

Hadronic contributions to the muon $g - 2$ in holographic QCD

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Abstract. We discuss the recent progress made in using bottom-up holographic QCD models in calculating hadronic contributions to the anomalous magnetic moment of the muon, in particular the hadronic light-by-light scattering contribution, where holographic QCD naturally satisfies the Melnikov-Vainshtein constraint by an infinite series of axial vector meson contributions.

1 Introduction

The anomaly of the magnetic moment of the muon $a_\mu = (g-2)_\mu/2$ [1] is currently known with an experimental uncertainty of only 41×10^{-11} when the E821/BNL measurement from 2006 [2] is combined with the concordant result obtained in 2021 by the Muon $g-2$ Collaboration at Fermilab [3], corroborating a long-standing discrepancy with the Standard Model prediction, which according to the 2020 White Paper (WP) of the Muon $g-2$ Theory Initiative [4] has a similar estimated error of 43×10^{-11} but a deviation of 4.2σ in its value. The theoretical error is completely dominated by hadronic contributions, which arise from hadronic vacuum polarization (HVP) and hadronic light-by-light (HLBL) scattering,

$$\begin{aligned} a_\mu^{\text{HVP,WP}} &= (6845 \pm 40) \times 10^{-11} \quad (0.6\% \text{ error}), \\ a_\mu^{\text{HLBL,WP}} &= (92 \pm 19) \times 10^{-11} \quad (20\% \text{ error}), \end{aligned} \quad (1)$$

in (effective) one-loop and two-loop contributions in the muon-photon vertex diagram.

The WP result for the HVP contribution is obtained by a data-driven, so-called dispersive approach, which however has been challenged by a recent lattice QCD result with comparable errors but a central result that is about 3% larger [5], almost eliminating the discrepancy with the experimental result for a_μ , but instead producing a $\sim 3\sigma$ deviation from results based on the R -ratio in $e^+e^- \rightarrow \text{hadrons}$.

The WP estimate of the HLBL contribution is only partially determined by data and involves input from hadronic models with larger uncertainties, albeit direct complete lattice evaluations are now gradually bringing down their errors [6, 7].

Holographic QCD (hQCD) is a conjectural approximation to strongly coupled QCD (in its large- N_c limit) based on the more well-established AdS/CFT correspondence [8, 9] and experience with top-down string-theoretical constructions of gauge/gravity dual theories

[10–12]. Using a minimal set of parameters, holographic QCD has proved to be capable of good qualitative and often quantitative predictions in hadron physics [13–15] with typical errors of (sometimes less than) 10 to 30%. This is clearly too crude to be of help with the currently most pressing issue of the theory result for a_μ^{HVP} , where the discrepancy between lattice and data-driven approaches is a few percent and where eventually sub-percent accuracy is needed. However, in the case of HLBL contributions, the error is dominated by two contributions, the estimate of the effect of short-distance constraints (SDC) and the contribution of axial-vector mesons, where the WP values are

$$\begin{aligned} a_\mu^{\text{HLBL,SDC}} &= (15 \pm 10) \times 10^{-11} \quad (67\% \text{ error}), \\ a_\mu^{\text{HLBL,axials}} &= (6 \pm 6) \times 10^{-11} \quad (100\% \text{ error}). \end{aligned} \quad (2)$$

It is here that hQCD can provide interesting results, whereas the rather precisely known HVP contribution can be used to assess potential deficiencies of the various hQCD models.

2 Top-down and bottom-up hQCD models

Exact holographic duals are known only in certain limits for highly symmetric theories such as the superconformal $\mathcal{N} = 4$ Yang-Mills theory in the limit of large- N_c and infinite 't Hooft coupling $g^2 N_c$. But already in 1998, Witten [10] succeeded in constructing a gauge/gravity dual to the low-energy limit of large- N_c Yang-Mills theory, “top-down” from type-IIA superstring theory, where supersymmetry and conformal symmetry are broken by compactification on one extra dimension beyond the holographic dimension dual to inverse energy. In 2004, Sakai and Sugimoto found a D-brane construction within Witten’s model that adds chiral quarks in the fundamental representation [11, 12], thus providing the so far closest dual theory to low-energy QCD in the large- N_c and chiral limit. With having only a mass scale and one dimensionless number (the 't Hooft coupling at the scale of the Kaluza-Klein compactification) the (Witten-)Sakai-Sugimoto model turns out to be remarkably successful

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both qualitatively and also quantitatively. Chiral symmetry breaking is a direct consequence of its geometry, while flavor anomalies are naturally realized. However, above the Kaluza-Klein scale $M_{KK} \approx 1$ GeV this model exhibits a different ultra-violet (UV) behavior than QCD, since its actual dual at high energy scales is a 5-dimensional superconformal theory instead of 4-dimensional QCD with asymptotic freedom.

