Double-frequency-comb-like source with PM passive fibre cavity and Gain Through Filtering.

Negrini Stefano[1*], *Perego* M. Auro[2], *Conforti* Matteo[1] and *Mussot* Arnaud[1]

[1]PhLam UMR 8523, University of Lille, 2 Av. Jean Perrin, 59650 Villeneuve-d'Ascq, France
[2]Aston University, Aston St, Birmingham B4 7ET, United Kingdom

Abstract. In this work, we present a theoretical and numerical study about the generation of double-frequency-comb-like source with Gain Through Filtering (GTF) in passive Polarization Maintaining (PM) fibre ring cavity.

INTRODUCTION

Frequency combs are spectra composed by equally spaced laser lines, and in the recent year have been object of many interesting applications: from telecommunication to Lidar to spectrometry and spectroscopy [1]. In the latter case, several advantages are obtainable with a technique known as double-frequency-comb spectroscopy [2], which imply the use of two coherent frequency combs.
In this work we propose a theoretical study which illustrate the formation of a double-frequency-comb-like spectrum in a single PM cavity, by exploiting a phenomenon called GTF [3]. Even if a fibre cavity does not reach the number of spectral lines of an OFC, this work is a valid physical study of the phenomenon, which can be scaled down to sizes and performances typical of micro-cavities.

1.1 System model: LLE model and GTF

1.1.1 Lugiato-Lefever Equation

The system can be described with a set of two LLE coupled in the non-linear phase term. In the following, subscript character x and y specify the polarization axis correspondent to the parameter. If not specified, the parameter is supposed to be equal in each polarization:

$$L\frac{\partial A_x}{\partial z} = [-\alpha + i\,\phi_0 + \Phi_x \star + i\,\Psi_x \star]A_x + \left[+L\,\Delta\beta_1\frac{\partial}{\partial t} - \frac{iL\beta_2}{2}\frac{\partial^2}{\partial t^2} + i\,L\,\gamma\left(|A_x|^2 + \sigma|A_y|^2\right)\right]A_x + \theta E_x, \quad (1)$$

$$L\frac{\partial A_y}{\partial z} = [-\alpha + i\,\phi_0 + \Phi_y \star + i\,\Psi_y \star]A_y + \left[-L\,\Delta\beta_1\frac{\partial}{\partial t} - \frac{iL\beta_2}{2}\frac{\partial^2}{\partial t^2} + i\,L\,\gamma\left(|A_y|^2 + \sigma|A_x|^2\right)\right]A_y + \theta E_y. \quad (2)$$

Where $A_{x,y}$ are the intracavity fields, $E_x = E_{IN}\cos(\chi)$ $E_y = E_{IN}\sin(\chi)$ are the projection of the input fields E_{IN} into the two polarizations axis with χ the angle of injection respect to the x axis. $\alpha = 1-\rho$ accounts for all the losses of the cavity, with ρ and θ being the reflectivity and transmission parameters of the coupler, respectively. ϕ_0 is the linear phase detuning, $\Phi_{x,y}$ and $\Psi_{x,y}$ are the Fourier anti-transform of the filter's loss profile and phase, defined in Eqs. (4) and (5). L is the length of the cavity, β_2 is the group velocity dispersion parameter of the fibre, and $\Delta\beta_1 = \frac{\beta_{1x}+\beta_{1y}}{2}$ accounts for the group velocity mismatch between the two polarizations. Finally, $\sigma = \frac{2}{3}$ is the phase coupling parameter of the two components.

1.1.2 Filter description and GTF

One could describe the filter with its transfer function $H(\omega)$:

$$H(\omega) = e^{F(\omega)+i\psi(\omega)}, \quad (3)$$

$$F(\omega) = b\frac{a^4}{\left(\omega-\omega_f\right)^4 + a^4}, \quad (4)$$

$$\psi(\omega) = ba\frac{\left(\omega-\omega_f\right)\left[\left(\omega-\omega_f\right)^2 + a^2\right]}{\sqrt{2}\,[(\omega-\omega^4) + a^4]}. \quad (5)$$

With $F(\omega)$ and $\psi(\omega)$ being the loss and phase profile, respectively. a and b are two parameters which controls the depth and the width of the loss profile and ω_f is the central frequency of the filter. This model can be used to describe fibre Bragg gratings (FBG), typically used as spectral filters. A PM FBG can be easily modelled by defining an x and y component, with the same shapes, i.e. same a and b parameters, but two different ω_f shifted by a given $\Delta\omega = 2\pi\Delta f$. This shift in the Bragg frequencies is due to the different refraction index of the two polarizations.

