Following the Flux in diffracted fields – An efficient numerical method for tracing the Eikonal function

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Abstract. In a complex optical system, a ray is guided by the derivative of the Eikonal function, which is calculated at each plane to navigate through the intricate vectorial model. We present an approach we call ‘intricate vectorial model’ to calculate the phase derivative of the field, which can be more accurately calculated even for numerical apertures <0.7.

1 Introduction

The objective of this work is to develop computationally efficient methods that accurately trace the flux lines, also known as non-linear rays, in the focal region of a lens. The first step is the computation of the 3D sample distribution of the converged wavefield in the focal region of the lens over a three-dimensional grid. The sampling requirements and computational efficiencies of numerical methods that accurately calculate a propagating complex wavefield over a three-dimensional volume must be considered in detail due to the high computational load involved in generating this three-dimensional representation of the diffracted wavefield.

In order to implement this first step we employ the Angular Spectral Method (ASM) [1,2] as the numerical propagation algorithm of choice, making use of recent advances in sampling theory to optimize the computational memory efficiency of the ASM for focused fields. The ASM is iteratively applied to generate the sampled 3D wavefield over a uniform grid in the focal region of the lens. This is the precursor for the second step, whereby a "ray" is guided by the derivative of the phase, which is calculated at a sequence of points traveling through this grid.

2. The first step: calculating the 3D Grid

Numerical propagation algorithms that can accurately calculate diffraction patterns at variable distances within a homogeneous isotropic medium are an active area of research, with far-reaching applications, including in quantitative phase microscopy, computer-generated holography, confocal microscopy, computation of the point spread function in lens design and lithography, and digital holography. Algorithms based on nonparaxial scalar optics are preferable since they offer superior performance in terms of accuracy, and can be numerically implemented at least as efficiently as paraxial-based algorithms. The Angular Spectrum method (ASM) and the Rayleigh-Sommerfeld method (RSM) are two such algorithms that provide highly accurate solutions for numerical apertures up to at least 0.6. [1,2] For higher numerical apertures, algorithms based on the more intricate vectorial model are required. The ASM is defined using the following equations:

\[ u_x(x, y) = \text{FT}^{-1}\{\text{FT}(u_0(x_0, y_0))H_z(f_x, f_y)\} \]  \hspace{1cm} (1)

\[ H_z(f_x, f_y) = \exp\left(j2\pi z \frac{1}{\lambda^2} - f_x^2 - f_y^2\right) \]  \hspace{1cm} (2)

Where FT denotes the Fourier transform operator which is implemented numerically using the discrete Fourier transform, \( u_0 \) is the initial wavefield (for example the thin lens function) and \( u \) is this function having propagated a distance \( z \) in free space. In our algorithm, the ASM is applied iteratively over a sequence of distances generating the 3D grid of samples representing the focusing light.
wavefront propagation, which can be defined in terms of the local gradient of the phase of as follows:[4]

\[
\begin{align*}
    f_{lx} &= \frac{1}{2\pi} \frac{\partial}{\partial x} \phi_z(x,y) \\
    f_{ly} &= \frac{1}{2\pi} \frac{\partial}{\partial y} \phi_z(x,y)
\end{align*}
\]

and it is specifically this quantity that we measure for the wavefield in order to trace the direction of the Eikonal function through space. Tracking the movement of the wavefront's intensity, or flux, involves tracing these local wavevectors through space.

The crux of the proposed method is that we iteratively trace the flux between subsequent z-planes making use of the local spatial frequencies to guide the ray angle at each plane. This is illustrated in Fig.1 for which case several rays, with initial points uniformly aligned in a focusing (a) Gaussian laser and (b) Laguerre-Gaussian (TEM01) laser, have been traced through the focal plane. It is important to note that field \( u_z \) will have been sampled over a 3D grid with uniform sampling intervals; however, the 3D positions at which the ray is sampled will not, in general, be coincident with the points on this grid. Therefore, interpolation is required in order to determine the complex amplitude at the desired points. It is particularly interesting to note the movement of the flux lines for the case of the Laguerre-Gaussian laser, which appear to spiral around the optical axis.

4 Conclusion

Although not shown here we have found that the proposed method is particularly useful for investigating the behaviour of light focused by convergent lenses that contain aberration, several of which cannot readily be interpreted in terms of linear geometrical ray. It also provided fascinating insights into the movement of light carrying orbital angular momentum. We believe that the method may have applications in various areas, including lens design and lithography. More details on the method are provided in Ref 5.

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References


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