Skipping a string-theoretic top-down derivation, phenomenologically interesting “bottom-up” models of hadron physics were obtained in [13–15] that combine certain features also present in the Sakai-Sugimoto model with a simpler geometry that is asymptotically AdS_5 (and therefore conformal in the UV), breaking conformal symmetry in the infrared (IR) by a hard-wall (HW) cutoff with appropriate boundary conditions or by a soft wall (SW) provided by a nontrivial dilaton [16–24], which has similarities with light-front holographic QCD [25].

A common feature of the various bottom-up and also the top-down Sakai-Sugimoto model is that vector and axial vector mesons as well as pions are described by 5-dimensional Yang-Mills fields $\mathcal{B}_M^{L,R} = \mathcal{B}_M^V \mp \mathcal{B}_M^A$ for the global $U(N_f)_L \times U(N_f)_R$ chiral symmetry of boundary theory with 5-dimensional action

$$S_{YM} \propto \frac{1}{g_5^2} \text{tr} \int d^4x \int_0^{z_0} dz e^{-\Phi(z)} \sqrt{-g} g^{PR} g^{QS} \times (\mathcal{F}_{PQ}^{(L)} \mathcal{F}_{RS}^{(L)} + \mathcal{F}_{PQ}^{(R)} \mathcal{F}_{RS}^{(R)}), \quad (3)$$

where $P, Q, R, S = 0, \dots, 3, z$ and $\mathcal{F}_{MN} = \partial_M \mathcal{B}_N - \partial_N \mathcal{B}_M - i[\mathcal{B}_M, \mathcal{B}_N]$ with conformal boundary at $z = 0$, and either a sharp cut-off of AdS_5 at z_0 (the location of the hard wall) or $z_0 = \infty$ (SW) with nontrivial dilaton.

Chiral symmetry is broken either by introducing an extra bifundamental scalar field X dual to quark bilinears $\bar{q}_L q_R$ as in the original HW model of Erlich et al. [13, 14] (HW1), or through different boundary conditions for vector and axial vector fields at z_0 as in the Hirn-Sanz model [15] as well as in the top-down Sakai-Sugimoto model [11, 12].

Vector meson dominance (VMD), in a form necessarily involving an infinite tower of vector mesons, is naturally built in. Photons are described by boundary values of the appropriate combination of the vector gauge fields as those are sourced by quark currents, and they couple through bulk-to-boundary propagators to mesonic degrees of freedom described by normalizable modes.

Of particular importance for the following is that flavor anomalies follow uniquely from 5-dimensional Chern-Simons terms $S_{CS}^L - S_{CS}^R$ with

$$S_{CS} = \frac{N_c}{24\pi^2} \int \text{tr} \left(\mathcal{B} \mathcal{F}^2 - \frac{i}{2} \mathcal{B}^3 \mathcal{F} - \frac{1}{10} \mathcal{B}^5 \right) \quad (4)$$

in differential-form notation.

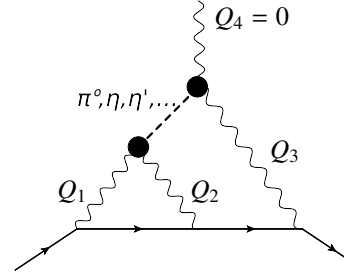


Figure 1. Hadronic light-by-light scattering contribution to a_μ from single meson exchange

3 Anomalous TFFs from hQCD and HLBL contribution to the muon $g - 2$

3.1 Holographic pion TFF

The leading HLBL contribution to a_μ comes from the π^0 exchange diagram shown in fig. 1 due to the anomalous coupling of the pion to two photons. This involves both a singly-virtual transition form factor (TFF) in the upper vertex and a doubly-virtual one in the interior of the diagram. The holographic prediction is given by

$$F_{\pi^0 \gamma^* \gamma^*}(Q_1^2, Q_2^2) = -\frac{N_c}{12\pi^2 f_\pi} \int_0^{z_0} dz \mathcal{J}(Q_1, z) \mathcal{J}(Q_2, z) \Psi(z), \quad (5)$$

