Core-valence absorption in breakup and stripping reactions and its isospin dependence

Mario Gómez-Ramos\(^1\), Joaquín Gómez-Camacho\(^1,2\), and Antonio Matías Moro Muñoz\(^1,3\).

\(^1\)Departamento de Física Atómica, Molecular y Nuclear, Facultad de Física, Universidad de Sevilla, Apartado 1065, E-41080 Sevilla, Spain
\(^2\)Centro Nacional de Aceleradores (U. Sevilla, J. Andalucía, CSIC), Tomás Alva Edison, 7, 41092 Sevilla, Spain
\(^3\)Instituto Interuniversitario Carlos I de Física Teórica y Computacional (iC1), Apdo. 1065, E-41080 Sevilla, Spain

Abstract. In this work, the effect of nucleon-core absorption on nucleon removal reaction is explored through the use of complex nucleon-core interactions. Results are presented for exclusive breakup reactions, where absorption is explored through the use of a binormal basis in the continuum-discretized coupled-channel method, and for stripping nucleon knockout reactions, where absorption is considered through the application of an effective density in the eikonal approximation. Both methods show an increased effect of nucleon-core absorption when removing a deeply-bound nucleon, which leads to smaller cross sections, a reduction that is larger than in the weakly-bound case.

1 Introduction

Nucleon-removal reactions with medium-mass targets (\(^{9}\)Be, \(^{12}\)C) have a long and successful history in the study of the single-particle properties of nuclei [1, 2]. In this kind of reactions, a projectile \(P\) is made to collide with a target \(T\) so that a nucleon \(N\) is removed from the projectile, and a residual core \(C\) is detected. If the nucleon \(N\) and the target \(T\) (in its ground state) are detected, \(P(N+C)+T\rightarrow N+C+T\), the reaction corresponds to elastic (sometimes called diffractive) breakup, while the reaction process is called stripping if the nucleon is absorbed by the target and only the core is detected \(P(N+C)+T\rightarrow C+X\). Elastic breakup is the main nucleon-removal process for the removal of halo and weakly-bound nucleons, while for more deeply-bound nucleons the importance of stripping increases with the binding energy of the removed nucleon, being the dominant contribution for removal from stable nuclei and for deeply-bound nucleons.

For the study of elastic breakup for weakly-bound nuclei, the state-of-the-art method is the Continuum-Discretized Coupled Channels method (CDCC) [3] while stripping reactions, which are usually measured at an energy of \(\sim 100\) MeV per nucleon, are usually analyzed using an eikonal sudden description [4]. Both methods, in their standard form, require a real interaction between the nucleon and the core in the final state, in order to ensure the properties of orthogonality and closure for the nucleon-core eigenstates. However, the final-state-interaction between removed nucleon and core can lead to excitation and breakup of the latter, leading to a loss of flux (and therefore cross section) that is usually described by an imaginary part in the interaction, so an extension in these methods to consider complex interactions is required to describe this processes.

In this work we describe briefly two such extensions for the CDCC and eikonal methods and present results for cross sections corresponding to the removal of weakly- and deeply-bound nucleons.

2 Elastic breakup

2.1 Theoretical framework: CDCC

In the CDCC method, the full three-body wavefunction for the \(C+N+T\) system \(\Psi(\mathbf{r})\) is expanded in eigenstates of the \(C+N\) Hamiltonian \(\phi_j\), with a discretization procedure (usually a binning procedure) to express the infinite continuum eigenstates in a finite discrete basis [3].

\[
\Psi(\mathbf{R}, \mathbf{r}) = \sum_j \chi_j(\mathbf{R})\phi_j(\mathbf{r}),
\]

(1)

where \(\mathbf{R}\) is the relative coordinate between projectile and target, \(\mathbf{r}\) is the one between nucleon and core and \(\chi_j(\mathbf{R})\) is the coefficient of state \(\phi_j(\mathbf{r})\) in \(\Psi(\mathbf{R}, \mathbf{r})\). Provided the eigenstates \(\phi_j\) are orthogonal, one can obtain from the Schrödinger equation a set of coupled equations for \(\chi_j(\mathbf{R})\),

\[
\sum_j \left((T_R - E_j)\langle \phi_i|\phi_j \rangle + \langle \phi_i|U_{CT} + U_{NT}\phi_j \rangle\right)\chi_j(\mathbf{R}) = 0,
\]

