The evaluation of the fission mode and fragment yields of neutron-rich nuclei by the dynamical model

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Abstract. Nucleosynthesis by the rapid neutron-capture process (r-process) produces elements heavier than iron via neutron-rich nuclides, observed in the solar system and stars with various metallicities. In the r-process, fission plays a fundamental role by recycling the matter during neutron irradiation and by shaping the final r-abundance distribution. Nevertheless, due to the difficulty of experimental approaches, most of the fission data available for r-process calculations are based on theoretical predictions with phenomenological models. In this study, we focused on the transition of fission mode from asymmetric to symmetric in neutron-rich isotopes, which has been suggested in recent experiments for fermium isotopes. We investigated the fission of neutron-rich nuclei by a theoretical calculation based on the dynamical model and employed Langevin equations.

1 Introduction

Nucleosynthesis by the rapid-neutron-capture process, r-process, represents a significant contribution to the abundance beyond the iron-group peak, and the main process for the heaviest elements (thorium, uranium). Although several astrophysical scenarios have been proposed, the mechanism of the r-process is not completely understood. One of the main reasons is large uncertainty in the nuclear-physics properties of very neutron-rich nuclei, e.g., neutron capture rates and several decay half-lives. To determine nucleosynthesis flows on the r-process “path”, the \(\beta\)-decay and the neutron capture (strongly depending on theoretical mass prediction) are significant. Nuclear fission is a key ingredient of the termination of and the final abundance distribution if r-processing is strong enough to reach actinoids. Nuclear fission, therefore, plays an essential role under certain r-process conditions, in particular, in very neutron-rich environments, e.g., neutron star mergers. The nucleosynthesis path goes into the very neutron-rich trans-uranium region. The effects of fission are significant to shape the r-process abundances due to fission recycling, of which fission products become seed nuclei \((A < 200)\) for the next r-processing during a single nucleosynthesis process. Besides the abundance prediction, fission is also a key role as the heating source of kilonovae at late

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times (∼ 10 days to months) electromagnetic transients of neutron star mergers. A sign of fission heating may have been observed in the light curve of the kilonova associated with the gravitational wave, GW170817. The precise understanding of fission becomes much crucial in the era of gravitational astronomy.

In this study, we calculate the fission-fragment mass distributions (FFMDs) of very neutron-rich nuclei. Fission product distributions are important for the r-process, but experimental data are not available. We adopt the Langevin method [1], widely adopted in the study of low-energy fission.

2 Model

We use the fluctuation-dissipation model and employ Langevin equations to calculate the evolution of nuclear shape with time [2, 3]. The nuclear shape is defined by the two-center parametrization [4, 5], which has three deformation parameters, $z$, $\delta$ and $\alpha$ to serve as collective coordinates, abbreviated as $q = \{z, \delta, \alpha\}$. The symbol $z$ is the distance between two potential centers, the symbol $\delta$ denotes the deformation of the fragments, and $\alpha = (A_1 - A_2)/(A_1 + A_2)$ is the mass asymmetry of the two fragments [2], where $A_1$ and $A_2$ denote the mass numbers of heavy and light fragments.

For a given value of the temperature of a system $T$, the potential energy is defined as a sum of the liquid-drop (LD) part and a microscopic (SH) part:

$$V(q, T) = V_{\text{LD}}(q) + V_{\text{SH}}(q, T),$$

$$V_{\text{LD}}(q) = E_S(q) + E_C(q),$$

$$V_{\text{SH}}(q, T) = [\Delta E_{\text{shell}}(q) + \Delta E_{\text{pair}}(q)] \Phi(T),$$

$$\Phi(T) = \exp \left( -\frac{aT^2}{E_d} \right).$$

(1)

Here, the potential energy $V_{\text{LD}}$ is calculated with the finite-range liquid drop model [6], given as a sum of the surface energy $E_S$ and the Coulomb energy $E_C$. The microscopic energy $V_{\text{SH}}$ at $T = 0$ is calculated as the sum of the shell correction energy $\Delta E_{\text{shell}}$, evaluated by the Strutinski method [7, 8], and the pairing correlation correction energy $\Delta E_{\text{pair}}$ [9]. The shell correction energy has a temperature dependence expressed by a factor $\Phi(T)$ in which the shell damping energy $E_d$ is chosen as 20 MeV [10] and $a$ is the level density parameter [3, 11]. To define the potential of the two-center shell model a neck parameter of $\epsilon = 0.35$ ($0 \leq \epsilon \leq 1$) has been routinely used. However, this value is not appropriate for heavier actinide nuclides as pointed out in We adopt the optimal $\epsilon$ values following the empirical relation

$$\epsilon(A_c) = 0.01007A_c - 1.94,$$

(2)

where $A_c$ is the mass of the fissioning nucleus [12].

