

# Quark susceptibilities, transport properties and heavy quark production in an extended Quasi-Particle Model with $N_f = 2 + 1 + 1$ flavors

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**Abstract.** The quark susceptibilities are a very useful tool to understand the nature of the degrees of freedom in the vicinity of the QCD phase transition while Heavy-Quarks (HQs) transport coefficients give us information on their thermalization time in the Quark-Gluon Plasma (QGP). Recently, new lattice results for the equation of state of QCD with  $2 + 1 + 1$  dynamical flavors have become available. Therefore, we extend our QPM approach for  $N_f = 2 + 1$  to  $N_f = 2 + 1 + 1$  where the charm quark is included. We also explore the extension of QPM approach to a more realistic model in which partonic propagators explicitly depend on the three-momentum with respect to the partonic matter at rest in order to match pQCD at high momenta following Dyson-Schwinger studies in the vacuum. In this context, we evaluate and correctly reproduce both EoS and quark susceptibilities which are underestimated in the simple QPM approach. Therefore, we study the impact of the extended QPM approach on the spatial diffusion coefficient  $D_s$  of charm quark making a comparison with the results in the standard QPM approach.

## 1 Introduction

The description of heavy quarks diffusion in the QGP through the lower order perturbation theory for quantum chromodynamics has been extensively studied in the literature [1]. The use of pQCD was initially considered a reference point for the study of the heavy quarks dynamics since the coupling constant in the typical energy range of the formed system assumes intermediate values: perturbation analysis has allowed to describe QGP as a system of weakly interacting particles. The experimental results of the last decades have however shown that the QGP, produced in collisions between heavy ions at high energies, is a system characterized by a greater interaction than the one predicted by a perturbative treatment so that the created medium can therefore be described as a strongly many-body system influenced by non-perturbative effects even at very high temperatures ( $\sim 3 - 4 T_c$ ). In particular, lattice QCD calculations predict that the energy density of the system created deviates from the Stefan-Boltzmann limit of a perfect gas by about 15 – 20 % resulting in a value of the interaction measures  $\langle \Theta_{\mu\mu} \rangle = \epsilon - 3P$  significantly different from zero [2]. In this scenario of course perturbative QCD treatment fails in reproducing lattice QCD (lQCD) expectations

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and the observables related to the dynamics of heavy quarks such as  $R_{AA}$  and  $v_2$ , are not well reproduced by the model in comparison with the experimental data. In particular, these calculations considerably underestimate the energy loss and the elliptic flow of heavy quarks and, therefore, in general, the strength of the HQs interaction with the bulk partons especially in the low  $p_T$  region. A successful way to account the non-perturbative QCD effects at non-zero temperature is to encode the IQCD thermodynamical expectations within effective temperature dependent particle masses by prescription of a Quasi-Particle Model (QPM) [3] [4]. The origin of quark and gluon masses can be explained as the energy contribution coming out from the particle correlations within the strongly interacting QCD medium: if a large part of the interaction can be included into the effective masses, quasi-particles move freely or interact only weakly. We provide the expressions of these partonic masses according to their thermal pQCD evaluation:

$$\begin{aligned}
 m_g^2 &= \frac{1}{6}g(T)^2 \left[ \left( N_c + \frac{1}{2}N_f \right) T^2 + \frac{N_c}{2\pi^2} \sum_q \mu_q^2 \right], \\
 m_{u,d}^2 &= \frac{N_c^2 - 1}{8N_c} g(T)^2 \left[ T^2 + \frac{\mu_{u,d}^2}{\pi^2} \right]
 \end{aligned} \tag{1}$$

where  $N_c$  is the number of colors and  $N_f$  is the number of flavors. The presence of temperature dependent masses give rises an extra term in the energy density of the system which does not have the ideal gas form. Therefore, in order to ensure the thermodynamic consistency, pressure and energy density contain additional medium temperature dependent contribution, which we will call  $B(T)$ : this bag contribution takes into account further non-perturbative effects and preserve the entropy expression of an ideal gas. In this approach, the strength coupling  $g$  is a general T-dependent function whose parametrized form is obtained by fitting the energy density  $\epsilon$  of IQCD thermodynamics. For more details see Ref.[3].