(2)

where \(U_{ij} = \langle \phi_i|U_{CT} + U_{NT}\phi_j \rangle\) is the coupling potential and \(U_{CT} , U_{NT}\) are the core-target and nucleon-target
interactions respectively. This method relies on the orthogonality of states $\phi_i$, which is no longer fulfilled when the nucleon-core interaction is complex and energy-dependent, as required to describe nucleon-core absorption. This can fortunately be solved by the introduction of the binormal basis $\tilde{\phi}_j$, which, for a set of non-orthogonal states $\varphi_i$, verifies $\langle \tilde{\phi}_i | \varphi_j \rangle = \delta_{ij}$ and so for a finite basis can be computed analytically [5]. The use of the binormal basis leads to a set of coupled equations analogous to the standard CDCC which can be solved through standard methods, where the only change is that the coupling potential is substituted by $\tilde{U}_{ij} = \langle \tilde{\phi}_i | U_{CT} + U_{NT} | \varphi_j \rangle$.

$$
\sum_j \left( (T_R - E_i) \langle \tilde{\phi}_i | \varphi_j \rangle + \langle \tilde{\phi}_i | U_{CT} + U_{NT} | \varphi_j \rangle \right) \chi_j(R) = 0
$$

It should be noted that the introduction of complex potentials doubles the basis of eigenstates, as states with outgoing $\varphi_i^{(+)}$ and incoming $\varphi_i^{(-)}$ boundary conditions are no longer equivalent. For the description of knockout reactions the states with incoming boundary conditions $\varphi_i^{(-)}$ is the adequate basis, which may come as a surprise, as they have larger-than-one norms for absorptive potentials. However, we note that for weakly-energy-dependent potentials one can verify that $\tilde{\phi}_i^{(-)} \cong \varphi_i^{(+)}$ [6], as is shown in Fig. 1 for a reference bin, so the norm for $\tilde{\phi}_i^{(-)}$ is smaller than unity and the DWBA matrix element $\tilde{U}_{i0} = \langle \tilde{\phi}_i^{(-)} | U_{CT} + U_{NT} | \varphi_0 \rangle$ between bound and continuum states leads to smaller cross sections, as expected of absorptive processes.

![Figure 1](image1.png)

**Figure 1.** Real part for a bin computed for an absorptive potential. The red and green curves correspond to bins with incoming and outgoing boundary conditions respectively. The blue curve corresponds to the conjugate of the binormal of the state with incoming boundary conditions.

### 2.2 Results

We have applied this formalism to the neutron removal reaction with a $^{12}$C target at 70 MeV per nucleon for the nuclei $^{11}$Be (weakly-bound nucleon) and $^{41}$Ca (deeply-bound nucleon). For the interaction between neutron and core, the Köning-Delaroche[7] interaction was used for $^{41}$Ca, while for $^{11}$Be the real part was taken from [8] and the imaginary part was adjusted to reproduce the systematics from [9]. Further details on the interactions and calculations can be found in [10]. The results of the calculations are presented in Fig. 2.

![Figure 2](image2.png)

**Figure 2.** Energy distribution for the elastic breakup of a neutron for $^{11}$Be (left panel) and $^{41}$Ca (right panel) at 70 MeV per nucleon. The black curve corresponds to standard CDCC calculations while the red curve corresponds to CDCC extended to include neutron-core absorption. For $^{11}$Be, experimental data from [11] are presented.

As can be seen in the figure, the reduction due to the absorption is moderate in the weakly-bound $^{11}$Be case, reducing the overall cross section by a factor of ~10% and affecting mostly the contribution of the $5/2^+$ resonance while for the more deeply-bound $^{41}$Ca nucleus the reduction is much more significant, of ~50%. It should be noted that in [12] it was found that the cross section for elastic neutron breakup in $^{11}$Be has a significant contribution associated to the excitation of the $^{10}$Be core, which is not considered in this calculation. Therefore it is not surprising that the calculations do not describe well the experimental data, particularly at higher excitation energies. The larger reduction in $^{41}$Ca can be understood as the energies explored when removing the more deeply-bound neutron are larger, as seen in Fig. 2, so the imaginary potential is larger at these higher energies. Also, for the weakly-bound neutron the number of open channels is smaller, particularly at low energies, so the loss of flux, and hence the imaginary potential, is smaller.

