EPJ Web of Conferences **276**, 06002 (2023) https://doi.org/10.1051/epjconf/202327606002
SQM 2022

Causality violations in realistic simulations of relativistic nuclear collisions

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Abstract. We show that relativistic causality is violated in the early stages of state-of-the-art heavy-ion hydrodynamic simulations of nuclear collisions. Up to 75% of the initial fluid cells violate nonlinear causality constraints, while superluminal propagation is observed by up to 15% the speed of light. Only after 2-3 fm/c of evolution, do 50% of the fluid cells become definitely causal. Inclusion of pre-equilibrium evolution significantly reduces the number of acausal cells, but it does not eliminate them. Relativistic causality thus imposes constraints on the model parameter space of heavy-ion collision simulations.

1 Introduction

Relativistic viscous hydrodynamics is vital for the phenomenological modeling of ultrarelativistic heavy ion-collisions and provides strong evidence for the formation of a fluid-like state of matter known as the quark-gluon plasma (QGP) in high-energy nuclear collisions. Comparisons to experimental data require modeling all the stages of a heavy-ion collision: the initial stages, the pre-equilibrium stages, relativistic hydrodynamics, and hadronic interactions. Relativistic viscous fluid-dynamics is currently determined by equations of motion for an extended set of dynamical variables which include both equilibrium and non-equilibrium quantities.

Traditionally, hydrodynamics is only expected to accurately describe the behavior of systems close to equilibrium. In the far-from-equilibrium domain, dissipative contributions to the energy-momentum tensor of the system can become comparable to the equilibrium pressure P. Recently, new constraints have been derived [1] involving the magnitude of the viscous currents and the QGP's transport coefficients, which ensure that causality holds in the nonlinear regime of the class of second order hydrodynamic equations of motion [2–4] used in heavy-ion simulations. These constraints define the physically allowable space of out-of-equilibrium corrections to the initial state, providing new theoretical guidance for relativistic viscous hydrodynamics.

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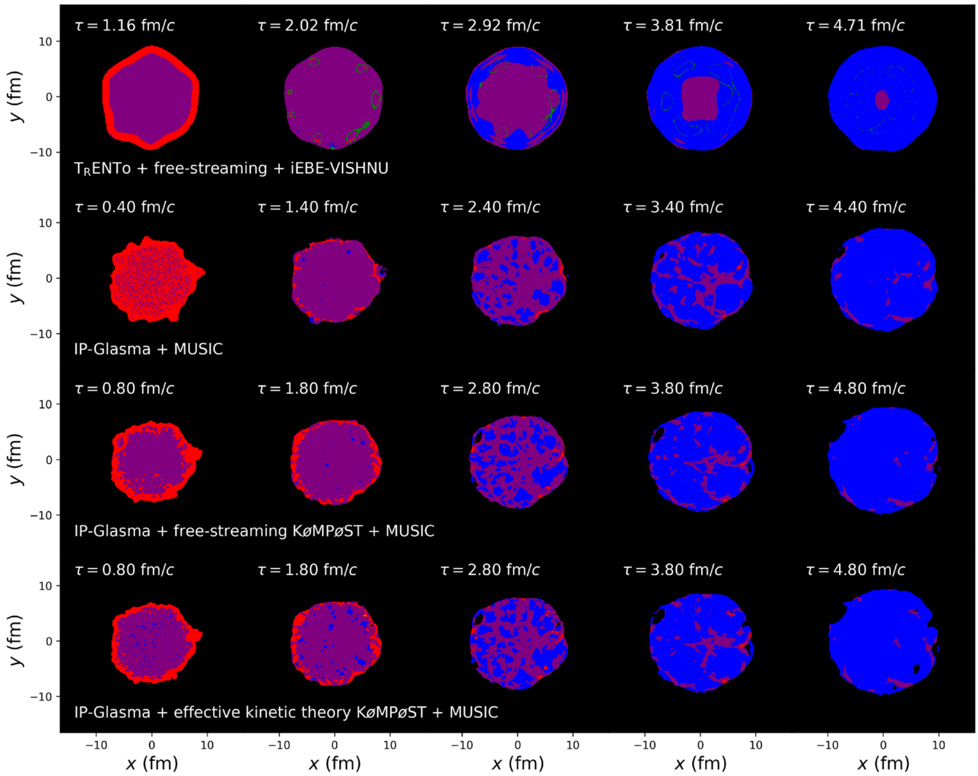

Figure 1. From top to bottom: the TFV scenario and the three IKM scenarios (no KøMPøST, free-streaming KøMPøST, EKT KøMPøST). Colors correspond to the following cell classifications: certainly causal (blue), certainly acausal (red), and purple (indeterminate). Taken from Ref. [5].