where $\mathcal{J}(Q, z)$ is the bulk-to-boundary propagator of a photon with virtuality $Q^2 = -q^2$ and a holographic pion profile function $\Psi(s)$. This has been studied by Grigoryan and Radyushkin in [26–28], who noticed that hQCD models with asymptotic AdS_5 geometry reproduce the asymptotic momentum dependence obtained by Brodsky and Lepage [29–31],

$$F_{\pi^0 \gamma^* \gamma^*}(Q_1^2, Q_2^2) \rightarrow \frac{2f_\pi}{Q^2} \left[\frac{1}{w^2} - \frac{1-w^2}{2w^3} \ln \frac{1+w}{1-w} \right] \quad (6)$$

with $Q^2 = \frac{1}{2}(Q_1^2 + Q_2^2) \rightarrow \infty$, $w = (Q_1^2 - Q_2^2)/(Q_1^2 + Q_2^2)$, which is not achieved by conventional VMD models.

With certain simplifications the holographic results for the pion TFF have been employed in ref. [32, 33] to evaluate the a_μ contribution given by the diagram in fig. 1. In ref. [34] we have carried out a complete evaluation and found good agreement with the data-driven (dispersive) result, which is bracketed by the HW1 and HW2 results when these models are fitted to f_π and m_ρ . With such a fit, the HW2 model, which has one parameter less than the chiral HW1 model, actually undershoots the asymptotic limit of (6) by 38% (fitting the asymptotic limit would instead lead to an overweight ρ meson with mass of 987 MeV). Its prediction of $a_\mu^{\pi^0} = 56.9 \times 10^{-11}$ is correspondingly on the low side. The HW1 model, which satisfies (6) exactly, predicts $a_\mu^{\pi^0} = 65.2 \times 10^{-11}$, which is consistent with, but somewhat larger than, the WP value $62.6_{-2.5}^{+3.0} \times 10^{-11}$.

However, at not too large, phenomenologically relevant energy scales, the asymptotic behavior of the TFF is

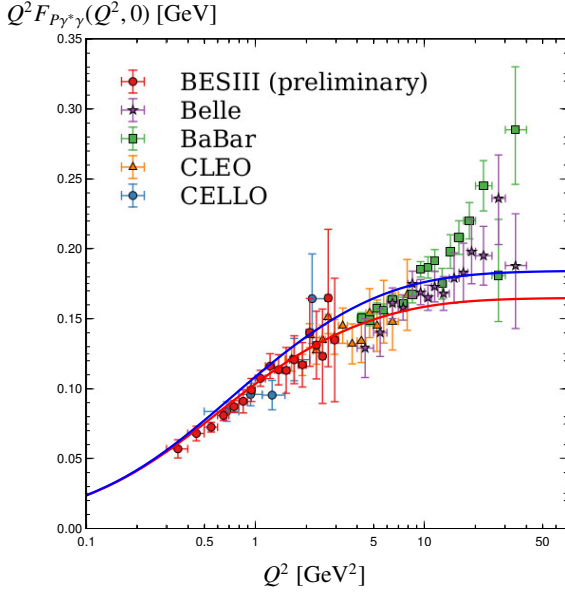


Figure 2. Holographic results for the single virtual TFF $Q^2 F(Q^2, 0)$ for π^0 , plotted on top of experimental data as compiled in Fig. 53 of Ref. [4] for $g_5 = 2\pi$ (OPE fit, blue) and the reduced value (red) corresponding to a fit of F_ρ . (Taken from [41])

modified by gluonic corrections of the order of 10% [35, 36], while hQCD models with a simple AdS₅ background approach the asymptotic limit somewhat too quickly. Similar corrections but with opposite sign appear in the vector correlator appearing in HVP. Indeed, the HW1 model with complete UV fit underestimates HVP. Reducing g_5^2 by 10% (which is achieved by fitting instead the decay constant of the ρ meson) brings the HVP in line with the dispersive result (to within 5%) [37] and also makes the HLBL result $a_\mu^{\pi^0}$ completely coincident with the central WP value [38].

In fig. 2 the HW1 result (with finite quark masses) for the singly virtual pion TFF is compared with experimental data, where the result with reduced g_5^2 is seen to agree somewhat better at the relevant energy scale $Q^2 \lesssim 10 \text{ GeV}^2$; in fig. 3 the doubly virtual result for $Q_1^2 = Q_2^2$ is compared with the dispersive result of ref. [39] and the lattice result of Ref. [40]. Here the HW1 result with reduced coupling coincides with the central result of the dispersive approach within line thickness and it also happens to be in the center of the lattice result.

