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Universal Models for Heavy-Ion Fusion Cross Section Above-Barrier

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Abstract. The paper discusses a recent re-investigation of a large body of heavy-ion fusion cross section data with the aim of deriving a simple phenomenological model able to describe data from the Coulomb barrier up to the onset of nuclear multifragmentation. To this end, we adopted two complementary approaches: a first universal phenomenological model was derived exploiting a novel artificial intelligence tool for the formal modelling of large datasets. This tool is capable of advanced feature selection and is ideal to drive the discovery process even using traditional methods. A second phenomenological model was derived using a sum-of-difference approach and achieved an unprecedented accuracy in describing above-barrier fusion excitation functions data. Future perspectives and opportunities arising from the present models are also discussed in the text.

1 Introduction

During the past decades, extensive experimental campaigns were performed to measure the cross section of the fusion between heavy ions at relatively low energies [1–5]. This gave a crucial boost also to many experimental techniques conventionally adopted in nuclear physics (gamma-ray analysis, time-of-flight and magnetic spectrometers, charged particle detection with telescope arrays). Data were collected especially above the Coulomb barrier and up to the onset of multi-fragmentation phenomena.

Understanding the fusion between heavy ions at energies above-barrier is anything but trivial. A number of models, mainly semi-classical, have been developed to attempt to describe large systematics of data [6]. For light-to-medium-mass systems, i.e. $A_{tot} \simeq 20-140$, the fusion cross section, as a function of the inverse center-of-mass energy $1/E_{cm}$, exhibits peculiar trends that can be schematically described in three main regions[2, 7]. Region I starts approximately at the Coulomb barrier, and shows a quasi linear increase of the cross section for decreasing $1/E_{cm}$ values [1, 2, 7–9]. In this region, the fusion mechanisms is in competition with multi-nucleon-transfer processes [10–12] and quasi-fission (especially for heavy systems) [13–15]. Region II is instead characterized by an almost stagnant trend of the fusion cross section. This is due to the competition with deep inelastic phenomena [16, 17] and quasi-fission. Finally, region III is the highest energy region, where the fusion cross section has a sharp decrease for decreasing values $1/E_{cm}$ giving rise to incomplete fusion phenomena, [18–20], multi-fragmentation [21–23], and other complicated reaction scenarios [5]. At higher energies, the fusion cross section is negligibly small, due to mechanical and thermodynamical instabilities of the transient system formed in the collisions [22–28]. From a theoretical point of view, the fusion cross section between heavy ions has been deeply investigated. To interpret experimental data in regions II and III, where the existence of a limiting angular momentum was previously suggested [29], two main families of macroscopic models have been developed: critical distance models [7, 30], and models based on limitations to compound nucleus [31]. In addition to macroscopic models, also microscopical (as Time-Dependent Hartree-Fock, TDHF, see e.g. [32, 33]), molecular dynamics (see e.g. [34, 35]), and phenomenological models (see, e.g., [1, 27, 28, 36–38]) have been proposed. For example, Refs. [28, 36–39] report phenomenological models capable to describe complete fusion data from several collision systems in the I-II regions, while the model of Ref. [27] is capable to describe the data in the highest energy region (III). Despite the relevant experimental and theoretical efforts done so far, there are yet unanswered questions, especially regarding possible effects of nuclear structure parameters (i.e. characteristics of projectiles, targets and compound nuclei) on fusion cross sections, or the occurrence of shell effects in the fusion between light-to-medium-mass systems. To try to contribute in the field, we analyzed a very large dataset of complete fusion cross section data, with the aim to derive universal models to describe the entire dataset and to inspect the correlation of the fusion cross section on nuclear structure variables. In this paper, we describe the derivation of new computationally simple models, for the description of the fusion cross section between heavy

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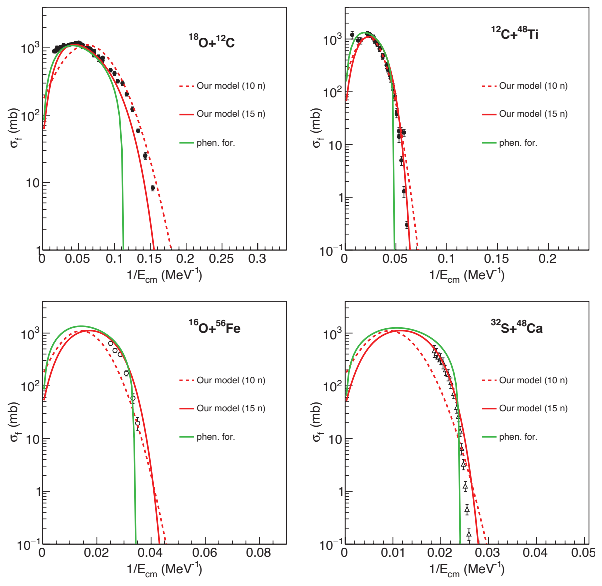

Figure 1. Results of the models of eq. (1), red dashed line, and (2), red solid line, for some of the collisions systems analyzed in this work. The results of the model from Ref. [39] is shown with the green line.

ions, with two complementary approaches: (1) the first exploits novel artificial intelligence algorithms for symbolic regression and advanced feature selection techniques; (2) the second is a modified sum-of-differences (mSOD) approach, which exploits the dataset and the variables refined with the artificial intelligence analysis.

