

Computing Three-Dimensional Freeform Reflectors with a Scattering Surface

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Abstract. We present a novel approach to computing reflectors with a scattering surface in illumination optics. A scattering model governed by a Fredholm integral equation is derived. Solving this integral relation yields a virtual specular target distribution, which we insert into a Monge-Ampère least-squares numerical solver to get a scattering reflector that yields the desired illumination.

1 Introduction

Computing freeform reflectors within the field of illumination optics constitutes solving the so-called *inverse problem*, i.e., finding a reflector such that a given source light distribution is transformed into a given target distribution. Contemporary methods include solving Monge-Ampère differential equations directly yielding the shape of the reflector [1]. Whilst very powerful, these methods assume that the reflector surface is perfectly smooth so that the reflected rays always abide the specular law of reflection. We have previously successfully shown that two-dimensional reflectors with scattering surfaces can be computed using specular computational methods by pre-processing the target distribution [2]. We now propose an extended scheme allowing three-dimensional freeform reflectors to be computed in an analogous manner.

Specifically, the scattered light is given by a Fredholm integral equation. Solving said integral equation yields a so-called *virtual target distribution* which can be inserted into the specular design problem to compute the reflector surface. The scattering model is introduced in Sec. 2, followed by some brief words about the specular design problem in Sec. 3; the proposed solution algorithm is then outlined in Sec. 4, and some examples are shown in Sec. 5.

2 Scattering Model

Consider Fig. 1 showing a source domain $S \subset \mathbb{R}^2$, a source ray travelling along $\hat{s} \equiv \hat{e}_z = (0, 0, 1)^\top$ striking the reflector at a point $\mathcal{P}$, a specular ray along $\hat{t} = \hat{s} - 2(\hat{s} \cdot \hat{n})\hat{n}$ ($\hat{n}$ is the normal at $\mathcal{P}$ – not shown) and a scattered ray along $\hat{u}$. The orange cone constitutes equally valid directions of $\hat{u}$ for a fixed deviation $\alpha \in [0, \pi/2]$ of $\hat{u}$ w.r.t. $\hat{t}$. The scattered direction $\hat{u}$ traverses the circle at the base of the cone via an angle $\beta \in [0, 2\pi]$.

Specifically, let the specular direction $\hat{t} := \hat{t}(\psi, \chi) := R_z(\chi)R_y(\psi)\hat{e}_z$, where R_y and R_z represent rotations around

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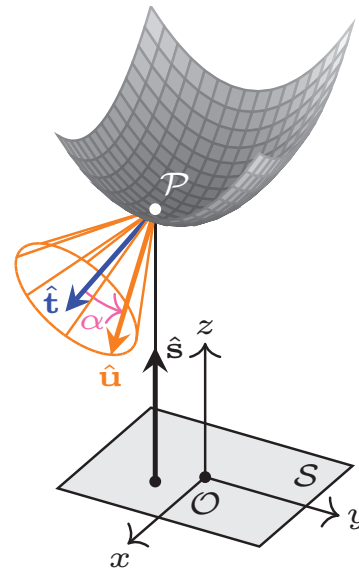


Figure 1. Scattering from a reflector.

the y -axis and z -axis, respectively, and where $\psi \in [0, \pi]$, $\chi \in [0, 2\pi]$. Then, the scattered direction is given by $\hat{u} = R_z(\chi)R_y(\psi)R_z(\beta)R_y(\alpha)\hat{e}_z$. Equivalently, $\hat{u}$ can be parametrised using two angles $\gamma \in [0, \pi]$ and $\nu \in [0, 2\pi]$ by $\hat{u} := R_z(\nu)R_y(\gamma)\hat{e}_z$.

We now define the source, virtual specular target and scattered target distributions. Let $f(\mathbf{x}) > 0$, $\mathbf{x} \equiv (x, y) \in S$ be the source *exitance* [$\text{W} \cdot \text{m}^{-2}$]. Let $g(\psi, \chi) > 0$, $(\psi, \chi) \in \mathcal{T}$ so that $\hat{t}(\psi, \chi) \in S^2$, be the virtual specular target *intensity* [$\text{W} \cdot \text{sr}^{-1}$]. Let $h(\gamma, \nu) > 0$, $(\gamma, \nu) \in \mathcal{U}$ so that $\hat{u}(\gamma, \nu) \in S^2$, be the scattered target *intensity*. Additionally, let $p(\alpha) > 0$, $\alpha \in [0, \pi/2]$ be a rotationally symmetric probability density function on S^2 representing the scattering properties of the surface. Consider a *density* of the form $\rho(\psi, \chi, \gamma, \nu) := p(\alpha(\psi, \chi, \gamma, \nu))g(\psi, \chi)$, which gives the scattered light into the spherical direction (γ, ν) when integrated w.r.t. (ψ, χ) , and the specularly reflected light

