Non-local property of single-photon states: illustration with spontaneously emitted photon from a Hydrogen atom

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Abstract. We discuss a new proof of the non-locality of single-photons using a concrete physical observable such as the local energy density. As an illustration of this non-locality, we compute the mean value of the local energy observable for the spontaneous emission from a Hydrogen atom. We find that, in the subspace of single-photon states, the mean value of the local energy density observable is non-zero everywhere and decreases as a power of the distance.

1 Photon localization properties

The ability to produce pulse-shaped single-photons [1] has brought back the importance of their description in position space [2,3]. Our recent works show that instead of defining photons through the decomposition of the field into plane waves, one can write the theory with bosonic creation-annihilation operators directly on pulses of arbitrary shape

$$|1_{\vec{\psi}}>=\hat{B}^{\dagger}_{\vec{\psi}}|\emptyset>,$$

where $\vec{\psi}$ is a classical complex representation of the pulse in position space [2,3]. The dynamics associated to this single-photon pulse is then given directly by $\hat{B}^{\dagger}_{\vec{\psi}(t)}|\emptyset>$ where $\vec{\psi}(t)$ is the classical dynamics deduced from Maxwell's equations. Such a description allows to consider photon properties from a spatial point of view like for instance their detection. Indeed, since photons are no longer considered as plane waves, their shape may influence detection processes.

Motivated by the important growing of nanophotonics, one can ask what would happen if a photon were carried by a pulse whose size is bigger than the detector which absorbs it [1]. To construct a model able to describe such situation, we consider as an observable, the local electromagnetic energy density

$$\hat{\mathcal{E}}_{\rm loc}(\vec{x})=\frac{\varepsilon_0}{2}:\left(\vec{\hat{E}}^2(\vec{x})+c^2\vec{\hat{B}}^2(\vec{x})\right):,$$

which is clearly local and could describe detectors sensitive to photon's energy like e.g. superconducting nanowire single-photon detectors [4].

However, single-photons are known to be non-local [5,6] i.e. even if $\vec{\psi}$ is zero outside a finite subset $\mathcal{V}$, the mean value of some observable $\hat{A}$ does not equal the mean value of the vacuum outside $\mathcal{V}$:

$$<1_{\vec{\psi}}|\hat{A}|1_{\vec{\psi}}>\neq<\emptyset|\hat{A}|\emptyset>.$$

This result can be explicitly shown using the local electromagnetic energy density observable whose mean value in a general single-photon state reads [5]

$$<1_{\vec{\psi}}\left|\hat{\mathcal{E}}_{\rm loc}(\vec{x})\right|1_{\vec{\psi}}>=\hbar\left|\Omega^{1/2}\vec{\psi}^{(h+)}\right|^2+\hbar\left|\Omega^{1/2}\vec{\psi}^{(h-)}\right|^2,$$

where the exponents $(h\pm)$ denote positive and negative helicity parts of the field and we have introduced the frequency operator Ω which is proportional to the positive square root of the Laplacian operator i.e. $\Omega=c(-\Delta)^{1/2}$. Our argument to show the non-locality of $|1_{\vec{\psi}}>$ is based on the anti-local property of Ω [7] which allows to show that

$$<1_{\vec{\psi}}|\hat{\mathcal{E}}_{\rm loc}(\vec{x})|1_{\vec{\psi}}>\neq 0$$

everywhere and for any $\vec{\psi}$ — including those that are zero outside $\mathcal{V}$.

2 Spontaneous emission

To illustrate such behavior of single-photons, we will discuss the localization of a photon spontaneously emitted by a Hydrogen atom. Indeed, we consider the Lyman- α transition in a Weisskopf-Wigner type model for which we use an interaction Hamiltonian of the form $\vec{\hat{p}}.\vec{\hat{A}}(\vec{x})$, i.e. without the dipole approximation since it allows to compute a finite Lamb shift [8,9]. Therefore, in the subspace of single-photons, we reconstruct the state of the emitted photon, and estimate $\langle\hat{\mathcal{E}}_{\rm loc}(\vec{x})\rangle$. We find, as expected, a non-vanishing result everywhere in space and we also characterize the decreasing for large positions which is far from an exponential decay and rather proportional to a power of the distance.

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