3.2 Axial vector meson TFF

In hQCD, the coupling of axial vector mesons to photons is determined by the same action (4) that gives rise to the anomalous TFFs of the pseudoscalars. There is an infinite tower of axial vector mesons and their decay amplitude has the form [42, 43]

$$\mathcal{M}_{\mathcal{A}\gamma^*\gamma^*} = i \frac{N_c}{4\pi^2} \text{tr}(Q^2 t^a) \epsilon_{(1)}^\mu \epsilon_{(2)}^\nu \epsilon_{\mathcal{A}}^{*\rho} \epsilon_{\mu\nu\rho\sigma} \times [q_{(2)}^\sigma Q_1^2 A_n^a(Q_1^2, Q_2^2) - q_{(1)}^\sigma Q_2^2 A_n^a(Q_2^2, Q_1^2)], \quad (7)$$

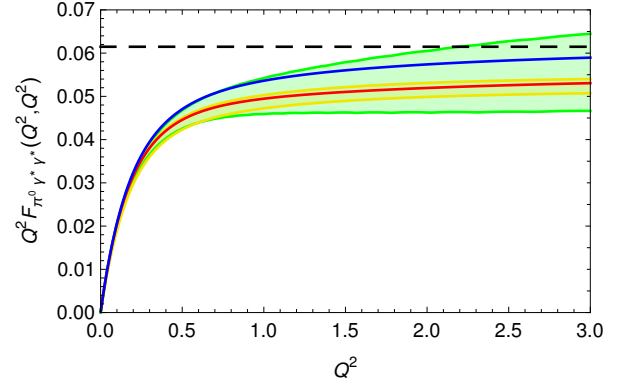


Figure 3. Holographic results for the doubly virtual $F_{\pi^0 \gamma^* \gamma^*}$ compared to the dispersive result of Ref. [39] (green band) and the lattice result of Ref. [40] (yellow band); the OPE limit is indicated by the dashed horizontal line. The blue line corresponds to $g_5 = 2\pi$ (OPE fit) and the red one to the reduced value $g_5 (F_\rho\text{-fit})$. (Taken from [41])

involving an asymmetric structure function

$$A_n(Q_1^2, Q_2^2) = \frac{2g_5}{Q_1^2} \int_0^{z_0} dz \left[\frac{d}{dz} \mathcal{J}(Q_1, z) \right] \mathcal{J}(Q_2, z) \psi_n^A(z) \quad (8)$$

where $\psi_n^A(z)$ is the holographic wave function of the n -th axial vector meson. The most general decay amplitude of axial vector mesons would permit a second asymmetric structure function [44–46], but this does not appear in the simplest versions of hQCD considered here.

The holographic result satisfies the Landau-Yang theorem [47, 48], which forbids the decay of an axial vector meson in two real photons, due to the fact that $\mathcal{J}'(Q, z) = 0$ for $Q^2 = 0$.

The asymptotic form of $A_n(Q_1^2, Q_2^2)$ for large virtualities is given by [42]

$$A_n(Q_1^2, Q_2^2) \rightarrow \frac{12\pi^2 F_n^A}{N_c Q^4} \frac{1}{w^4} \left[w(3 - 2w) + \frac{1}{2}(w + 3)(1 - w) \ln \frac{1 - w}{1 + w} \right], \quad (9)$$

$w = (Q_1^2 - Q_2^2)/(Q_1^2 + Q_2^2)$, which agrees with the independent pQCD result obtained recently in [49].

At moderate virtualities, the shape of the singly virtual axial TFFs turns out to be consistent with the dipole fit of data obtained by the L3 experiment for $f_1(1285)$ [51], as shown in fig. 4. The overall magnitude, which for HW1 and HW2 models is found as 21.04 and 16.63 GeV^{-2} is in roughly the right ballpark compared to experimental data, albeit somewhat too high for HW1 which overestimates the equivalent photon widths of $f_1(1285)$ and $f_1' = f_1(1420)$ [51, 52], but consistent with the phenomenological value $A(0, 0)_{a_1(1230)} = 19.3(5.0) \text{ GeV}^{-2}$ obtained in [45, 53].