2 Dataset and Modeling

The data used to derive the models were originally extracted from the Nuclear Reaction Video Project (NRV) database [40, 41], taking into account the results of Ref. [27]. We gathered about 4500 data points for 124 light-to-medium-mass collision systems ($Z_p \cdot Z_t < 250$), and additional data from 63 medium-mass systems ($Z_p \cdot Z_t > 250$). We considered systems with $Z_{p,t} \geq 6$, and with $A_{tot} \leq 130$, thus excluding from the dataset too heavy systems, for which fusion-fission and quasi-fission can be the dominant reaction modes, and also too light systems, where the presence of break-up and transfer reactions complicates the analysis. In addition, we included in the dataset only fusion data at incident energies from approximately 1.2 times the Coulomb barrier [2].

2.1 Modeling with advanced artificial intelligence tools

The models derived using artificial intelligence techniques exploited a novel hybridization of genetic programming and artificial neural networks for the formal modelling of data called Brain Project (BP) [42, 43]. BP was previously used in a number of investigations involving different research fields (see, e.g., [44–47]) and adopts a well-known programming scheme inspired by the natural selection in biological systems [48, 49]. In the scheme implemented by BP, analytical models for the description of a set of experimental data points form a *population* of individuals. Each individual is associated with a *fitness* value, which measures the degree to which an individual is a good solution for the modelling problem to solve. At each iteration, new individuals are generated by a suitable crossover of other existing individuals, and the new individuals are inserted into the population replacing previous individuals. Individual selection and replacement criteria usually account for the individual fitness, simulating natural selection rules. In a similar scheme, the average fitness of the population is typically improved at each iteration. In the end, after a suitably large number of iterations, the best individual of the population is taken to represent the model to reproduce the experimental data.

A crucial point of BP is the *feature selection*, i.e. the automatic selection of the variables required to describe the data, neglecting poorly relevant variables. This is typically a complicated task with traditional statistical analyses, especially in the presence of a large number of variables. In the present application, in order to inspect possible dependencies of the fusion cross section on nuclear structure variables, we used 31 variables, described in Ref. [50], to represent colliding nuclei, their structure, and the collision energy of each given data point to model. Following the procedure discussed in detail in Ref. [50], we derived two models for the description of the fusion cross section between heavy ions, with different trade-offs between accuracy and complexity:

$$\sigma_{fus}^{(simple)}(E_{cm}) = 1103 \cdot \exp\left[-\left(1.387 - 0.468 \cdot \frac{Z_2 \cdot Z_1}{E_{cm}}\right)^2\right] \tag{1}$$

$$\sigma_{fus}^{(complex)}(E_{cm}) = 1116 \cdot \\ \cdot \exp\left[-\sinh^2\left(-1.359 + \mathrm{erf}\left(\frac{S_{2n}}{E_{cm}}\right) + 0.061 \cdot \frac{A_1 \cdot A_2}{E_{cm}}\right)\right] \tag{2}$$

The simplest model, eq. (1), is found to exploit exclusively 3 variables, namely the center-of-mass energy, and the charges of projectile and target. This is a very important fact, because it indicates that, given the presently published experimental data, E_{cm}, Z_1, and Z_2 suitably contain the whole informative content of the entire set of 31 variables required for their description. The most complex model, eq. (2), foresees 4 variables, namely E_{cm}, A_1, A_2, and the two-neutron separation energy of the compound system S_{2n}. In this case, a fourth variable is needed to slightly enhance the accuracy. It is found that further enhancing the complexity of the model does not help to further improve the accuracy in a statistically significant way.

The newly derived models help to improve the description of the entire set of fusion cross section data compared to former phenomenological models. Fig. 1 shows the results of our models (red lines) against some typical data, alongside the results of the phenomenological model from Ref. [39] (green line). Compared to the previously published model, our new models crucially improve the de-

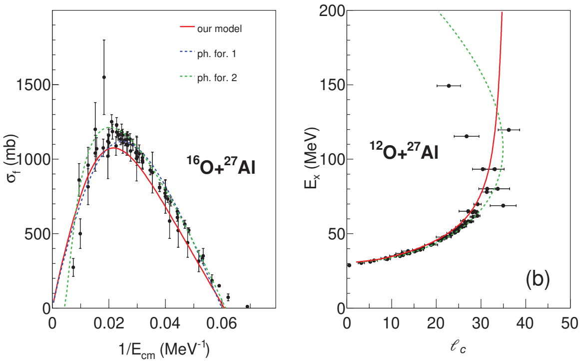

Figure 2. Phenomenological model descriptions of the ^{16}O+^{27}Al fusion cross section and critical angular momentum (red lines). For comparisons, analogous results obtained with models of Refs. [39] (blue) and [38] (green) are reported.

scription of the data in the vicinity of the Coulomb barrier. Close to the maximum of the cross section, our models achieve a reasonable description of data, comparably to the results of Ref. [39].