The multidimensional Langevin equations [2] are given as

$$\frac{dq_i}{dt} = (m^{-1})_{ij}p_j,$$

$$\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2} \frac{\partial}{\partial q_i} (m^{-1})_{jk}p_jp_k - \gamma_{ij}(m^{-1})_{jk}p_k + g_{ij}R_j(t),$$

(3)
where \( q_i = \{z, \delta, \alpha\} \) and \( p_i = m_{ij} dq_i / dt \) is a momentum conjugate to coordinate \( q_i \). In the Langevin equation, \( m_{ij} \) and \( \gamma_{ij} \) are the shape-dependent collective inertia and the friction tensors, respectively. The wall-and-window one-body dissipation [13–15] is adopted for the friction tensor. The normalized random force \( R_i(t) \) is assumed to be that of white noise, i.e.,

\[
\langle R_i(t) \rangle = 0, \quad \langle R_i(t_1) R_j(t_2) \rangle = 2 \delta_{ij} \delta(t_1 - t_2).
\]

(4)

The strength of the random force \( g_{ij} \) is related to the friction tensor \( \gamma_{ij} \) by the classical Einstein relation,

\[
\sum_k g_{ik} g_{jk} = \gamma_{ij} T.
\]

(5)

We calculate the charge distribution based on the assumption of unchanged charge distribution (UCD). The charge distribution (charge density) remains unchanged during the whole fission process, i.e., the charge density of the compound nucleus is maintained.

3 Results and Discussion

3.1 A dramatic change of the fission mode at Fermium isotopes

The fission of fermium nuclides \(^{254–257}\)Fm at low excitation energy was studied using the dynamical model as shown in Fig. 1. We adopt lower excitation energy, \( E^* = 7 \) MeV, corresponding to low energy environments where the r-process occurs. The mass distributions of fission fragments show a dramatic change from an asymmetric shape for the lighter fermium isotopes to sharp symmetric fission for the heavier isotopes. This trend has been already observed experimentally [16]. The sudden change of the FFMD is strongly regulated by the structure of the second fission barrier and the dynamical motion of the nucleus in the second minimum [12].

Figure 1. Calculation results of a fission fragment mass distribution for \(^{254–257}\)Fm (\( E^* = 7 \) MeV).

3.2 Fission of neutron-rich nuclei at Uranium isotopes

We perform a series of fission calculations for U (\( Z = 92 \)) isotopes with 10–20 more neutron-rich from the \( \beta \)-stability line. The calculated FFMDs for \(^{250}\)U to \(^{256}\)U are shown in Fig. 2. A similar trend which is the dramatic change of FFMDs for fermium isotopes was also observed in uranium isotopes in this calculation. This trend did not appear in the results of the GEF model code [17].

3.3 Fission fragment charge distribution by the UCD assumption

For the application to the r-process simulations, we need fission properties for very neutron-rich nuclei far from the \( \beta \)-stability line. However, experimental data in the such region are
Figure 2. The calculation results of fission fragment mass distributions (FFMDs) for $^{250}$U to $^{256}$U with the excitation energy of $E^* =$7 MeV compared with the data from GEF code [17].

Figure 3. The calculation result of fission fragment distribution on the N-Z plane for $^{236}$U ($E^* =$10 MeV) is plotted. It is compared with the experimental data of $^{235}$U neutron-induced fission ($E_k =$ 500 KeV) from JENDL-4.0 [18].

not available yet. Therefore, we first carry out fission calculations for U isotopes near the $\beta$-stability line. Fig. 3 shows the results of the mass and charge distribution of fission fragments for $^{236}$U. The fission calculation is performed with the excitation energy, $E^* =$ 10 MeV. The distributions are compared with experimental data in JENDL-4.0 [18]. The calculated charge distribution with the current simple treatment, based on the UCD, shows a good agreement with experimental data. The charge distribution of fission products is a fundamental quantity for the determination of the production rate of individual isotopes. Thus, it is essential nuclear data to compare experiments with r-process calculations.

4 Summary

In the presented work, we have performed fission calculations of neutron-rich nuclei using a dynamical model for application to r-process simulations. We found that the fission fragment distribution changes from the two peak feature (asymmetric fission) to the one-peak (symmetric fission) with increasing the neutron number. We also found that the calculated fission mass distributions for uranium, of which the charge (Z) distribution is obtained by the UCD (unchanged charge density assumption), can well reproduce experimental data in JENDL (thermal neutron induced fission of $^{235}$U) [18]. Our results, including further improvements, are expected to contribute to the understanding of the r-process.
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