The results from the L3 experiment have been used by Pauk and Vanderhaeghen [50] to estimate the contributions of f_1 and f_1' to a_μ , assuming a symmetric factorized dipole moment $A^{\text{PV}}(Q_1^2, Q_2^2)/A(0, 0) = (1 + Q_1^2/\Lambda_D^2)^{-2}(1 +$

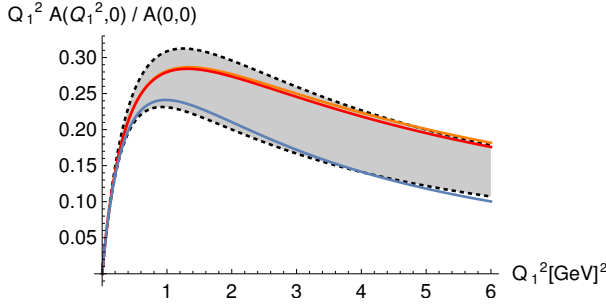


Figure 4. Single-virtual axial vector TFF from holographic models (SS: blue, HW1: orange, HW2: red) compared with dipole fit of L3 data for $f_1(1285)$ (grey band). The parameters of all models are fixed by matching f_π and m_ρ . The results for HW1 and HW2 almost coincide, with HW2 at most a line thickness above HW1. (Taken from [42])

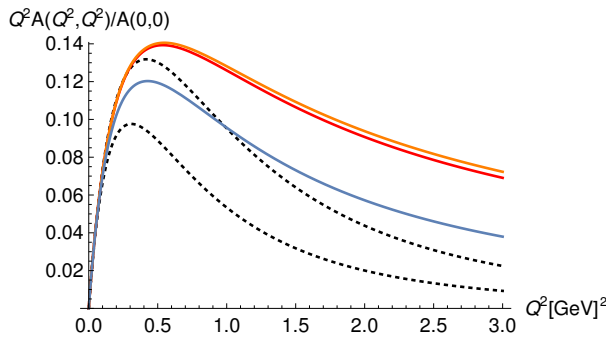


Figure 5. Double-virtual axial vector TFF for $Q_1^2 = Q_2^2 = Q^2$ from holographic models (SS: blue, HW1: orange, HW2: red). The black dashed lines denote the extrapolation of L3 data with a dipole model for each virtuality as used in the calculation of $a_\mu^{\pi^0, \eta, \eta'}$ in Ref. [50]. (Taken from [42])

$Q_2^2/\Lambda_D^2)^{-2}$. This drops too quickly in the doubly virtual case to match the short-distance behavior required by pQCD. In fig. 5 the hQCD results are shown and compared with this simple ansatz, exhibiting a considerable difference.

3.3 Melnikov-Vainshtein SDC

In [42, 43] it has been shown moreover that in hQCD the infinite tower of axial vector mesons is responsible for the correct implementation of the Melnikov-Vainshtein SDC [54] for the HLBL scattering amplitude, which is a consequence of the nonrenormalization theorem for the chiral anomaly. In terms of the tensor basis used in [55] it reads

$$\lim_{Q_3 \rightarrow \infty} \lim_{Q \rightarrow \infty} Q_3^2 Q^2 \bar{\Pi}_1(Q, Q, Q_3) = -\frac{2}{3\pi^2}. \quad (10)$$

Clearly, each single meson exchange contribution gives a vanishing result in this limit, because the propagator in fig. 1 goes like $1/Q_3^2$ and the two form factors go like $1/Q^2$ and $1/Q_3^2$.

In [54] Melnikov and Vainshtein proposed to estimate the impact of the MV-SDC by replacing the external TFF

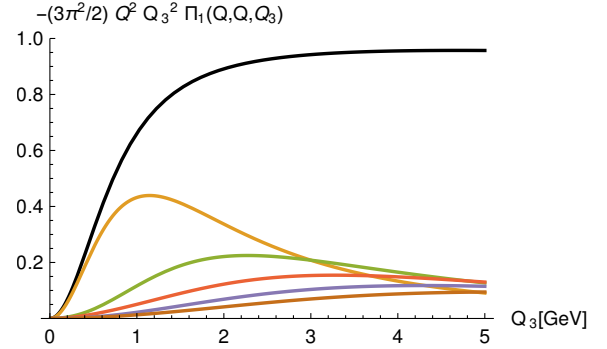


Figure 6. Axial-vector contribution to $Q_3^2 Q^2 \bar{\Pi}_1(Q, Q, Q_3)$ as a function of Q_3 at $Q = 50$ GeV in the HW2 model (with $g_5 = 2\pi$ so that the SDC is satisfied to 100%). The black line corresponds to the infinite sum over the tower of axial vector mesons, and the colored lines give the contributions of the 1st to 5th lightest axial vector mesons. (Taken from [42])