2.2 The improved SOD model

The artificial intelligence analysis described in the previous section helps to suggest the relevant variables required to describe the complete systematics of fusion cross section data, and to improve the dataset itself by identifying mislabelled data. Guided by these suggestions, and exploiting the improved dataset, we derived a phenomenological model starting from a sum-of-difference (SOD) approach.

In the SOD approach, the fusion cross section can be simplified as $\sigma_f \approx 2\pi \int_{\theta_f}^{\pi} \sigma_R(\theta) sin\theta d\theta$, which gives:

$$\sigma_f = \pi \left(\frac{\eta}{k}\right)^2 \cot^2 \frac{\theta_f}{2} = \frac{\pi}{k^2}(kD)^2 \left(1 - \frac{2\eta}{kD}\right) \quad (3)$$

where η is the Sommerfeld parameter, k the wave number, and D the distance of closest approach of the Coulomb trajectory linked to θ_f. By factorizing D as $D = d(\sqrt[3]{A_1} + \sqrt[3]{A_2})$, d being dependent on the energy and other characteristics of the collision, as discussed in Ref. [51], and introducing the quantity $F(\varepsilon) = \frac{kd}{A_{tot}}$, $\varepsilon \equiv \frac{E}{A_{tot}}$, one obtains:

$$\sigma_f = \frac{C_1}{E} F(\varepsilon) \left[C_2 F(\varepsilon) - \frac{C_3}{\sqrt{E}} \right] \quad (4)$$

where the constants C_1, C_2, C_3 depends only on the mass and atomic numbers of the reagents, as discussed in [50]. By inverting eq. (4), one finds an expression for kd as a function of σ_f. This allowed to build $(kd)_{exp}$ values for all data points in the dataset. The modeling procedure, described in detail in Ref. [51], consisted thus in finding the proper expression of kd to describe $(kd)_{exp}$ values for the entire dataset. We find the following expression for $\frac{kd}{A_{tot}}$:

$$\frac{kd}{A_{tot}} = F(\varepsilon, \Delta, n) = a \cdot f_A(\Delta, n) \cdot g_v(\varepsilon) \cdot h_C(\varepsilon) \quad (5)$$

where $f_A = 1 - \Delta^3 \cdot n$ is a factor related to the neutron richness of the total system formed by fusion, being $n \equiv N_{fin}/N_{tot}$, and $N_{fin} = 1.072 \cdot Z + 2.32 \cdot 10^{-3} Z^2$ [52]; $g_v(\varepsilon) = 1 - e^{-b\varepsilon}$ is a term related to friction-like forces occurring during the collision ($b = 2.26 \pm 0.01 MeV^{-1}$ [39]); and $h_C(\varepsilon) = 1 + \kappa_1 e^{-\kappa_2 \varepsilon}$ is used to reproduce the trend of data in the proximity of the Coulomb barrier. $\kappa_1 = 4.22 \pm 0.04$ and $\kappa_2 = 9.40 \pm 0.04$ are free parameters, whose value is determined from a fit to the experimental data.

The improved SOD model described in the present section achieves an unprecedented accuracy in describing previously published fusion cross section data. Fig. 2 (left panel) shows the results of the model (red line) compared to the experimental points for one of the collisions systems, alongside the results of the phenomenological models from Refs. [38, 39]. An interesting feature of the newly derived model is the presence of limiting values of the critical angular momentum ℓ_c, as visible in the right panel of Fig. 2, where we show the excitation energy versus angular momentum as derived from data in the *sharp cut-off* approximation [2, 53] (black dots) and from our model (red lines). This is a feature not commonly shown by phenomenological models (see, e.g. the dashed green line, obtained with the model of Ref. [38]). The present model leads to a saturation of ℓ_c values that is, at least for not too heavy systems ($A_{tot} < 90$), in reasonable agreement with the trend of experimental data and also with the prediction of the liquid drop model [29].

3 Conclusions and Perspectives

In this manuscript, we describe recent efforts to model the fusion cross section between heavy ions from the Coulomb barrier to the onset of multifragmentation phenomena.

Universal data-driven models, with different trade-offs between complexity and accuracy, have been derived using a novel hybridization of genetic programming and artificial neural networks, with advanced feature selection capabilities. The underlying analysis shows that a few variables allow to describe the entire dataset with analytically simple formulas.

A more refined model is derived with a SOD approach, exploiting the entire dataset. The new SOD model is capable to improve the state-of-the-art and shows saturation of the critical angular momentum in agreement with the liquid drop model.

The present analysis evidences the need for new accurate experiments in the regions II and III of the fusion cross section. These studies could benefit from the availability of modern arrays for charged particles being developed in the past decades [55–62].

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