* Corresponding author: stefano.negrini@univ-lille.fr

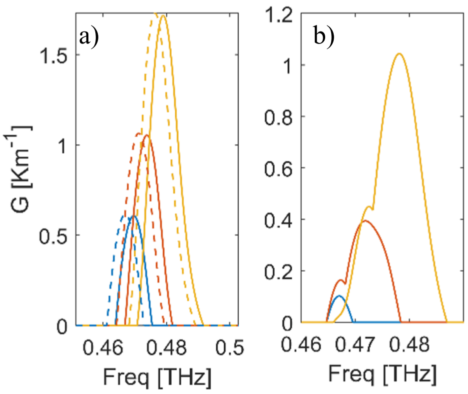

Fig. 1 Two example of positive sideband of parametric gain computed from a stability analysis of the LLE model: a) Gain for an injection angle $\chi = 0\ rad$ (dashed lines), $\chi = \pi/2\ rad$ (solid lines), at increasing power; b) Gain for an injection angle $\chi = 0.86\ rad$. In both cases, the input power is $P_{in} = 0.05\ W, P_{in} = 0.15\ W$ and $P_{in} = 0.65\ W$. The following parameters has been used: $\Delta\beta_1 = 0.5\ ps\ Km^{-1}$, $\beta_2 = 0.5\ ps^2 km^{-1}$, $\gamma = 2.5\ W^{-1}km^{-1}$, $\phi_0 = 0\ rad,\ \rho = \sqrt{0.95}$, $\theta = \sqrt{0.05}$, $\omega_{pump} = 2\pi * 194.1$THz . The parameters used for modelling the filters are: $a = 85\ rad/ns$, $b = -2.45$, $\omega_{fx} = 2\pi * 193.4\ THz$, $\omega_{fy} = 2\pi * (193.4\ THz + \Delta f)$, with the frequency shift of the two polarization being $\Delta f = 3.2\ GHz$.

By adding the filter to the cavity, the phase accumulated by the signal during the propagation is modified. Within the right conditions, this allows the destabilization of Modulation Instability (MI) bands at frequencies which depends on the Bragg wavelength of the filter. This process is known as Gain Through Filtering [3], and it's basically an additional degree of freedom for MI in fibre cavity. In this work, we exploit this degree of freedom for generate spectra with slightly different FSR in the two polarizations axis, creating a double-frequency-comb-like source.

2 Theoretical and Numerical results

By performing a stability analysis to Eqs. (1) and (2), we were able to compute the gain of MI stimulated by GTF. In Fig. 1 we present two plots of the evolution of the gain for different injection angles χ. In Fig.1 a) $\chi = 0\ rad$ for the dashed lines and $\chi = \pi/2\ rad$ for the solid lines. In these cases, the pump is aligned with one of the axes, thus the resulting gain bands position depends only on the *x* or *y* component of the filter. When the injection angle is different, Fig.1 b), a double peak is visible. The shift between the two peak is equal to Δf, i.e., the frequency shift between the *x* and *y* components of the filter. To better understand the meaning of the double peak in the gain, the model has been numerically integrated with a split-step-Fourier method, and the results are illustrated in Fig. 2 and Fig. 3. The simulation shows that the frequency of the first band's peak of spectra on the two polarizations axis differs for Δf. This because the position of those bands is dictated by GTF, which brings to the formation of instability bands near the Bragg wavelength of the filter. Given the PM nature of the filter, and the consequent shift in the Bragg wavelengths, the spectra in

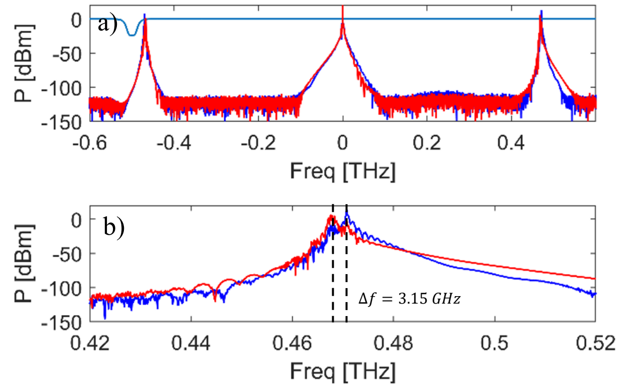

Fig. 2 a) Example of the intracavity spectra; b) zoom on the first bands of the spectra. Red traces are the *x* polarization axis signal, blue traces are the *y* polarization axis signal. The injection angle is $\chi = 0{,}86\ rad,$ *and the power* $Pin = 0.15\ W$, the other parameters are listed in the caption of Fig.1. The charming trace represent the amplitude profile of the filter (not in scale).

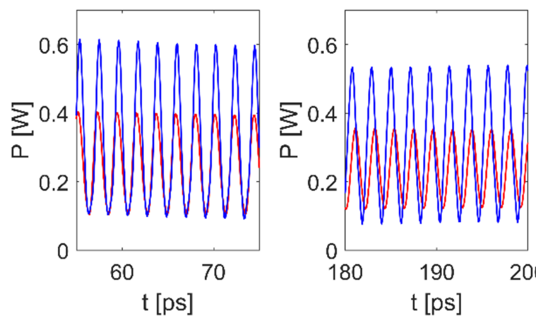

Fig. 3 Two sample from the simulated time frame which illustrate the difference in the period of the modulation on the two polarizations. Red traces represent the signal on the *x* polarization axis, blue traces the *y* polarization axis.

the two polarizations manifest the same frequency shift. Finally, in Fig.3, we report the signal in time domain: given the difference in the FSR of the spectra in the two polarizations, the period of the modulation on the two polarization is not the same. This demonstrate that it's indeed possible to exploit GTF process, within the context of PM fibre cavity, to create a double frequency comb-like resonator. A major advantage of exploiting GTF is the possibly of tailoring the filter *ad-hoc* depending on the necessities, with the possibility of tuning the Δf and the free spectral range of the spectrum.

References

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