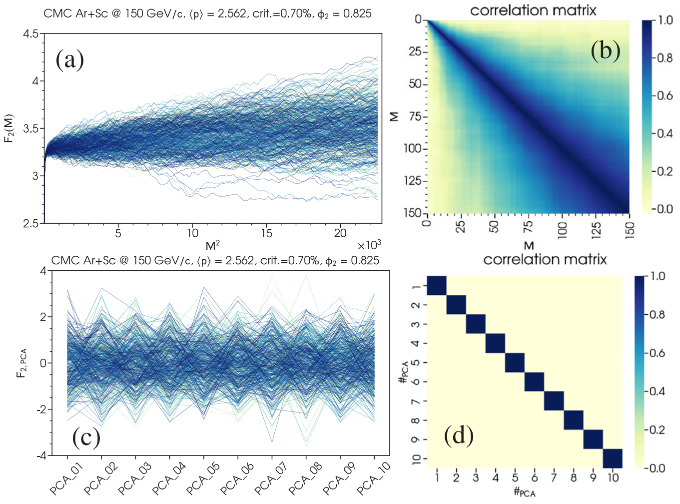

Figure 4. *(a)* $F_2(M)$ for 1000 iterations of CMC+background, for 0.70% critical protons and $\phi_2 = 0.825$, along with *(b)* the corresponding correlation matrix. *(c)* Principal Component Analysis (PCA) transformation of CMC $F_2(M)$; transformed correlation matrix *(d)* is diagonal by construction.

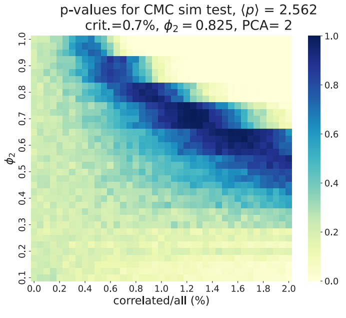

Figure 5. Exclusion plot comparing various models of ϕ_2 strength and critical proton percentage against the CMC model shown in Fig. 4. Color map corresponds to the p-value of each model vs the data, with high p-values corresponding to better fits. A narrow band of "best fit" models runs diagonally across the plot.

References

[1] T. Anticic et al. (NA49), Eur. Phys. J. C **75**, 587 (2015), `1208.5292`

[2] N. Abgrall et al., Journal of Instrumentation **9**, P06005 (2014)

[3] Y. Hatta, M.A. Stephanov, Phys. Rev. Lett. **91**, 102003 (2003), `hep-ph/0302002`

[4] T. Vicsek, *Fractal Growth Phenomena* (World Scientific, 1989), ISBN 9789971508302, `https://books.google.com/books?id=N8XSq7rTzTkC`

[5] N.G. Antoniou et al., Nucl. Phys. A **693**, 799 (2001), `hep-ph/0012164`

[6] N.G. Antoniou et al., Nucl. Phys. A **761**, 149 (2005), `hep-ph/0505185`

[7] N.G. Antoniou et al., Phys. Rev. Lett. **97**, 032002 (2006), `hep-ph/0602051`

[8] A. Bialas, R.C. Hwa, Phys. Lett. B **253**, 436 (1991)

[9] W.J. Metzger (2004), HEN-455 (unpublished), `https://repository.ubn.ru.nl/handle/2066/60397`

[10] B. Efron, The Annals of Statistics **7**, 1 (1979)

[11] B. Wosiek, Acta Phys. Polon. B **21**, 1021 (1990)

[12] C. Michael, Phys. Rev. D **49**, 2616 (1994), `hep-lat/9310026`

[13] N.G. Antoniou et al., Nucl. Phys. A **1003**, 122018 (2020), `2004.12133`

[14] N. Davis (NA61/SHINE), Acta Phys. Polon. Supp. **13**, 637 (2020), `2002.06636`