EPJ Web of Conferences **282**, 01004 (2023) https://doi.org/10.1051/epjconf/202328201004
SSP 2022

The Standard-Model Extension

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Abstract. Lorentz and CPT symmetry represent cornerstones of our present understanding of nature, but may be violated in various theoretical approaches to underlying physics. Testing these symmetries therefore establishes a promising avenue to search for physics beyond the Standard Model. The canonical theoretical tool to identify possible experimental signatures of such violations is an effective-field-theory framework known as the Standard-Model Extension. This talk provides an overview of this topic with focus on efforts involving low-energy atomic and subatomic systems.

1 Introduction

The concept of symmetries—and their breakdown—plays a key role in understanding, modeling, testing, and advancing physics. Due to their relative simplicity, atomic and subatomic systems are exceptionally well positioned for investigations of various discrete and continuous symmetries, and the ultra-high sensitivity attainable in these systems offers real discovery potential for physics beyond the Standard Model (SM).

Lorentz and CPT invariance are no exception to this assessment: past atomic and subatomic tests of these spacetime symmetries are among the most precise ones available with some of them reaching sensitivities that may be interpreted as probing the Planck scale [1]. Conceptual motivations for such studies arise from theoretical ideas that seek to answer questions beyond established physics because they often accommodate small deviations from Lorentz and CPT symmetry [2–6].

For the identification and interpretation of currently feasible Lorentz and CPT tests a theoretical framework has been developed. This framework, called the Standard-Model Extension (SME), classifies all possible perturbative departures from these spacetime symmetries that are compatible with coordinate independence and standard effective field theory [7]. All measurements of Lorentz and CPT symmetry tabulated in the aforementioned Ref. [1] are given as constraints on SME coefficients.

The outline of this work is as follows. A brief overview of SME basics is contained in Sec. 2. Section 3 discusses as an example a recent Lorentz test in a subatomic system, namely Tritium beta decay. Measurements of Lorentz and CPT symmetry in an atomic system, (anti)hydrogen, are the topic of Sec. 4. Throughout this work, we employ natural units $\hbar = c = k_B = 1$.

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2 SME Basics

In view of the various theoretical ideas for Lorentz and CPT violation in underlying physics cited in the introduction and the ambitious goal to cover a broad range of experimental searches for these effects, the SME test framework needs to describe departures from Lorentz and CPT symmetry with great generality at presently attainable energies. To this end, the SME is constructed by hand rather than derived as the low-energy limit of one of the underlying models incorporating Lorentz and CPT breakdown. At the same time, the SME maintains as many other fundamental physics principles as possible. What follows is an overview of the key ideas behind the construction of the SME framework.

One of these ideas is an effective field theory (EFT) approach. This approach is widely—and with great success—employed in a number of different areas in physics such as elementary-particle, nuclear, and condensed-matter physics. Moreover, it is theoretically well understood and provides a general and flexible description of dynamical systems with large numbers of degrees of freedom. It is thus a natural assumption that the low-energy leading-order physics of a potential breakdown of Lorentz and CPT symmetry originating in underlying physics can be described within EFT. This assumption leads to a lagrangian formulation of the SME as an EFT.

A second key idea concerns the characterization of departures from Lorentz and CPT symmetry. A common feature of all of the aforementioned sample mechanisms for Lorentz and CPT violation is the emergence of preferred directions in the vacuum; these are the physical objects causing the symmetry breakdown. Based on this insight, Lorentz and CPT breaking is parametrized by non-dynamical external vector or tensor fields $b^\mu, c^{\mu\nu}, \ldots$ in the SME. In the flat-spacetime limit, these are usually taken to be constant ensuring energy–momentum conservation. Note the analogy to the spontaneous breakdown of a conventional gauge symmetry, which also generates a constant background field carrying the indices of some representation of the corresponding symmetry group.

A third key idea is coordinate independence: although Lorentz and CPT invariance are violated, it should still be possible to select any suitable reference frame for the mathematical description of physical laws. In other words, coordinates are a pure product of human thought and should not acquire physical significance. Coordinate independence is guaranteed if physics is formulated in terms of geometrical quantities, such as scalars, vectors, tensors, and spinors. This implies that the SME background vectors and tensors $b^\mu, c^{\mu\nu}, \ldots$ must enter the SME Lagrangian with all their indices contracted. Note again the analogy to the spontaneous breakdown of a conventional gauge symmetry: Each term in the Lagrangian is a gauge singlet with all group indices contracted. This features is then trivially maintained, when one object in such a term acquires a vacuum expectation value.

