Exotic Particles and heavy ion collisions

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Abstract. We discuss the structures of exotic candidates and why it is interesting to measure them in heavy ion collisions. We take the $X(3872)$ and T_{cc} to illustrate our point.

1 Introduction

Observations of exotic hadrons opened a new ear in hadronic physics. This started with the observation of the $X(3872)$[1] and then followed by a series of exciting discoveries such as the pentaquark states[2, 3], the tetraquark states X(5568)[4], the T_{cc}[5] and the recent strange pentaquark states. Of particular interest is the flavor exotic T_{cc} state with quark content $(cc\bar{u}\bar{d})$, which suggests that a meson can be composed of two quarks and two anti-quarks. Despite the excitement, we are still left with the pressing question of whether their structures are molecular types composed of loosely bound hadrons or compact multiquark states where the quarks and anti-quarks are confined inside a compact configurations with the typical hadron size. Discriminating their structures will give us important clues to understanding confinement phenomena.

Previously, we have pointed out that for exotic hadrons to be strongly bound and compact, they have to include multiple heavy quarks and therefore could be easily produced in relativistic heavy ion collisions[6, 7]. We have then formed the ExHIC theory collaboration and through the collaborative work found that one could discriminate whether an exotic is a molecular configuration or a compact multiquark state, by studying their production characteristics in a heavy ion collision[8–10]. The CMS collaboration at CERN has recently measured the $X(3872)$ for the first time in a heavy ion collision, and found that its production ratio to $\psi(2S)$ is anomalously larger than that measured in pp collisions[11]. We have shown in a recent work[12], that the ratio of the total yield of molecular configuration of $X(3872)$ to the total yield of $\psi(2S)$ in the statistical hadronization model with charm (SHMc) is 0.806 ± 0.234. The experimental measurement is at high transverse momentum, nevertheless, considering suppression of $\psi(2S)$ compared to the SHMc, the observed ratio seems consistent with a molecular picture for $X(3872)$.

In this talk, we first argue why the $X(3872)$ can not be a compact configuration and thus should be a molecular configuration while T_{cc} could either be a compact multiquark configuration or a molecular configuration. The argument is based on short distance properties using

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a quark model[13] combined with expectations from the pion exchanged model[14, 15] for long range interactions. We then predict the transverse momentum distribution of the molecular configuration of the $X(3872)$ and T_{cc} in Pb-Pb collisions at $\sqrt{s_{NN}}$ =5.02 TeV using the coalescence formula that well reproduces the deuteron data at similar collision energies.

2 Short distance interaction from a quark model

A compact multiquark state that is composed of two hadrons can form a stable multiquark configuration if the short distance interaction between the two composing hadrons is attractive. As is well known, the short distance nucleon-nucleon interaction is repulsive and thus we do not expect to have a dibaryon state with the deuteron quantum number. On the other hand, it is well known that when extended to SU(3), baryon-baryon interaction is attractive in the flavor singlet channel leading to the possibility of a stable H-dibaryon.

To understand the short distance interaction and what happens when two hadrons approach each other within a quark model perspective, let us consider the following typical nonrelativistic Hamiltonian for n quarks that contains the confinement and hyperfine potential for the color and spin interaction.

$$H = \sum_{i=1}^{n}(m_i + \frac{\mathbf{p}_i^2}{2m_i}) - \sum_{i<j}^{n} \lambda_i^c \lambda_j^c V_{ij}^C(r_{ij}) - \sum_{i<j}^{n} \frac{(\lambda_i^c \lambda_j^c)(\sigma_i \sigma_j)}{m_i m_j} V_{ij}^{SS}(r_{ij}). \tag{1}$$

Here, m_i, $\lambda_i^c/2$ and σ_i are the mass, color and spin operators of the i'th quark, respectively. V_{ij}^C and V_{ij}^{SS} are the confinement and hyperfine potential, respectively with r_{ij} being the distance between quarks. The above model is known to well reproduce the properties of the ground state hadrons[16]. Let us now consider what happens to each contribution in Eq. (1) as we bring two hadrons respectively with n_1 and n_2 quarks into a compact configuration of n quarks.

