**Soliton-number measurement in lossy waveguides**

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**Abstract.** A general technique for obtaining the soliton number, and hence the nonlinear coefficient, in waveguides with high dispersion and loss is derived and demonstrated numerically and experimentally in a kilometer-long standard silica fiber pumped close to 2 μm.

1 Introduction

Loss management is becoming increasingly important in areas of nonlinear optics exploiting plasmons, PT-symmetry [1], 2D materials [2] or dispersive nonlinearities [3]. In these systems, the interplays between nonlinear, dispersive (or diffusive), and loss (or gain) effects are instrumental. As such, none of them should be dismissed when determining nonlinearities experimentally. Neglecting either dispersion [4] or losses [5], current methods fail to provide accurate nonlinearities measurements in these cases. This limitation may also hinder the experimental assessment of the different theoretical expressions proposed so far for the nonlinear coefficient of lossy waveguides [6]. Here this problem is solved extending a conservation law of the nonlinear Schrödinger equation (NLSE), instead of making use of its approximated solutions. This technique is proved numerically and experimentally in a kilometer-long SMF28e+ fiber pumped at 1.951 μm, where it features high dispersion and losses.

2 Method

Let us consider the propagation of optical pulses along a lossy waveguide governed by the generalized NLSE,

\[
\frac{\partial}{\partial z} A(z, T) = -\frac{\alpha}{2} A - i\frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} + i\gamma A(z, T)|^2 A, \quad (1)
\]

where \(A\) represents the complex envelope of the electric field, \(T\) denotes the time in the retarded frame, \(\alpha\) provides the linear loss, \(\beta_2\) indicates the group-velocity dispersion, and \(\gamma\) is the nonlinear coefficient. Targeting a solution for \(A\) as a means to obtain \(\gamma\) leads generally to additional constraints on Eq. (1), e.g., neglecting the dispersion term [4]. In contrast to this traditional approach, we propose to work with the two \(z\)-dependent variables \(\rho(z) = (1/2) \int_{-\infty}^{z} |A(z, T)|^2 dT / \int_{-\infty}^{z} |A(z, T)|^2 dT,\) which is proportional to the intensity autocorrelation at zero time delay, and the spectral variance, \(\Delta_2(z) = \int_{0}^{\infty} \Omega |\tilde{A}(z, \Omega)|^2 d\Omega / \int_{0}^{\infty} |\tilde{A}(z, \Omega)|^2 d\Omega,\) where \(\tilde{A}\) and \(\Omega\) are the Fourier transform of \(A\) and the relative angular frequency, respectively [5]. These functions satisfy the ordinary differential equation

\[
\frac{d}{dz} \rho(z) - \alpha \rho(z) = \frac{\beta_2}{2\gamma} \frac{d}{dz} \Delta_2(z), \quad (2)
\]

which integration leads to the algebraic equation

\[
\alpha e^{-\alpha T} - \Delta_2(z) = \frac{\beta_2}{2\gamma} \Delta_2(z), \quad (3)
\]

where \(0 < \ell < L\), with \(L\) being the output distance, \(\Delta_0 = e^{i\ell}\rho(L) - \rho(0)\), and \(\Delta_2 = \mu_2(L) - \mu_2(0)\) [8]. When losses are negligible, Eq. (3) provides \(\beta_2/\gamma\) straightforwardly, and hence the soliton number for a given input pulse and \(\gamma\) if \(\beta_2\) is available, measuring \(\Delta_0\) with an intensity autocorrelator (or an oscilloscope if relatively long pulses are employed) and \(\Delta_2\) using an optical spectrum analyzer (OSA) at different powers [5]. When losses cannot be disregarded, however, \(\ell\) is needed, in addition, to extract \(\beta_2/\gamma\) from \(\Delta_0\) and \(\Delta_2\) measurements. The integration of Eq. (2) also shows that

\[
e^{-\alpha T} \Delta_0 + \alpha \int_{0}^{\ell} \rho(z)dz = \Delta_2, \quad (4)
\]

which indicates that, under high losses, information of \(\rho\) over the propagation length, and not only at the input and output [5], is required to evaluate \(\ell\), and hence \(\beta_2/\gamma\).