These ideas inspire the following general structure of the SME Lagrangian $\mathcal{L}_{\mathrm{SME}}$:

$$\mathcal{L}_{\mathrm{SME}} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{EH}} + \delta\mathcal{L}_{\mathrm{SME}}\,, \tag{1}$$

where $\mathcal{L}_{\mathrm{SM}}$ and $\mathcal{L}_{\mathrm{EH}}$ are the ordinary SM and Einstein–Hilbert pieces, and $\delta\mathcal{L}_{\mathrm{SME}}$ contains small Lorentz- and CPT-violating corrections constructed according to the reasoning above:

$$\delta\mathcal{L}_{\mathrm{SME}} = -\overline{\psi} b^\mu \gamma_5 \gamma_\mu \psi + i\overline{\psi} c^{\mu\nu} \gamma_\mu \partial_\nu \psi + \ldots\,, \tag{2}$$

where b^μ and $c^{\mu\nu}$ are the aforementioned examples for preferred directions assumed to be generated by underlying physics. Within the present context, they represent phenomenological coefficients that govern the type and extent of Lorentz violation. We remark that the b^μ coefficient also controls certain types of CPT breaking, while $c^{\mu\nu}$ is CPT even.

Other desirable physics features, such as the ordinary U(1) × SU(2) × SU(3) gauge symmetry, are usually imposed on the SME. The overview above has primarily focused on the flat-spacetime SME. However, these ideas can be generalized to curved-spacetime situations involving gravity. We finally remark that the SME has provided the basis for numerous theoretical and mathematical investigations of Lorentz and CPT violation [8–13], and that the framework has also found applications in adjacent research fields [14].

As an EFT, the SME lends itself to a natural classification of Lorentz- and CPT-violating contributions according to their mass dimension d. The restriction to $d = 3, 4$ is usually called the minimal SME (mSME), and it is expected that this restriction contains the effects dominant at low energies. However, for broadest possible generality, the full SME also contains nonminimal contributions $d > 4$. For example, specific underlying models, such as noncommutative field theory, are indeed known to generate Lorentz-violating corrections starting at $d = 5$. This classification can be refined in various ways including by considering the C, P, and T properties of a given SME term.

3 Tritium beta decay

The neutrino remains one of the least understood particle species, and there is correspondingly large experimental and theoretical interest in neutrino investigations including Lorentz- and CPT-symmetry studies. Measurements involving flavor oscillations are interferometric in nature and thus offer unparalleled sensitivity to the neutrino's SME coefficients [1]. The long propagation distances of astrophysical neutrinos can also lead to tight constraints on Lorentz and CPT violation in the neutrino sector via time-of-flight measurements [1].

There are, however, some SME effects that leave both the oscillation pattern and the neutrino's velocity unaffected. The corresponding coefficients are termed 'countershaded.' The difficulty of accessing them experimentally leaves open the possibility of relatively large countershaded SME contributions that may have thus far evaded detection. Such effects therefore need to be searched for using other observables and experimental techniques.

One type of these countershaded contributions is generated by the SME's a^μ coefficients. They are associated to fermions and may have independent values for each species. In situations in which gravity can be disregarded, these coefficients generate a constant shifts by a^μ in the particle's 4-momentum and are known to be undetectable in systems involving only a single species of fermions. For this reason, previous experimental studies have employed systems with multiple species of fermions [15–21] or gravitational backgrounds [22, 23], and they have constrained a^μ coefficients for various quark flavors as well as the proton and the electron [1].

The SME's a^μ coefficients are also known to alter the phase space available to the outgoing fermions in particle reaction processes. In this context, the β-decay spectrum in particular has been the topic of prior theoretical studies [24, 25]: it involves an outgoing neutrino and thus provides experimental access to this fermion's a^μ coefficient. Moreover, the endpoint energy of this decay process has already been measured in various high-precision experimental investigations, and continuing efforts in this context are slated for future improvements in sensitivity [26–33]. What follows is a brief review of the most recent update of this particular type of measurement at KATRIN [34, 35].