by its constant on-shell value. With current input data, this would lead to an increase of almost 40% of the pseudoscalar contribution $a_\mu^{\pi^0, \eta, \eta'}$ by 38×10^{-11} . However, the MV-SDC can also be satisfied by having an infinite tower of single-meson contributions. In Ref. [56, 57] Colangelo et al. showed that a Regge model for a tower of excited pseudoscalars can be constructed that saturates the MV-SDC with a contribution $\Delta a_\mu^{\text{PS}} = 13(6) \times 10^{-11}$ when using the available experimental constraints for the photon decay amplitudes of excited pseudoscalars. However, as also noted in [57], excited pseudoscalar states decouple in the chiral large- N_c limit, which is not the case for the infinite tower of axial vector mesons. Indeed, in holographic QCD it is the latter which are responsible for the implementation of the MV-SDC [42, 43]. This is illustrated in fig. 6 for the simple HW2 model, which permits a closed form solution for the full series, for $\bar{\Pi}_1(Q, Q, Q_3)$ with large $Q = 50$ GeV and increasing $Q_3 \ll Q$: while each axial vector meson mode gives an asymptotically vanishing contribution, their infinite series satisfies the MV-SDC constraint.

In [38] two of us have shown that also in the HW1 model with massive quarks the infinite tower of axial vector mesons is responsible for the realization of the MV-SDC. The infinite tower of excited pseudoscalars, which does not decouple from the anomaly relation away from the chiral limit, contributes only subleading terms $\propto \ln(Q_3^2)/Q_3^4 Q^2$ to (10).

The hQCD models generally satisfy the MV-SDC to the same level as they satisfy the SDCs on the TFFs (see the remarks in sect. 3.1.) In the limit where all Q_i become large simultaneously, another SDC can be derived [54, 58] for $\bar{\Pi}$, $\lim_{Q \rightarrow \infty} Q^4 \bar{\Pi}_1(Q, Q, Q) = -4/9\pi^2$. In hQCD models with correct short-distance limit of the TFFs, the coefficient on the right-hand side comes out 19% smaller [38, 43].

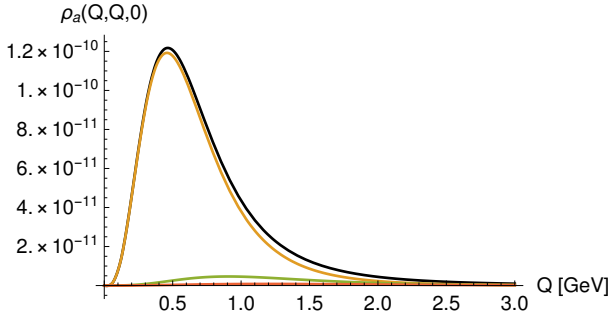


Figure 7. The integrand $\rho_a(Q_1, Q_2, \tau)$ in (11) in units of GeV^{-2} for $Q_1 = Q_2 = Q$ and $\tau = 0$ in the case of the HW2 model. The black line is the result from the infinite sum over the tower of axial vector mesons, the colored lines give the contributions of the 1st to 3rd lightest axial vector meson multiplets. (Taken from [42])

3.4 a_μ contributions

Fig. 7 shows the individual contributions of axial vector meson excitations to the integrand in

$$a_\mu^{\text{AV}} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \rho_a(Q_1, Q_2, \tau) \quad (11)$$

for $Q_1 = Q_2$ and $\tau = 0$. The ground-state axial vector meson mode clearly dominates, but the infinite sum of excited modes does contribute non-negligibly. For chiral models, this is shown quantitatively in table 2. In the Sakai-Sugimoto model, where the SDCs are not satisfied at all, excited modes contribute only a few percent. In the HW2 model, which satisfies the SDCs qualitatively, but quantitatively only at the level of 62% when f_π and m_ρ are fitted to physical values, the excited modes add +25% to the contribution of the ground-state mode ($j = 1$). This percentage increases to about 30% in the HW1 model, where the SDCs are matched exactly. Also shown in table 2 are two modified versions of the HW1 model with g_5^2 reduced by 10% and 15%, which can be viewed as taking into account next-to-leading order corrections at large but still physically relevant energy scales. Also in these models, the excited modes add around 30%, if not more, to $a_\mu^{\text{AV}}(j = 1)$.