The variables \(\rho(z)\) and \(\mu_2(z)\) have recently found application to distinguish dynamic regimes in the parameter space of the NLSE based on their propagation equation [7]. Attending to this theory, \(f(z) = (\rho(z)/\rho(0))^{-1}\) propagates approximately according to the equation

\[
f(z) e^{d^2f/dz^2} = \text{sign}(\beta_2) \eta^2 \rho_0 \mu_20 e^{-\alpha z}, \quad (5)
\]
where $\eta$ is a fitting parameter, $\rho_0 = \rho(0)$ and $\mu_2(0) = \mu_2(0)$ [8], which will allow determining $\ell$, and thus $\beta_2/\gamma$, as follows. For each $\rho_0$ and $\mu_2(0)$ values, $\eta$ will be fitted so that Eq. (5) recovers the experimental value for $f(L)$. Being $\eta$ fixed, $\ell$ will be computed using first Eq. (5) and later Eq. (4). Subsequently, $\beta_2/\gamma$, and hence $\gamma$ is $\beta_2$ had previously been measured, will be obtained from Eq. (3). Such a $\gamma$ value could even speed an iterative calculation to improve the accuracy of $\gamma$ solving Eq. (1) to evaluate $\ell$, as the next numerical test reproducing our experimental conditions confirms.

Let us consider 90 ps long sech pulses propagating 1.16 km along a fiber with $\alpha = 2.5 \times 10^{-3}$ m$^{-1}$, thus $\alpha L \sim 3$, $\beta_2 = -79$ ps$^2$ km$^{-1}$ and $\gamma = 0.68$ W$^{-1}$ km$^{-1}$. These parameters correspond to a SMF28e+ fiber at 1.951 $\mu$m [8]. To check the proposal numerically, Eq. (1) is solved for input peak powers of 1 W, 20 W and 30 W to obtain output values for $\rho$ and $\mu_2$, namely, the eventual experimental data. Following the steps outlined above, the value of $\gamma$ is recovered after a few iterations in all cases, see Fig. 1.

3 Proof-of-concept experiment

In our experiment, see Fig. 2(a), ~90 ps pulses generated by a passively mode-locked polarization-maintaining (PM) thulium-doped fiber (TDF) laser at 1.951 $\mu$m with a repetition rate of 18.45 MHz and amplified in a PM TDF were injected into a 1.16 km long SMF28e+ Corning fiber. The spectra and temporal pulses were measured using an OSA and a digital sampling oscilloscope together with a 2 $\mu$m InGaAs photodetector, respectively. A variable optical attenuator was employed to fix the operation of the PM amplifier and select input peak powers between 14 W and 37 W [8].

After processing the experimental data as explained in the previous section, $\beta_2/\gamma = -102 \pm 13$ ps$^2$ W was measured from the results plotted in Fig. 2(b), corresponding to the first iteration. This value already agrees with $\beta_2/\gamma = -110 \pm 6$ ps$^2$ W obtained using $\beta_2 = 75 \pm 4$ ps$^2$ km$^{-1}$, which we directly measured using a standard method [8], and $\gamma = 0.68 \pm 0.06$ W$^{-1}$ km$^{-1}$, which we calculated numerically based on the experimental $\gamma$ value at 1.55 $\mu$m [5]. No more iterations were carried out since the experimental errors due to the relatively small $\Delta \rho$ values produced in this proof of concept set the attainable accuracy.

In conclusion, $\beta_2/\gamma$ has been successfully measured accounting for high dispersion and losses exploiting a balance law of the generalized NLSE. This approach can particularly benefit nonlinear optical systems involving plasmons, $PT$-symmetry, 2D materials or dispersive nonlinearities.

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**References**