The KATRIN experiment measures the β spectrum of Tritium decay

$$ {}^{3}_{1}\mathrm{T} \to {}^{3}_{2}\mathrm{He}^{+} + e^{-} + \bar{\nu}_e \,. \tag{3}$$

The dynamics of this low-energy decay is primarily governed by the involved two nuclei, the electron, and the neutrino. Correspondingly, we treat all these particles effectively as spin-$\frac{1}{2}$ fermions, as usual [36]. Countershaded a^μ Lorentz and CPT breaking in this system is then given by the lagrangian contributions for each of these four particles

$$\delta\mathcal{L}^{a}_{\text{SME}} = -\bar{\psi}_w \, a^{\mu}_{w}\gamma_{\mu}\psi_w \, . \tag{4}$$

Here, the species subscript $w \in \{T, H, e, n\}$ denotes the tritium, the helium, the electron, and the neutrino, respectively. There are also numerous other SME coefficients that can affect the decay (3), but many of these have already been bounded in other physical systems at sensitivities that render them effectively zero in the present context.

It turns out that in the absence of interactions, all four a^{μ}-type coefficients can be removed from the Lagrangian via a field redefinition, so that they would be undetectable [7]. However, this reasoning no longer holds when there are interactions. One can show that in the present situation, for example, only

$$a^{\mu} \equiv a^{\mu}_{T} - a^{\mu}_{H} - a^{\mu}_{e} + a^{\mu}_{n} \tag{5}$$

is observable [34]. This fact represents one of the reasons why this type of SME effect is countershaded.

With this input the leading-order a^{μ} correction to the conventional tree-level shape of the β spectrum can be determined [34]. A particularly useful observable is its endpoint energy. In general, it depends on the direction of the emitted electron due to the presence of a^{μ}. The actual KATRIN setup involves the collection of electrons emitted in all directions inside an acceptance cone of opening angle κ and pointing in direction $\hat{c}$, precluding the notion of a unique, single endpoint. However, averaging over the momenta inside the cone, one may introduce a 'mean' endpoint

$$\bar{E}^{a}_{m}(\hat{c}) \equiv E_m + a^0 + M_T^{-1}\sqrt{E_m^2 - m_e^2}\,\hat{c}\cdot\vec{a}\,\cos^2\tfrac{1}{2}\kappa \tag{6}$$

valid at first order in a^{μ}. Here, we have linearized in E_e by dropping terms of order $E_m(E_m - E_e)/(E_m^2 - m_e^2)^{1/2}$. The conventional endpoint is denoted by E_m, and the Tritium mass is M_T. Intuitively, $\bar{E}^{a}_{m}(\hat{c})$ represents the endpoint corresponding to the average cone momentum and therefore depends on the cone direction $\hat{c}$.

As a terrestrial experiment, KATRIN's orientation and velocity is time dependent due to the motion of the Earth. On the other hand, a^{μ} is presumed to be spacetime constant. Result (6) therefore predicts in general a time-dependent mean endpoint energy $\bar{E}^{a}_{m}$. Focusing solely on the orientation change due to the Earth's rotation about its axis, the direction of the cone vector $\hat{c} = \hat{c}(t)$ will exhibit a periodic time dependence determined by the length of a sidereal day. For this situation, Result (6) forecasts a constant shift by a^0 in the endpoint energy relative to its conventional value, as well as sinusoidal variations of the endpoint with a sidereal-day period and an amplitude governed by $\vec{a}$.

Available time-stamped KATRIN data together with information on KATRIN's location and orientation on Earth can be used to perform an experimental search for such effects. Such an analysis has indeed been recently performed by KATRIN leading to a constraint on a^{μ} at the level of 10^{-6} GeV [35]. This result represents the world's first, and thus best, measurement of this type.

4 Tests involving antihydrogen

The availability of slow antiprotons at CERN's AD facility has permitted the production of cold antihydrogen in recent years. This achievement, in turn, has opened an avenue for experimental hydrogen–antihydrogen comparisons, and thus atomic tests of CPT symmetry. This sections lists some ongoing efforts in this field within the SME context.

The lagrangian SME terms $\delta\mathcal{L}_{\text{SME}}$ lead for example to contributions δH_{SME} to one-particle Hamiltonians, which in turn generate corrections δE_{SME} to the energy spectrum of bound states. Spectroscopic investigations of atomic spectra therefore represent a well-suited experimental tool for ultra-high precision Lorentz and CPT tests. Paralleling the conceptual underpinnings of β-decay studies discussed in the previous sections, one type of atomic-spectroscopy measurements of SME coeffcients monitors transition frequencies over time and analyzes them for sidereal variations. But the availability of matter–antimatter conjugates for hydrogen also allows the instantaneous comparison of a given transition in H with that in $\overline{\text{H}}$. Such a direct comparison is typically sensitive to those SME coefficients that violate both Lorentz and CPT symmetry.

