Design of a reflectionless optical amplifier through broken-supersymmetry

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Abstract. A very interesting class of metamaterials is characterized by supersymmetry (SUSY). What makes SUSY very attractive for the design of new optical devices is the possibility to define different spatial refractive index distributions (superpartners) having the same scattering spectra (angularly and spectrally). In this study we explore the possibilities offered by the generation of superpartners of vacuum, being reflectionless and possessing unit transmission by definition. In particular, broken-SUSY is used to define a reflectionless active cavity capable of amplifying electromagnetic radiation in the visible. The approach is analytical through the use of the Darboux transform (a type of supersymmetric transformation) for the generation of the optical potential and the calculation of the field, while the transmission/reflection spectra evaluation is done with the Transfer Matrix method. Interestingly, we show that the Darboux transform allows to define 1D materials that are reflectionless for a continuum of frequencies. Moreover, the proposed device behaves as a dynamic optical filter amplifying radiation arriving at large angles while for other directions is almost completely transparent. Thus, simply by rotation different functionalities can be obtained. In addition, the active filter is reflectionless for all wavelengths and angles of incidence.

1 Introduction

Initially, it has been proposed to use supersymmetric transformations [1] to generate PT-symmetric potentials (metamaterials), however in the context of quantum mechanics making their experimental realization very difficult [2–4]. On the other hand, in optics in the context of optical waveguides, it has been recently realized that supersymmetry can be used in order to restore PT-symmetry or to produce classes of optically equivalent potentials (in terms of reflection and transmission amplitudes) with extremely different spatial distribution of optical properties [5–7]. What makes SUSY very attractive for the design of new optical devices is the possibility to define different spatial refractive index distributions (superpartners) having the same scattering spectra (angularly and spectrally). For this work we explore the possibilities offered by the generation of superpartners of vacuum, being reflectionless and possessing unit transmission by definition.

Interestingly, we show that the Darboux transform (DT), a type of supersymmetric transformation, introduces (indirectly) dispersion analytically in media that are spatially heterogeneous along one direction, thus allowing to define materials that are reflectionless for a continuum of frequencies. This scheme is very different from traditional methods in metamaterials where the mathematical transformations are applied directly on the optical properties so that the potential is by definition frequency dependent. This has a heavy consequence on the Darboux generated optical properties i.e. both the real and the imaginary parts of the refractive index share the same dispersion. This sort of artificial electromagnetic wave dispersion imposed on the refractive index is compatible with physical materials (as represented by the Drude model [8]) only in the limit of high frequencies i.e. far from absorption resonances thus for very small absorption. Indeed, in order to satisfy the necessary Kramers-Kronig relations [9], the imaginary part of the refractive index should be kept sufficiently small (few orders of magnitude smaller than the real part). To summarize, our study is directed towards the calculation of transmission and reflection spectra in order to make use of SUSY to design optimal optical filters. In this framework we take into account spatial distributions of the refractive index possessing resonances that can be calculated as bound states of a Schrödinger-like equation (transversal modes). Interestingly, when SUSY is broken (for complex energy states) anomalous transmission is observed i.e. the transmission coefficient becomes larger than one [10, 11]. This is a manifestation of the fact that the system is open towards the external environment i.e. external energy is used to generate anomalous transmission. This behavior is typical of open systems [12]. Thus, a broken SUSY active optical cavity is proposed which can be used as a dynamic optical amplifier. Indeed, for large incidence angles amplification is obtained, while for small angles the device is almost transparent (due to SUSY breaking). Notably, being the superpartner of vacuum, the device is reflectionless for all wavelengths and angles of incidence.
2 Results

Given some $\epsilon$, we consider the supersymmetric partner of the null potential $V(X) = 0$ which can be obtained through a single DT of $\psi(X, \epsilon) = C_1 \cos(\sqrt{\epsilon}X) + C_2 \sin(\sqrt{\epsilon}X)$, giving

$$V_{\text{DT}}(X) = 2\epsilon \left[ 1 + \left( \frac{C_2 - C_1 \tan(\sqrt{\epsilon}X)}{C_1 + C_2 \tan(\sqrt{\epsilon}X)} \right)^2 \right]$$

(1)

where for the present study we took $\epsilon = 0.00005i$ and $C_1 = 1, C_2 = 0.01$ with the refractive index spatial distribution given by

$$n_{\text{DT}}(X, \omega) = \sqrt{1 - V_{\text{DT}}(X)k_{sc}^2 / k_0^2}$$

(2)

for a fixed $k_{sc}$ and for an incoming wavenumber $k_0$ [13].

We note that the obtained index distribution has very small imaginary part as it is required by the Kramers-Kronig relations. In addition, the profile has no spatial symmetry and has a typical width of few micrometers. To evaluate its spectral response, the analytical Transfer Matrix method is used as explained in [14] using the analytical solution for the field as obtained from the DT. Considering the angular spectrum, the device is almost transparent for incident angles lower than 70° (with transmission slightly larger than unity). For incoming directions around 85°, the device behaves as an amplifier for the visible spectrum. As already mentioned, the proposed filter remains reflectionless for all angles and wavelengths. Thus, the device can be considered as a reflectionless active cavity capable of amplifying radiation impinging at large angles. Interestingly, if amplification is not required, simply by rotating the device, it is possible to bypass the filter using its transparency in the rest of the angular spectrum.

3 Conclusions

We demonstrate how using supersymmetry we can define refractive index spatial distributions that can be used as a dynamical optical amplifier. In particular, the proposed filter is reflectionless and is amplifying the incoming radiation for large angles close to 85° while it is almost transparent for smaller angles. The device can be rotated to achieve dynamism i.e., by changing the incoming direction, the same filter can be used as an amplifier or it can be bypassed exploiting its transparency for smaller angles. This is valid in transmission while reflection is always completely suppressed for all wavelengths and angles due to the fact that vacuum is transformed through a Darboux transform.

References