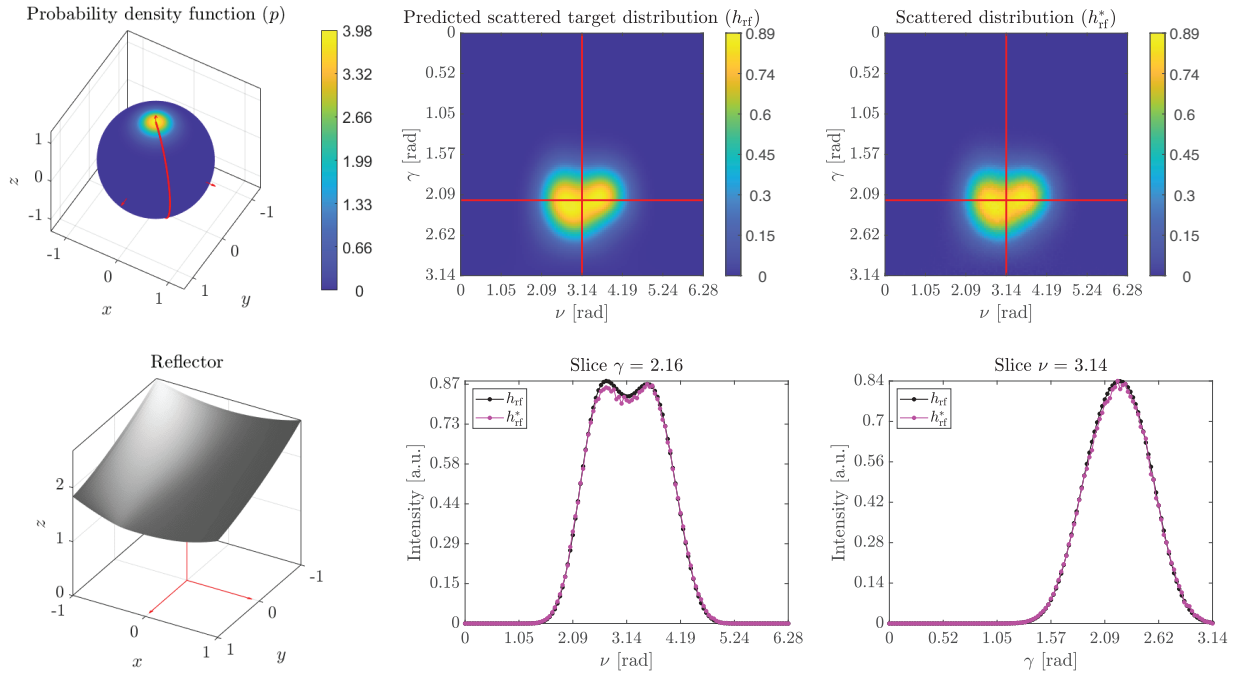


Figure 2. Verification of a reflector designed with our scheme using a custom raytracer which implements our model of scattering.

into direction (ψ, χ) when integrated w.r.t. (γ, ν) . Then, the following Fredholm integral relation governing the scattering in the system can be derived [3]:

$$h(\gamma, \nu) = \int_{\mathcal{T}} p(\alpha(\psi, \chi, \gamma, \nu)) g(\psi, \chi) \sin(\psi) d\psi d\chi. \quad (1)$$

3 Specular Reflector Design

Let the reflector be given by a function $z = u(\mathbf{x}) > 0$, $\mathbf{x} \in \mathcal{S}$. The Monge-Ampère differential equation turns out to be easier to solve in stereographic coordinates. Let $\tilde{g}$ be the stereographic representation of the virtual specular target distribution g . Then, the Monge-Ampère differential equation needed to be solved for u is given by

$$\det(D^2 u(\mathbf{x})) = \frac{1}{4} \left(1 + \|\nabla u(\mathbf{x})\|^2 \right)^2 \frac{f(\mathbf{x})}{\tilde{g}(\nabla u(\mathbf{x}))}, \quad (2)$$

where $D^2 u(\mathbf{x})$ denotes the Hessian matrix. We solve Eq. (2) using a least-squares solver developed in our group by multiple authors and outlined in [1, Ch. 6].

4 Solution Algorithm

The solution algorithm we propose to compute freeform reflectors with a scattering surface using Eq. (2) is as follows. Given the source exitance f , target intensity distribution h and scattering probability density function p ,

1. Solve Eq. (1) for g — we used a custom Richardson-Lucy iterative deconvolution implementation in Matlab. Call this “**unfolded**” distribution g_{uf} .
2. Compute the reflector using f and g_{uf} by solving Eq. (2) with $\tilde{g}$ being the stereographic representation of g_{uf} .

3. Compute the “**refolded**” scattered distribution h_{rf} by inserting g_{uf} back into Eq. (1). This is our predicted scattered light.

4. Verify the reflector using raytracing.

5 Example

An example is shown in Fig. 2. The source (not shown) is uniform with $f(\mathbf{x}) = 1/4$, $\mathcal{S} = [-1, 1]^2$, and p takes the shape of a rotationally symmetric Gaussian in the stereographic plane projected back onto the sphere. The raytraced distribution h_{rf}^* is very close to our predicted h_{rf} , as can be seen — especially from the slices at fixed γ or ν . The raytracer implements scattering in accordance with our model, i.e., rotating the specular vector $\hat{\mathbf{t}}$ by stochastic parameters α and β .

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References

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