While being completely absent in the chiral HW2 model, excited pseudoscalars, which decouple from the axial current in the chiral limit, always contribute to a_μ in the HW1 model and its variants. This has been studied in [38] in the flavor-symmetric case with and without modified scaling dimension of the bifundamental scalar and also for different boundary conditions on the hard wall. The various models show some differences in the composition of the contributions to a_μ , but hardly deviate from the sum total in the chiral model (where the physical pion mass is inserted manually in the pion propagator).

In the flavor-U(3)-symmetric case the total axial vector sector is four times the contribution of the a_1 meson. With a 10% reduction of g_5^2 and correspondingly the level of saturation of the SDCs, the massive HW models together

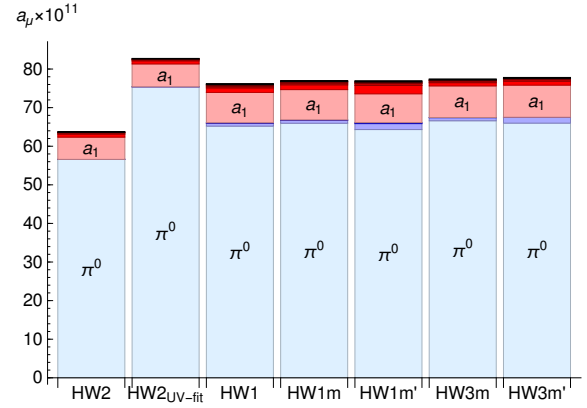


Figure 8. Bar chart of the individual contributions to a_μ from the isotriplet sector in the Hirn-Sanz model fitted to IR (HW2) and UV data (HW2_{UV-fit}), the chiral HW1 model and various extensions to the massive case, with excited modes given by increasingly darker colors, blue for the π^0 's, red for the a_1 's. (Taken from [38])

yield $a_\mu^{\text{AV}} = 4a_\mu^{a_1} = 39.5(1.6) \times 10^{-11}$, where approximately 58% are due to longitudinal contributions entering $\tilde{\Pi}_1$. The contribution of excited pseudoscalars varies somewhat more, as can be seen from fig. 8. In models where their contribution is larger, the ground-state yields less, but we have $a_\mu^{\text{PS}^*} \gtrsim 3.3 \times 10^{-11}$ (again with U(3) symmetry and with 10% reduction of g_5^2). Even without the latter contribution, the holographic result is somewhat higher than the WP estimate for the combined effect of axial vector mesons and SDCs, $a_\mu^{\text{axials}+\text{SDC(WP)}} = 21(16) \times 10^{-11}$.

3.5 Katz-Schwartz model: HW1 with solved $U(1)_A$ problem

In the above estimates obtained from flavor-U(3)-symmetric HW models, the effects of the large strange quark mass and the anomalous breaking of $U(1)_A$ symmetry has been ignored (in [42, 43] the contribution of η and η' mesons, where U(3) symmetry clearly is no good approximation, has been estimated by manually setting their masses and decay constants to physical values).

In [41] we have recently considered the minimal extension¹ of the HW1 proposed by Katz and Schwartz [60, 61] to implement the $U(1)_A$ anomaly through an additional complex scalar Y whose magnitude and phase are dual to the gluon operators $\alpha_s G_{\mu\nu}^2$ and $\alpha_s G\tilde{G}$. There a coupling term $\kappa Y^{N_f} \det(X) \subset \mathcal{L}$ in the 5-dimensional theory accounts for $U(1)_A$ anomalous Ward identities. It turns out that all results are weakly dependent on the new coupling κ when $\kappa \gg 1$. The only new free parameter is the value of the gluon condensate Ξ which can be prescribed in the choice of the background field $\langle Y \rangle = C + \Xi z^4$, while C is fixed by OPE and the $U(1)_A$ anomaly to $C \propto \alpha_s$. The latter

¹In [59] an extension of the HW1 model was considered which in contrast to the minimal models leads to nonvanishing HLBL contributions of scalar mesons. While results were obtained that are consistent with previous estimates of these contributions, the scalar TFFs obtained therein do not agree with pQCD SDC constraints [49].