The first terms are the kinetic terms and show that the compact configuration has an additional term corresponding to the extra kinetic energy that one has to overcome to bring the two hadrons into a compact configuration. Assuming the compact configuration to be the size of a typical hadron ($\sim$ 0.64fm) and the typical reduced mass $\mu \sim 0.5$ GeV, the extra term roughly amounts to the following.

$$\frac{p_{n_1,n_2}}{2\mu} \sim \frac{1}{1\text{GeV}(0.64\text{fm})^2} \sim 100 \text{ MeV}. \tag{2}$$

This suggests that the attraction has to be sufficiently larger than this value for the configuration to be compact.

The color-color attraction appearing in the second set of terms in Eq. (1) can not give such a large attraction. This can be seen by considering the compact configuration where all the quarks occupy a similar size. Then, the effects of V_{ij}^C are the same for all quarks pairs so that the overall strength is given by the color-color factor. However, assuming that both the compact configuration and individual hadrons are color singlets, we find that the overall factor should be the same as the sum of color spin factor of the two hadrons.

$$\begin{aligned} \sum_{i<j}^{n}(\lambda_i^c \lambda_j^c) &= \frac{1}{2}\Big((\lambda_1^c + ... + \lambda_n^c)^2 - \lambda_1^2 - .. - \lambda_n^2\Big) \\ &= 0 - \frac{8}{3}(n_1 + n_2) = \sum_{i<j}^{n_1}(\lambda_i^c \lambda_j^c) + \sum_{i<j}^{n_2}(\lambda_i^c \lambda_j^c). \end{aligned} \tag{3}$$

Hence, the additional interaction between quarks coming from different hadrons should be small.

Now, let us consider the color spin interaction. To identify the important attractive channel, we again assume that all quarks occupy the same configuration and consider the prefactor, which we define as

$$K = -\sum_{i<j}^{n} (\lambda_i^c \lambda_j^c)(\sigma_i \sigma_j). \tag{4}$$

This factor is responsible for many of the characteristics of hadron interaction at short distance. First of all, the nucleon delta mass difference comes from the difference in the spin orientations, which lead to different K factors. For the nucleon and delta the K factors are, respectively, -8 and $+8$ leading to the $\Delta K = 16$ which corresponds to the mass difference of 290 MeV. Assuming that the quarks occupy similar size as those inside a nucleon, one can conclude that a unit factor of K corresponds to about 18 MeV. The ratio of the positive K factors between a dibaryon states in the (spin,isospin) $(0,1)$ and $(1,0)$ channels is 1.29, which is similar to the ratio between the corresponding nucleon-nucleon repulsions at short distance <0.3 fm observed in a lattice calculation. In SU(3) the ratio between flavor 27 and flavor 1 is -3, which is also consistent with the lattice result at short distance. In a previous publication[13], we have shown that the short distance part of the baryon-baryon interactions for various quantum numbers from the recent lattice calculation can be well reproduced using a constituent quark model with color-spin interaction. In fact, the full constituent quark model calculations are found to be comparable to the simple estimates based on the K factors among the multiquark configurations[13].

Therefore, we expect that a compact multiquark configuration will only exist if there is a strong negative short range interaction between composing hadrons which means a large negative K factor of the compact configuration. The negative K factor was the original motivation for a possible H-dibaryon state[17]. One could also use this K factor to search for possible pentaquark configurations[18].

2.1 The $X(3872)$ and T_{cc}

Let us consider the K factor to the first and latest observed tetraquark states. The quantum number of $X(3872)$ is $I^G(J^{PC}) = 0^+(1^{++})$. This leads to two allowed color-spin wave functions which are most conveniently written in the quark-antiquark basis $(c\bar{c}) \otimes (q\bar{q})$ with c, q being the charm and light quarks. In such basis, the two states are $(1 \otimes 1)_{C=1}(V \otimes V)_{J=1}$ and $(8 \otimes 8)_{C=1}(V \otimes V)_{J=1}$, where the first brackets denote two color singlets (1) or octets (8) combined into color singlet and the second two spin 1 (V) into spin 1. The K factor, after subtracting the lowest $D\bar{D}^*$ threshold, in the above basis becomes

$$K_{X(3872)} - K_D - K_{\bar{D}^*} = \begin{pmatrix} \frac{16}{3m_c^2} + \frac{16}{3m_q^2} + \frac{32}{3m_q m_c} & 0 \\ 0 & -\frac{2}{3m_c^2} - \frac{2}{3m_q^2} - \frac{4}{3m_q m_c} \end{pmatrix}. \tag{5}$$