Among the transitions in (anti)hydrogen that are affected at leading order is the ground-state hyperfine splitting. In H, experimentally attainable relative sensitivities to these splittings are on the order of 10^{-12}. A correspondingly high sensitivity in $\overline{\text{H}}$ would therefore allow an excellent test of CPT invariance. An experiment along these lines is currently being performed by the ASACUSA collaboration at CERN's AD/ELENA facility [37]. The goal is an in-flight measurement of the π transition $(1,1) \rightarrow (0,0)$. Here, the expressions (F, M_F) denote a state with total angular momentum F and projection M_F onto the trapping magnetic field. Within the mSME, this transition can acquire a shift $\delta\nu$ in H and a different shift $\delta\bar{\nu}$ in $\overline{\text{H}}$ with [38]

$$\delta\nu - \delta\bar{\nu} = -\frac{1}{\pi}\left(b_3^e - m_e g_3^{e(A)} + m_e g_{120}^{e(M)} + b_3^p - m_p g_3^{p(A)} + m_p g_{120}^{p(M)}\right) + \dots\,, \tag{7}$$

where m_e and m_p are the electron and proton mass, respectively, and the various b and g quantities are certain components of CPT-violating SME coefficients. The ellipses indicate nonminimal SME contributions.

Measurements of the 1S–2S transition represent another avenue for a Lorentz and CPT test in H and $\overline{\text{H}}$. In H, this transition can be measured with a relative precision at the 10^{-15} level. As for the aforementioned HF transition, this number bodes well for a H–$\overline{\text{H}}$ comparison with ultra-high precision. Such a type of measurement is the goal of the ALPHA collaboration, which is also located at CERN's AD/ELENA facility [39]. For this situation, an SME analysis reveals a transition-frequency correction $\delta\nu_{1S2S}$ in H and again a different correction $\delta\bar{\nu}_{1S2S}$ in $\overline{\text{H}}$ with [38]

$$\delta\nu_{1S2S} - \delta\bar{\nu}_{1S2S} = -\frac{8}{\pi}\sum_w \left[12(\alpha m_{\text{r}})^2\, \overset{\circ}{a}{}^{\text{NR}}_{w,2} + 67(\alpha m_{\text{r}})^4\, \overset{\circ}{a}{}^{\text{NR}}_{w,4}\right]. \tag{8}$$

In this expression, α denotes the fine-structure constant and m_{r} the reduced mass. The species label w runs over the values e and p, and $\overset{\circ}{a}{}^{\text{NR}}_{w,2}$ and $\overset{\circ}{a}{}^{\text{NR}}_{w,4}$ are SME coefficients. We remark that there are no leading-order effects predicted within the mSME; the $\overset{\circ}{a}{}^{\text{NR}}_{w,2}$ and $\overset{\circ}{a}{}^{\text{NR}}_{w,4}$ coefficients are part of the nonminimal SME.

Another experimental question that can be addressed with $\overline{\text{H}}$ concerns the interaction of antimatter with gravity. Various efforts are currently underway to perform such a measurement including AEGIS [40], GBAR [41], ALPHA [42], and AGE [43]. These efforts are focused in particular on studying the free fall of $\overline{\text{H}}$ in Earth's gravitational field. Since the gravitationally coupled SME contains Lorentz- and CPT-violating interactions that can affect free-fall trajectories, such experimental efforts are well positioned to place constraints on the corresponding SME coefficients. One particular isotropic SME effect consists of a correction

$$\delta F_z = -2g\alpha\left(\overline{a}^{\text{T}}_t + \overline{a}^{\text{S}}_t\,\frac{m^{\text{T}}}{m^{\text{S}}}\right) \tag{9}$$

to the gravitational force exerted by the source mass m^{S}, taken to be Earth, on the test mass m^{T} [22]. Here, α is a parameter that captures dynamical effects of the underlying physics generating Lorentz and CPT violation, the two $\overline{a}$ quantities are SME coefficients, and g is the usual gravitational acceleration on Earth.

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