In this work we investigate these causality constraints for the most well-behaved scenario in heavy ions collisions simulations: central LHC Pb+Pb collisions. Two state-of-the-art open-source frameworks are used in our study: The first couples $\mathrm{T_RENTo}$+free-streaming+VISHNU ("TFV") and the second framework couples IP-Glasma+(KøMPøST)+MUSIC ("IKM"). Further details about the individual components of each framework are provided in Ref. [5]. Both frameworks have constrained parameters through a Bayesian analysis and have been extensively compared to experimental data.

2 Modeling

In the TFV framework, we adopt the Bayesian tune to LHC p+Pb and Pb+Pb data which combines $\mathrm{T_RENTo}$ initial conditions with a conformal, pre-hydrodynamic free-streaming phase, a boost-invariant hydrodynamic phase, and a hadronic afterburner UrQMD. We use the maximum-likelihood parameters for the transport coefficients. In the IKM framework, the initial conditions are from IP-Glasma, coupled to classical Yang-Mills evolution, followed by a boost-invariant hydrodynamics (MUSIC), with a possible intervening phase of pre-hydrodynamic dynamics using KøMPøST: "FS" free-streaming or "EKT" effective kinetic theory. Three different scenarios are considered: (i) IP-Glasma + MUSIC; (ii) IP-Glasma + KøMPøST (FS) + MUSIC; (iii) IP-Glasma + KøMPøST (EKT) + MUSIC. Further

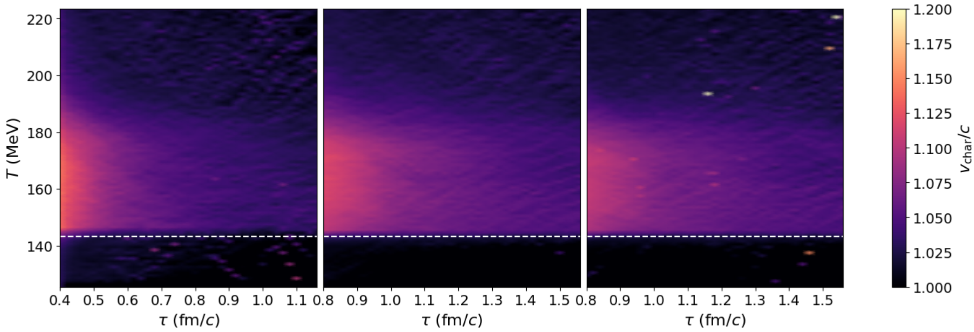

Figure 2. Characteristic velocities for PbPb collision in the IKM framework: Left: No KøMPøST, Middle: Free Streaming, Right: KøMPøST EKT. Taken from Ref. [5].

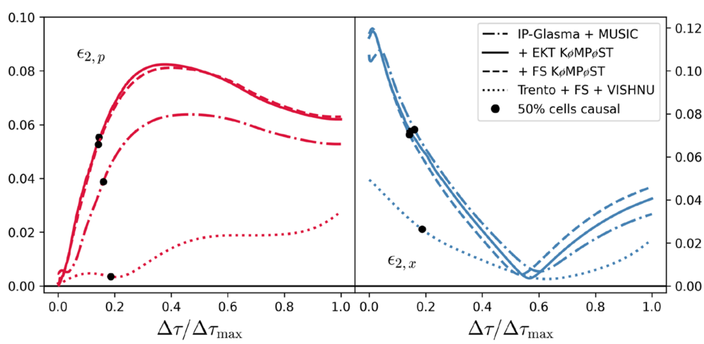

Figure 3. The momentum anisotropy $\epsilon_{2,p}$ (left) and spatial eccentricity $\epsilon_{2,x}$ (right) versus time. Black dots represent the points at which exactly half of the hydrodynamic cells (with $\varepsilon \geq \varepsilon_{FO}$) become explicitly causal. Taken from Ref. [5].

details of our simulations are provided in [5]. Both frameworks are used to generate a single, $\sqrt{s_{NN}} = 2.76$ TeV central Pb+Pb event to which the nonlinear causality constraints are then applied.