The factor $\frac{32}{3m_q m_c}$ is due to the subtracted threshold. One notes that the second diagonal component coming from the color octet element gives an attractive contribution. However, noting that the K factor coming from the delta nucleon mass difference is $\frac{16}{m_q^2}$, which gives a mass difference of around 290 MeV, we find that using a contitutent quark mass of $m_c = 1500$ MeV and $m_q = 300$ MeV, the attraction only amounts to about 17.4 MeV assuming that all quarks occupy the size of a nucleon. Considering the additional kinetic energy needed to bring the two mesons together, one can conclude that the attraction is not strong enough to confine the $X(3872)$ into a compact multiquark configuration. A detailed quark model calculation confirms this expectation. This result furthermore implies that the attraction between $D\bar{D}^*$ in this channel is small at short distance.

Assuming that $J^P = 1^+$ for the T_{cc}, a better way to describe the color-spin wave function is using the diquark-antidiquark $(ud)\otimes(\bar{c}\bar{c})$ basis, for which the two states are $(\bar{3}\otimes 3)_{C=1}(S\otimes V)_{J=1}$ and $(6\otimes\bar{6})_{C=1}(V\otimes S)_{J=1}$, where the first and second brackets show the color combination and spin with S denoting the spin zero scalar state, respectively. Using these bases, the K factor for the T_{cc} is given as follows.

$$K_{T_{cc}} - K_D - K_{D^*} = \begin{pmatrix} -\frac{8}{m_q^2} + \frac{8}{3m_c^2} + \frac{32}{3m_q m_c} & \frac{8\sqrt{2}}{m_c m_q} \\ \frac{8\sqrt{2}}{m_c m_q} & -\frac{4}{3m_q^2} + \frac{4}{m_c^2} + \frac{32}{3m_q m_c} \end{pmatrix}. \tag{6}$$

One now notes that with the same parameters as above, the attraction from the first diagonal component is found to be around 104.4 MeV, similar to what was first noted in Ref [19]. Diagonalizing the matrix, the attraction is expected to be larger. Detailed constituent quark model calculations agree on the deeply bound compact configurations for the T_{bb}, but are still controversial for T_{cc} [16, 20, 21].

Using the same analysis, one can estimate the attraction between $D - D^*$ at relative distance estimated within that configuration. Using a simple Gaussian wave function, one finds that the attraction is around 200 MeV at $r_{D-D^*} = 0.3$ f m, which is similar to what has been estimated in the lattice gauge calculation[22]. Hence, we will add the short distance attraction to the long range pion exchange potential and probe the possibility of a weakly bound molecular configuration of the T_{cc}.

3 Perspectives from the pion exchange

For molecular configurations, the most important physics is the pion exchange potential. We will use the pion exchange potentials between $D - \bar{D}^*$ and $D - D^*$ for the $X(3872)$ and T_{cc} quantum numbers, respectively, that were used in Ref.[14, 15].

$$V(r) = V_{short}(r) - \gamma V_0\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} C_\pi(r) + \begin{pmatrix} 0 & -\sqrt{2} \\ -\sqrt{2} & 1 \end{pmatrix} T_\pi(r)\right], \tag{7}$$

where $V_0 = 1.3$ and $\gamma = 3(-3)$ for $X(3872)$ (T_{cc}) channel[14]. $C_\pi(r)$ is small, so that the effect of $T_\pi(r)$ is important to the possible binding [14]. We have also added the V_{short}, which we take to be zero for $X(3872)$ but attractive for the T_{cc} as discussed before. For the $X(3872)$, the attractive diagonal component of the tensor potential and the D-wave mixing coming from the off diagonal component in the tensor potential lead to a loosely bound molecular configuration [14]. On the other hand, if we do not include V_{short}, because of the repulsion in the tensor part, T_{cc} is not bound despite of the common D-wave mixing Eq. (7) which lowers the ground state energy[14, 15]. Now with the additional attraction, which we parametrize as $V_{\rm short}(r) = V_\omega e^{-\mu_\omega^2 r^2}$ with $\mu_\omega^2 = m_\omega^2 - (m_V - m_P)^2$ to reproduce the r dependence in the lattice result [22] and normalized with an overall constant $V_\omega = -448$ MeV to reproduce the attraction extracted from the constituent quark model discussed previously, we obtain an weakly bound molecular configuration. The results, together with the result for the deuteron within the same formalism[14], are summarized in Table 1