## 2 The extended Quasi-Particle Model

Recently, new lattice results for the equation of state of QCD with  $2 + 1 + 1$  dynamical flavors have become available [5]. Therefore, we extend our quasiparticle model(QPM) approach for  $N_f = 2 + 1$  to  $N_f = 2 + 1 + 1$  where the charm quark is included. We also explore the extension of QPM approach to a more realistic model (in the following  $QPM^*$ ) in which partonic propagators explicitly depend on the three-momentum with respect to the partonic matter at rest in order to match pQCD at high momenta. In fact, the masses in the QPM increase with the temperature due to interactions with the medium and this is a good approximation if the particle has a low momentum but, if the momentum is large compared to the temperature, perturbation theory becomes accessible and the partons should behave like almost massless particles. This is not taken into account in the standard QPM approach. Following the Ref. [6], the essentially new elements in this extended approach with respect to the standard QPM are the momentum-dependent factor in the masses motivated by Dyson-Schwinger studies in the vacuum [7] which yields the limit of pQCD for  $p \rightarrow \infty$ .

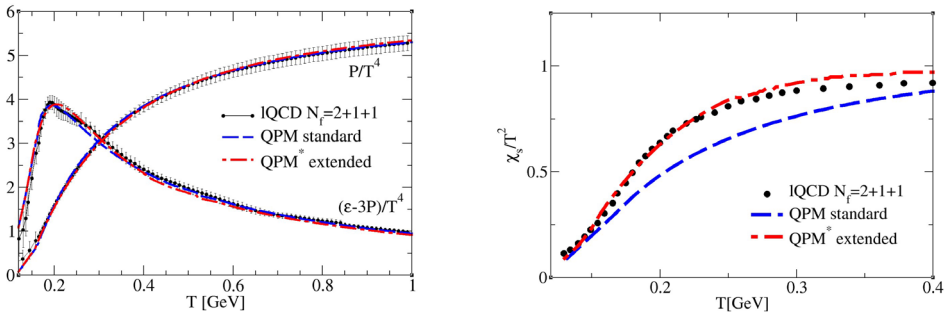
The functional forms for the partons masses at finite temperature  $T$ , quark chemical potential  $\mu_q$ , and three-momentum squared  $p^2$  are assumed to be given by:

$$\begin{aligned}
 M_g(T, \mu_q, p) &= \left(\frac{3}{2}\right) \left(\frac{g^2(T^*/T_c)}{6} \left[ (N_c + \frac{1}{2}N_f)T^2 + \frac{N_c}{2} \sum_q \frac{\mu_q^2}{\pi^2} \right] \left[ \frac{1}{1 + \Lambda_g(T_c/T^*)p^2} \right] \right)^{1/2} + m_{\chi g}, \\
 M_{q,\bar{q}}(T, \mu_q, p) &= \left( \frac{N_c^2 - 1}{8N_c} g^2(T^*/T_c) \left[ T^2 + \frac{\mu_q^2}{\pi^2} \right] \left[ \frac{1}{1 + \Lambda_q(T_c/T^*)p^2} \right] \right)^{1/2} + m_{\chi q} \quad (2)
 \end{aligned}$$

where  $T^{*2} = T^2 + \frac{\mu_q^2}{\pi^2}$  is the effective temperature used to extend the QPM to finite  $\mu_q$ ,  $\Lambda_g(T_c/T^*) = 5(T_c/T^*)^2 GeV^{-2}$  and  $\Lambda_q(T_c/T^*) = 12(T_c/T^*)^2 GeV^{-2}$ . Here  $m_{\chi g} \approx 0.5 GeV$  is the gluon condensate and  $m_{\chi q}$  is the light quark chiral mass ( $m_{\chi q} = 0.003 GeV$  for up and down quark and  $m_{\chi q} = 0.06 GeV$  for s quark). In Eq. 2,  $m_{\chi g}, (m_{\chi q})$  gives the finite gluon (light quark) mass in the limit  $p \rightarrow 0$  and  $T = 0$  or for  $p \rightarrow \infty$ . In our  $QPM^*$  calculations, likely in standard QPM approach, we consider  $\mu_q = 0$ . In the following we discuss the main results on the impact of the extended model on the EoS and charm quark coefficients.