### 3 Stripping reactions

#### 3.1 Theoretical framework: Eikonal model

For the description of nucleon removal reactions at intermediate beam energies the standard description is an eikonal one [4], where the stripping part of the cross section can be computed through a simple formula
where \( S_{NT} \) and \( S_{CT} \) are the nucleon-target and core-target \( S \)-matrices respectively and \( b_{NT} \) and \( b_{VT} \) the corresponding impact parameters, with \( b_{NC} \) and \( b \) being the nucleon-core and projectile-target impact parameters. \( \rho(b_{NC}) \) is a density function which in the standard eikonal model can be obtained as \( \rho(b_{NC}) = |\phi_{0}(b_{NC})|^{2} \), where \( \phi_{0} \) is the bound state nucleon-core wavefunction. This approximation of the density results from the application of closure over the continuum eigenstates of the \( V_{NC} \) interaction, which is no longer applicable when considering absorptive complex interactions between core and nucleon. Therefore, it is necessary to modify the formulas to include these effects. This modification results in a non-local density, which can be approximated to a local form by a judicious choice of the average local values of the coordinates:

\[
\rho(r_1, r_2) = \phi^{(+)}_{0}(r_1)\phi^{(+)}(r_2) \int dk e_{NC}^{(+)}(k, r_1)\phi_{NC}^{(+)}(k, r_2)
\]

(5)

\[
\rho^{\text{eff}}(x,y) = \int dr_{1} dr_{2} \delta(x - \frac{x_1 + x_2}{2})\delta(y - \sqrt{\frac{y_{1}^{2} + y_{2}^{2}}{2}})\rho(r_1, r_2),
\]

(6)

where \( \phi^{(+)}_{NC} \) are the continuum eigenstates of the complex \( U_{NC} \) interaction, \( x \) is the projection of \( b_{NC} \) in the direction of \( b \) and \( y \) the perpendicular component of \( b_{NC} \). This effective density \( \rho^{\text{eff}} \) can be used in Eq. (4) to obtain stripping cross sections with the effects of nucleon-core absorption. More information can be found in [13].

\[\sigma_{\text{str}} = \int db \int db_{NC} \rho(b_{NC}) S_{CT}(b_{CT})^{2} (1 - |S_{NT}(b_{NT})|^{2}),\]

(4)

3.2 Results

As an application of this formalism, we have considered a number of the reactions presented in [14], namely neutron and proton removal from \(^{40}\text{Si}, ^{12}\text{C} \) and \(^{24}\text{Si} \), where the \( S \)-matrices and real bound-state potential have been taken from the original references [1, 15, 16] and the absorptive interaction has been modelled as the imaginary part of the Morillon interaction. The black curve corresponds to densities using the imaginary part of the Morillon potential.

The larger absorption for more deeply-bound nucleons yields smaller theoretical cross sections, which would result in larger spectroscopic factors when comparing to experimental results. This would soften the dependence on \( \Delta S \) of the so-called “quenching factors”, the ratio between the theoretical and experimental cross sections \( R_{x} = \sigma_{\exp}/\sigma_{\text{theor}} \). This dependence was found to be rather steep in nucleon knockout reactions with \(^{8}\text{Be} \) and \(^{12}\text{C} \) targets [1, 14] but later analyses found the dependence to be flatter for transfer [18] and (\( \rho, 2p \)) [19] reactions, so the inclusion of this effect would help reduce this discrepancy. However, first tests show an overestimation of absorption...
in the weakly-bound cases, which results in unphysical \( R_s > 1 \) values for the “quenching factors” for weakly-bound-nucleon removal. The reason for this overestimation could be related to the fact that the potential by Morillon (as is usual for global potentials) presents a finite imaginary part even for weakly-bound nucleons at low energies, where the lack of open channels should lead to no absorption. This imaginary part is meant to describe compound-nucleus processes which are not well described in optical model calculations but that ultimately lead to the original elastic channel and thus do not result in absorption and therefore should not be considered in the present calculations. Methods to estimate these contributions to remove them from the calculations would help reduce this problem at low binding energies and are currently in consideration [13].

4 Acknowledgements

The authors are grateful to A. Di Pietro for her help and insights on compound-nucleus calculations. This work has been partially funded by MCIN/AEI/10.13039/501100011033 under I+D+i project No. PID2020-114687GB-I00 and under grant IJC2020-043878-I (also funded by “European Union NextGenerationEU/PRTR”), by the Consejería de Economía, Conocimiento, Empresas y Universidad, Junta de Andalucía (Spain) and “ERDF-A Way of Making Europe” under PAIDI 2020 project No. P20_01247, and by the European Social Fund and Junta de Andalucía (PAIDI 2020) under grant number DOC-01006.

References