Complex interactions of breathers

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\textbf{Abstract.} We present our recent theoretical and experimental advancements in studying complex multiple nonlinear interactions of coherent solitary wave structures on unstable background – breathers. We use the focusing one-dimensional nonlinear Schrödinger equation (NLSE) as a theoretical model. First, we describe the nonlinear mutual interactions between a pair of co-propagative breathers called breather molecules. Then with the novel approach of breather interaction management, we adjust the initial positions and phases of several breathers to observe various desired wave states at controllable moments of evolution. Our experiments carried out on a light wave platform with a nearly conservative optical fiber system accurately reproduce the predicted dynamics. In addition, we consider generalizations of the scalar breathers theory to the vector two-component NLSE describing polarized light and show examples of resonance vector breathers transformations.

\section{1 Introduction}

Dynamics of nonlinear wave groups propagating on unstable backgrounds represent a fundamental theoretical problem with many applications in optics, hydrodynamics, and other fields of physics. The interplay between the wave groups and the background, accompanied by the modulation instability, produces a rich spectrum of nonlinear phenomena \cite{1}. In general, such dynamics can be studied using linear analysis of modulation instability or numerically. Meanwhile, the model of the focusing one-dimensional nonlinear Schrödinger equation (NLSE) provides analytical approaches to investigate even nonlinear stages of the problem thanks to the Inverse Scattering Transform (IST) method. In particular, the IST approach allows for finding exact breather solutions to the NLSE and describing their interactions analytically. We write the NLSE in dimensionless form as,

\begin{equation}
\dot{\psi} + \frac{1}{2} \psi_{\tau \tau} + |\psi|^2 \psi = 0,
\end{equation}

where $\psi(\xi, \tau)$ describes complex-valued wavefield, $\xi$ and $\tau$ are the scaled propagation distance and time, respectively. Breathers are coherent nonlinear wave structures pulsating on an unstable background, which description represents a generalization of the soliton theory. In contrast to usual NLSE solitons, which are temporally localized traveling wave groups, the NLSE breathers can be either localized, in time, in space, or both in time and space. The family of fundamental breathers includes famous exact solutions of Kuznetsov-Ma, Peregrine, Akhmediev, and Tajiri-Watanabe, which properties have been previously studied in detail; see, e.g., \cite{2}. The fundamental breathers and their interactions have been recently observed experimentally in conservative optical fiber systems, deep water wave tanks, and other physical systems, e.g., \cite{3, 4}. The rapid growth of interest in describing arbitrarily shaped nonlinear wave fields promotes the development of novel approaches to describe complex multi-breather dynamics. Here we present a general view of complex interactions of breathers based on our recent theoretical and experimental works \cite{5, 7, 8}. We focus on the nonlinear dynamics of breather wave molecules, the novel concept of breather interaction management, and nontrivial generalizations of the scalar NLSE breathers theory to the vector two-component NLSE.

\section{2 Results and discussion}

Withing the NLSE model each breather is characterized by four real-valued parameters $R$, $\alpha$, $x_0$, $\theta$ and the background amplitude, which we set to unity. As such, the general $N$-breather solution $\psi^{NB}$ has $4N$ real valued parameters $\{R_1, \alpha_1, x_{0,1}, \theta_1; \ldots; R_N, \alpha_N, x_{0,N}, \theta_N\}$, where the subscripts distinguish each of the $N$ breathers. It fully describes propagations and interactions of the breathers and can be represented as exact solution to Eq. (1),

\begin{equation}
\psi^{NB}(R_1, \alpha_1, x_{0,1}, \theta_1; \ldots; R_N, \alpha_N, x_{0,N}, \theta_N),
\end{equation}

which explicit expression can be found for example in \cite{7}. We consider only temporary localized breathers (the case $R > 1$), which are characterized by group velocity,

\begin{equation}
V_{gr} = -\sin \alpha(R^2 + 1)/[R(R^2 - 1)],
\end{equation}

When the group velocities of two breathers coincide,

\begin{equation}
V_{gr,1} = V_{gr,2}
\end{equation}

they can form a pulsating quasi-periodic nonlinear complex called breather molecule, see example in figure 1.
For specific commensurate conditions, the molecule oscillations become precisely periodic; see details in [6].

When two breathers approach each other, they collide and then separate. Similar to solitons, the breathers interact elastically. After the collision, they restore their shape at large separation distances but acquire specific shifts of the position and phase parameters \( x_0 \) and \( \theta_N \). Recently we derived the shifts expressions in [5] and proposed a novel approach to manage the breathers interactions [7]. The latter allows adjusting the initial positions and phases of several breathers to observe various desired wave states at controllable points of the system evolution. For example, we obtain the exact expression for separation \( \tau_{sh} \) between two pairs of superregular (SR) breathers, which correspond to a further synchronized collision resulting in the formation of a rogue wave (RW) profile; see figure 2.

Our experiments carried out on a light wave platform with a nearly conservative optical fiber system accurately reproduce the predicted dynamics of breather wave molecules and synchronized interactions of breathers, providing experimental version of the dynamics similar to those shown in figure 1 and figure 2; see [6] and [7].

Finally, we consider the vector two-component extension of the NLSE – the Manakov system, which describes polarized light by two wave field components \( \psi_1 \) and \( \psi_2 \). The class of vector breather solutions is presented by three fundamental types I, II, and III. While the type I is a trivial vector generalization of the scalar NLSE breathers, see Eq. (2), the types II and III exhibit fundamentally different vector dynamics. The interactions of vector breather lead to complex wave field behavior, which we investigate in our recent work [8]. In particular, the vector breathers participate in resonance collision shown in figure 3. The experimental observation of vector breathers represents a challenging task for further studies.

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References