Our analysis shows that taking into account the short range attraction, the molecular structures of the $X(3872)$ and T_{cc} composed of heavy mesons are similar to that of the deuteron composed of the nucleons.

Table 1. Molecular structures obtained using Eq. (1)

Molecule	Content	V_{short}	V_π (S wave)	V_π (mixing)	$\langle r \rangle$ (fm)	E_b (MeV)
d	pn	repulsive	attractive	attractive	1.9	2.2
$X(3872)$	$D\bar{D}^*$	negligible	negligible	attractive	3.0	0.33
T_{cc}	DD^*	attractive	negligible	attractive	2.2	0.65

4 Production of molecular configurations in heavy ion collisions

Since both the molecular tetraquarks and the deuteron are weakly bound broad molecular states, their productions in heavy ion collisions should be formed and determined by the distributions of their respective constituents at the kinetic freeze-out point. Such descriptions can be well encoded in the coalescence model.

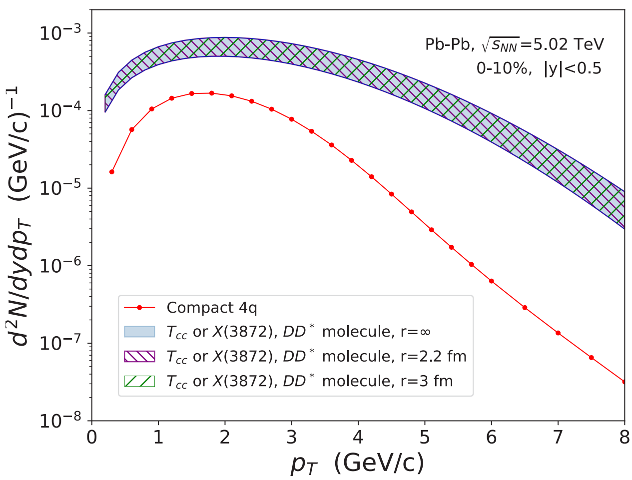

Figure 1. The transverse momentum distribution of $X(3872)$ and T_{cc} (0–10% event). The bands for molecular configurations are due to the uncertainties in the input data.

Using the same formula, we have parametrized the observed transverse momentum distribution of the D^0 and $\bar{D}^{*0}$ at centrality 0–10% for Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV[23]. Fig. 1 shows coalescence model result for the transverse momentum distribution of T_{cc} and $X(3872)$ assuming a molecular configuration. Since the chemical potential can be almost neglected, a molecular configuration of T_{cc} will have almost identical transverse momentum distribution of $X(3872)$ as given in Fig. 1. The band in Fig. 1 shows the statistical uncertainty of the input D, D^* data transferred into the coalescence result. One also notes that for the radii given in Table 1, the results are almost identical to the result given in the $\sigma \to \infty$ limit.

We also plot, the expected transverse momentum distribution as of T_{cc} given in Ref. [24], obtained assuming that it has a compact multiquark configuration. The result in Ref. [24] given for collision energy at 2.76 TeV, has been multiplied by 1.63 to take into account the difference in the charm quark number squared at 5.02 TeV as given in [10] to compare with the result for the molecular configuration calculated here. As can be seen in the figure, the expected transverse momentum distributions for two possible configurations of the T_{cc}

are markedly different in addition to the difference in the total yields. Integrating over the transverse momentum, we find that the total yields of molecular (compact) configuration is about a factor of 2.47 (0.625) times[12] that of the statistical model prediction[25]. Therefore, once the transverse momentum distributions of these tetraquarks are measured, we will be able to confirm/discriminate their structures.

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