EPJ Web of Conferences **287**, 01012 (2023) https://doi.org/10.1051/epjconf/202328701012
EOSAM 2023

Trajectory tracing dynamics in anisotropic microcavities

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Abstract. Ray-wave correspondence has proven a powerful tool in mesoscopic optics, in particular in the description of deformed microdisc cavities with a versatile application potential ranging from microlasers to sensors. New material classes such as graphene-based systems have enriched the field by adding Dirac Fermion optics as well as anisotropic material properties as further system parameters. The trigonally warped dispersion relation in bilayer-graphene billiards generalizes the concept of birefringence and opens unconventional ways of trajectory control in the interplay of dispersion relation and the cavity geometry as we illustrate in this contribution.

1 Introduction

The investigation of the real-space trajectory dynamics is a well-established field of nonlinear dynamics, quantum chaos, and mesoscopic physics in general. Depending on the symmetry properties of the system's geometry, its dynamics can be integrable, chaotic, or mixed. Ray-wave correspondence has proven a useful tool especially in mesoscopic optics and microlaser applications [1], [2].

Materials with anisotropic properties provide a new class of systems. They can be uniquely characterized by their dispersion relation. In the case of electronic systems such as graphene-based materials [3], the Fermi surface provides the properties needed. The directions normal to the Fermi line or surface can be associated with propagation directions in momentum space. This implies that as soon as the Fermi surface deviates from a sphere or, in two dimensions, from a circle, the material is not isotropic anymore. A system with such anisotropic material properties is for example bilayer graphene (BLG) with a so-called trigonally warped Fermi line that relates to three preferred wave propagation directions [4]. This replaces the isotropic emission properties of typical optical systems. At the same time, it generalises the phenomenon of birefringence.

Here we will discuss the interplay between non-circular geometries and/or dispersion relations in terms of optical and BLG microcavities of different geometric shapes and discuss its impact on the ray/particle dynamics of the system. We shall see that either symmetry breaking induces chaotic dynamics in the system. Anisotropic material properties can lead to deviations from the well-known law of reflection in optics – the angles of incidence and reflection are not equal any more – and induce new Fresnel coefficients [4].

2 Discussion

In Fig. 1 we show typical results for the various situations that can arise two-dimensional mesoscopic systems. The left column shows the dispersion relation, the system geometry is shown in the middle, and the resulting Poincaré surface of section (PSOS) is shown on the right. The intricate interplay of the symmetries in real and momentum space becomes evident in the PSOS and can be explained as follows.

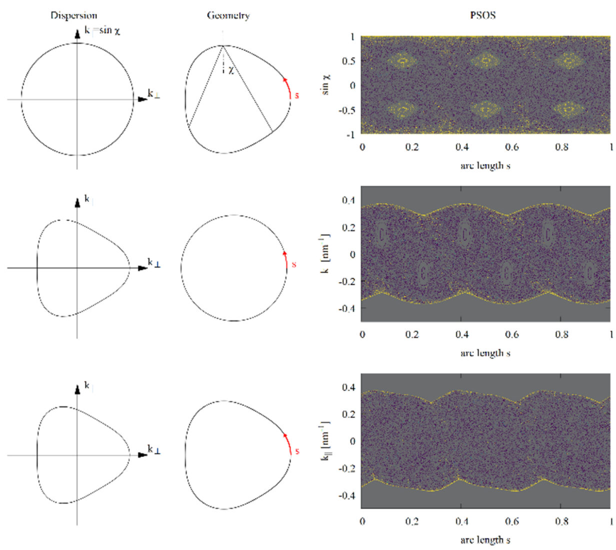

Fig. 1. Interplay of dispersion relation and geometrical symmetries in terms of the resulting particle dynamics depicted in terms of the Poincaré surface of section (PSOS). From top to bottom an optical system in so-called shortegg geometry [2], a circular BLG billiards [4], and a BLG billiards in shortegg shape are shown. Notice the change in the PSOS with the shift and eventual loss of stable islands. The color scale in the PSOS represents the intensity remaining inside the cavity (yellow indicating high values).

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We start our discussion with the well-known case of an optical microcavity with refractive index n=1.5 and with isotropic properties, thus a circular dispersion relation. We choose the so-called shortegg shape for the microcavity geometry as it resembles the BLG Fermi line that we will consider shortly. Its radius R reads in polar coordinates $R(\varphi) = R_0 \ (1 + \varepsilon_3 \cos (3 \ \varphi))$ with $\varepsilon_3 = 0.075$.

The resulting PSOS is the well-known [3] and the basis of such low-refractive index microlasers. It is spanned by the (normalised) arc length s and the sine of the angle of incidence χ at each reflection point (positive and negative χ indicating mathematically positive and negative sense of rotation). To this end rays are started uniformly in phase space with unit intensity, and their intensity is followed taking reflection and refraction according to Fresnel laws into account. Light colours in the PSOS indicate high intensities remaining inside the cavity. We observe that the three stable islands, related to the three-fold symmetry of the microcavity geometry, host long-lived trajectories for either sense of rotation.

In the middle panel row of Fig. 1, we reverse the situation and consider a dispersion relation with three-fold symmetry that is typical for BLG systems, and a circular cavity shape. We choose the BLG parameters defined in Ref. [4] with a Fermi energy E_F=100 meV, a potential step height U=160 meV, and an interlayer asymmetry gap Δ = 5 meV.

The first thing to notice is that the PSOS is now spanned by the arc length s as before, and by the $k_{||}$ momentum component aligned parallel to the billiard boundary in the respective point of reflection. The component $k_{||}$ replaces $\sin \chi$ as conserved quantity. In particular, we point out that $\sin \chi$, now defined as the angle between the normal to the dispersion line and the billiard boundary (along the $k_{||}$ axis) is not conserved in general any more upon a reflection at the billiard boundary, and we refer to Ref. [4] for details.

The consequence for the PSOS is evident in terms of missing whispering gallery orbits (WGO) that are so characteristic for isotropic circular billiards. Although we observe high intensity trajectories for the higher $k_{||}$ values, however, they eventually loose their WGO character. Even more surprising is the existence of regular islands whose position depends on their sense of rotation. They can be related to the preferred propagation directions read-off as the normal directions along the straight sections of the dispersion line and follow the reflection law of optics for symmetry reasons, but often carry little intensity. We point out that a chain of three unstable fixed point is situated in between the islands in agreement with the expectations from nonlinear dynamics.

Eventually, the last row of panels in Fig. 1 shows a BLG microcavity as before, but now in shortegg shape. We see that breaking the spatial symmetry destroys the island chains in phase space, such that the PSOS is now (almost) fully chaotic. Moreover, it exhibits a chiral symmetry breaking as generally the $k_{||} > 0$ and $k_{||} < 0$ regions can not be transformed into each other by a symmetry operation.

3 Conclusion

The interplay between geometric and material properties opens a wide field for tailoring and taming the trajectory dynamics and, via ray-wave-correspondence in the sense of mesoscopic optics, the resonance and transport properties in optical microcavities. The unique crystal lattice structure of single layer and bilayer graphene adds a potential anisotropy into the system that can be used to control the carrier dynamics via, in this case of electronic carriers, gate voltages [4]. The resulting unconventional phenomena can be used as an inspiration for novel optical devices.

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