### 3 Results

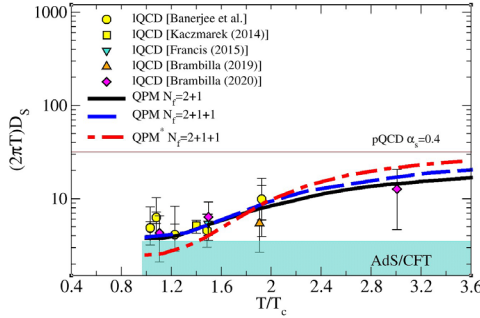
In this section we present our reproduction of EoS and quark susceptibility but also a discussion on the spatial diffusion coefficient  $D_s$  making a comparison between the standard QPM with 2 + 1 flavours, the standard QPM with 2 + 1 + 1 flavours including charm and the extended one with momentum-dependent propagators. In the left panel of Fig.1 we show the lattice data of Wuppertal-Budapest collaboration [5] for the pressure and trace anomaly as functions of the temperature, together with the quasi-particle model curves. Meanwhile, in the right panel the susceptibility  $\chi_s = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial \mu_s^2}$  is shown as function of temperature  $T$  both for standard  $QPM$  and extended  $QPM^*$ .



**Figure 1.** Pressure  $P/T^4$  and interaction measure  $(\epsilon - 3P)/T^4$  (left panel) and susceptibility  $\chi_s = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial \mu_s^2}$  (right panel) for standard  $QPM$  and extended  $QPM^*$  with  $N_f = 2 + 1 + 1$ . The lattice data of the Wuppertal-Budapest collaboration are taken from Ref. [5].

Differently from the standard QPM approach, in which the quark susceptibility is underestimated [3], in the  $QPM^*$  approach in which the parton masses have a momentum dependence, we are able to reproduce correctly both the IQCD quark susceptibility and the EoS. We finally discuss the impact of the extension of our QPM approach on the spatial diffusion coefficient  $D_s$  of charm quark. In fig. 2 we show the  $D_s$  of charm quark for standard QPM and the extended one both for  $N_f = 2 + 1$  and  $N_f = 2 + 1 + 1$ . The  $D_s$  for the extended QPM

is modified when  $T/T_c \leq 2$  due to the enhancement of the  $g(T)$  at low temperatures which corresponds to a decrease of the  $D_s$  coefficient of about 35%. This means that QPM describes a strong non-perturbative behaviour near to  $T_c$  similar to the one achieved in strongly coupled theory as AdS/CFT and in the high  $T$  region the  $D_s$  reaches the natural pQCD limit quickly than the standard QPM.



**Figure 2.** Spatial diffusion coefficient  $D_s(T)$  of charm quark for standard QPM with  $N_f = 2 + 1$  flavours (black solid line), standard QPM with  $N_f = 2 + 1 + 1$  flavours (blue dashed line) and extended QPM\* with  $N_f = 2 + 1 + 1$  (red dot-dashed line).

## 4 Conclusions

In this study we have upgraded our QPM approach with the recent IQCD calculations which include the charm quark. Therefore, we have also studied the extension of our QPM approach to a more realistic model in which partonic masses explicitly depend on the three-momentum of plasma particle in order to reach the pQCD limit in the high momentum regime. In this extended approach we reproduce correctly both the EoS and quark susceptibility which are underestimated in the standard QPM approach. Since this extended model better describes the most recent lattice QCD data and in particular the susceptibilities, it will be important to study if it leads to a better description of these dynamical observables lowering the global  $\chi^2$  coming from the comparison to the experimental data. An explicit treatment of the momentum dependence of quasi-particle masses and coupling has been often postponed, but it significantly affects the  $p$  dependence of  $R_{AA}$ ,  $v_2$ ,  $v_3$ . Given the new upgrade of ALICE and CMS will allow to access low  $p_T$  observables with high precision, the corresponding development of the QPM approach employed for realistic simulation is a necessary step toward a more solid and precise determination of  $D_s$ .

## References

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