The constraints from [1], which apply to the Israel-Stewart-like [2–4] equations of motion used in both frameworks, provide separate sets of *necessary* and *sufficient* conditions for causality. Both sets of conditions correspond to simple inequalities involving transport coefficients and viscous currents. For the explicit expressions of the constraints, see [1, 5]. On the basis of these constraints, we sort grid points in the simulations into three different categories: (i), points at which the sufficient conditions (and consequently the necessary conditions) hold, hence causality is certainly respected; (ii), points at which one or more necessary conditions (and consequently sufficient conditions) are violated, hence causality is unquestionably violated; and (iii), points at which all necessary conditions are satisfied but one or more sufficient conditions fail, hence the analysis cannot determine if causality is violated.

3 Results

The time evolution of the causality analysis for a typical Pb+Pb event is shown for the TFV framework and all scenarios of the IKM framework in Fig. 1. Note that only the fluid cells that have not yet frozen out are plotted. The hydrodynamic simulations are all characterized by pervasive violations of causality, particularly in the first 1-2 fm/c of the collision. For the TFV framework, most of the severe causality violation occurs near the edge of the system where Knudsen and inverse Reynolds numbers become large, though still above freeze out. For the IKM framework, without KøMPøST, approximately 75% of cells in the initial state violate causality. However, the inclusion of KøMPøST pre-equilibrium evolution significantly reduces the causality violation present in the IP-Glasma initial state, bringing it down to approximately 1/3 of fluid cells. EKT offers only a modest improvement over FS.

In Fig. 2, we show the times and temperatures over which characteristic velocities were found to propagate faster than the speed of light in the three cases of the IKM framework. The details concerning the calculation of these velocities are given in [1, 5]. These show that the characteristic velocities were calculated to be around 15% greater than the speed of light, most pervasive both at early times and near the transition temperature. One can note that while the calculated super-luminal speeds seem to decrease as pre-equilibrium is turned on, its duration in time is increased. Also, we note that the scenario with KøMPøST EKT has small regions of very high superluminal characteristic velocities.

We show in Fig. 3 how the momentum anisotropy $\epsilon_{2,p}$ and the spatial eccentricity $\epsilon_{2,x}$[1] evolve with time in different scenarios. The black dots indicate the point in time for each scenario when half of the fluid cells are certainly causal. Whereas most of the final $\epsilon_{2,p}$ in the TFV framework is built up after the majority of the system has become causal, in the IKM scenarios the majority of the $\epsilon_{2,p}$ anisotropy is built up at early times (up to $20-30\%$ of evolution time) and nearly half of the final anisotropy is built up when most of the system either explicitly violates causality or the sufficient conditions are not met. Although we cannot at this stage remove the effects of causality violations entirely from our simulations, these results suggest that both collective dynamics and spatial geometry will be affected once causality constraints are taken into account.

4 Conclusions

In this study we showed that there are sizable causality violations in state-of-the-art simulations of heavy-ion collisions. The TFV and IKM frameworks, with parameters constrained by experimental data, yield up to 75% of fluid cells explicitly violating causality in the earliest stages of central Pb+Pb collisions at the LHC. A pre-equilibrium phase prior to the hydrodynamic evolution significantly reduces the amount of causality violation, but it does not fully eliminate it.

Acknowledgements

J.N.H, T.D., D.A., and C.P. are supported by the US-DOE Nuclear Science Grant No. DE-SC0020633. J.N. is partially supported by the U.S. Department of Energy, Office of Science, Office for Nuclear Physics under Award No. DE-SC0021301.

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[1] The definitions for these quantities are provided in Ref. [5].