Table 1. Axial vector contributions in chiral HW models with different levels of saturation of SDCs. In the chiral, flavor U(3)-symmetric case we have simply $a_\mu^{\text{AV}} = a_\mu^{a_1+f_1+f'_1} = 4a_\mu^{a_1}$. Also given are the masses m_{a_1} and the photon coupling amplitudes $A_1(0,0)$ of the ground-state axial vector mesons ($j = 1$).

chiral model	m_{a_1}	$A_1(0,0)$	SDC	$a_\mu^{\text{AV}}(j=1)$	$(j \leq 2)$	$(j \leq 3)$	$(j \leq 4)$	$(j \leq 5)$	$a_\mu^{\text{AV}}(\text{all})$
HW1	1.3755	21.04	100%	31.4	36.2	37.9	39.1	39.6	41×10^{-11}
HW1-	1.295	19.65	90%	28.7	34.3	36.0	37.1	37.5	38×10^{-11}
HW1--	1.255	18.90	85%	27.3	33.2	34.9	35.9	36.3	37×10^{-11}
HW2	1.235	16.63	62%	23.0	26.2	27.4	27.9	28.2	29×10^{-11}
SS	1.1865	15.93	0%	13.8	14.5	14.7	14.8	14.8	15×10^{-11}

is modeled by $\alpha_s \rightarrow 1/\beta_0 \ln(\Lambda_{\text{QCD}} z)$, $\Lambda_{\text{QCD}} \rightarrow z_0^{-1}$. This model implements a Witten-Veneziano mechanism for the mass of isosinglet pseudoscalars, which are augmented by a pseudoscalar glueball mode mixing with the η and η' mesons.

In [60], Katz and Schwartz have found that their model can account for the masses of η and η' mesons with deviations of about 10%. By turning on the gluon condensate parameter Ξ , we have found [41] that this can be improved to the percent level.

In the case of axial vector mesons, this model is somewhat less successful: the masses of f_1 and f'_1 are raised too much compared to their experimental values, and also the experimental values of their mixing is not reproduced. Nevertheless, this provides a first glimpse of what a more complete hQCD model can give.

The results that we have obtained with the Katz-Schwartz model augmented by a gluon condensate are given in table 2. In the isotriplet (pion and a_1) sector, the results are essentially unchanged, with the pion contribution matching perfectly the dispersive result, in particular when g_5 is fitted to match F_ρ , corresponding to a 90% level of saturation of the OPE constraints. The contributions from η and η' mesons are then also completely in line with the WP estimates. Excited pseudoscalars (including a pseudoscalar glueball mode) contribute another 1.6×10^{-11} , so that the total contribution of pseudoscalars is at the upper bound of the WP estimate. The combined contribution of ground-state axial vector mesons equals around 3.5 times the a_1 contribution, which is a moderate reduction of the factor 4 in the flavor-U(3) symmetric case. Somewhat surprisingly, the contribution of excited f_1 and f'_1 axial vector mesons is more strongly reduced, so that the “best guess” for the axials + LSDC contributions in this model turns out to be $a_\mu^{\text{AV+LSDC}} = 30.5 \times 10^{-11}$, and $a_\mu^{\text{AV+PS*+LSDC}} = 32 \times 10^{-11}$ when excited pseudoscalars are included. Both results are somewhat above but now safely within the estimated error of the WP estimate $a_\mu^{\text{axials+SDC(WP)}} = 21(16) \times 10^{-11}$.

We should like to point out, however, that there are holographic QCD models that are closer to a string-theoretic top-down construction such as the typically more involved models of Ref. [62–64]. It would be interesting to see if they can achieve a better fit of masses and mixing angles of f_1 , f'_1 mesons, and what they would predict for their contribution to a_μ .

Table 2. Summary of the results [41] for the different contributions to a_μ in the massive HW model due to Katz and Schwartz (but augmented by a nonzero gluon condensate (v1)) in comparison with the White Paper [4] values. The mass of π^0 is fixed as input, the masses of η and η' are matched within 0.8 and 2.4%, while the axial vector mesons a_1 and f_1 are between 4% and 15% too heavy, f'_1 by 28%.

$a_\mu \times 10^{11}$	v1(OPE fit)	v1(F_ρ -fit)	WP
π^0	66.1	63.4	$62.6^{+3.0}_{-2.5}$
η	19.3	17.6	16.3(1.4)
η'	16.9	14.9	14.5(1.9)
G/η''	0.2	0.2	
$\sum_{PS^*}$	1.6	1.4	
PS poles	104	97.5	93.8(4.0)
a_1	7.8	7.1	
$f_1 + f'_1$	20.0	17.9	
$\sum a_1^*$	2.2	2.4	
$\sum f_1'^*$	3.6	3.0	
AV+LSDC	33.7	30.5	21(16)
total	138	128	115(16.5)

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